How to Achieve Society's Goals: The Mechanism Design Solution

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Outline

Motivation

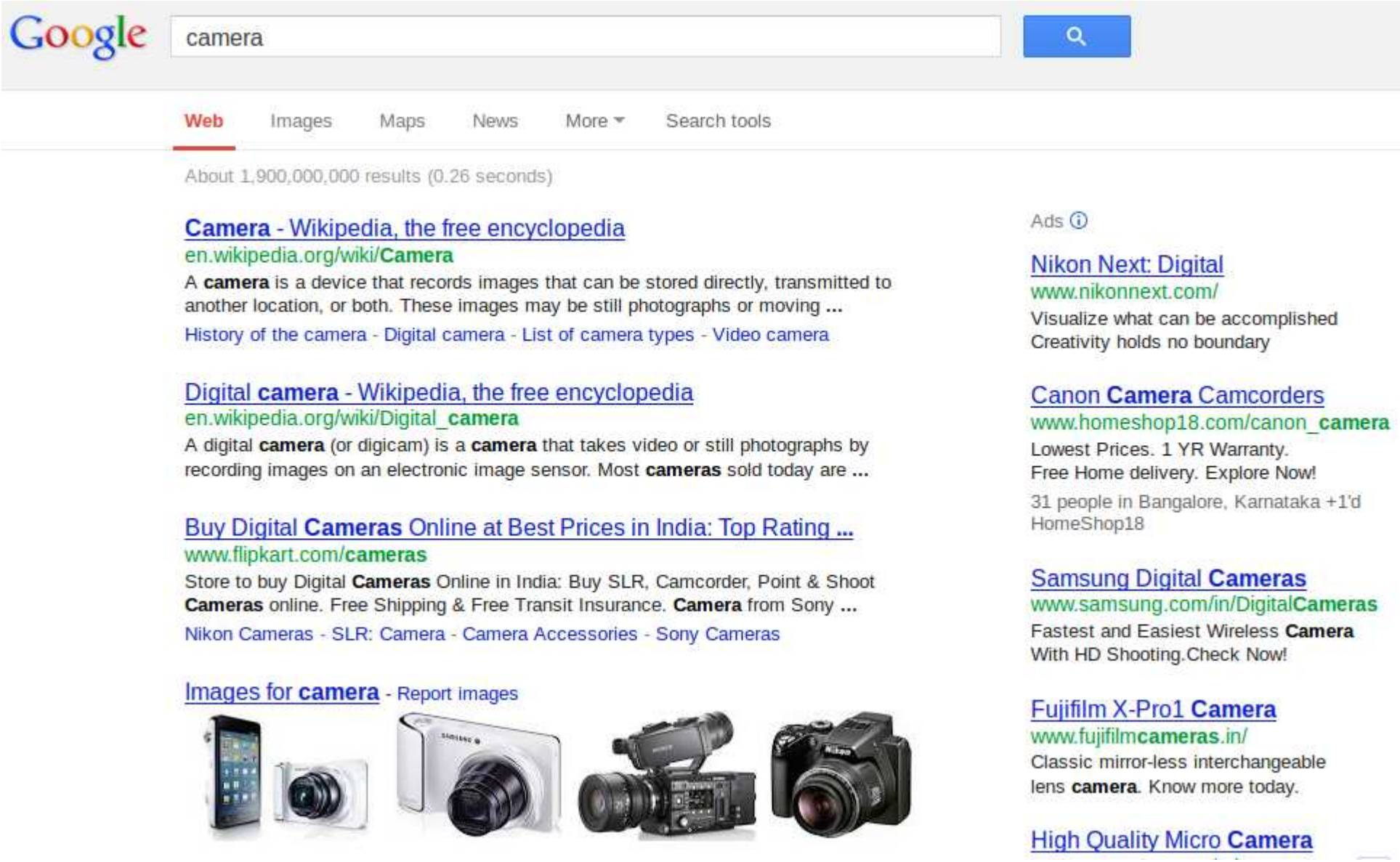
Game Theory Review

Mechanism Design

References

Sponsored Search Auction

Ever wondered how Google makes money?

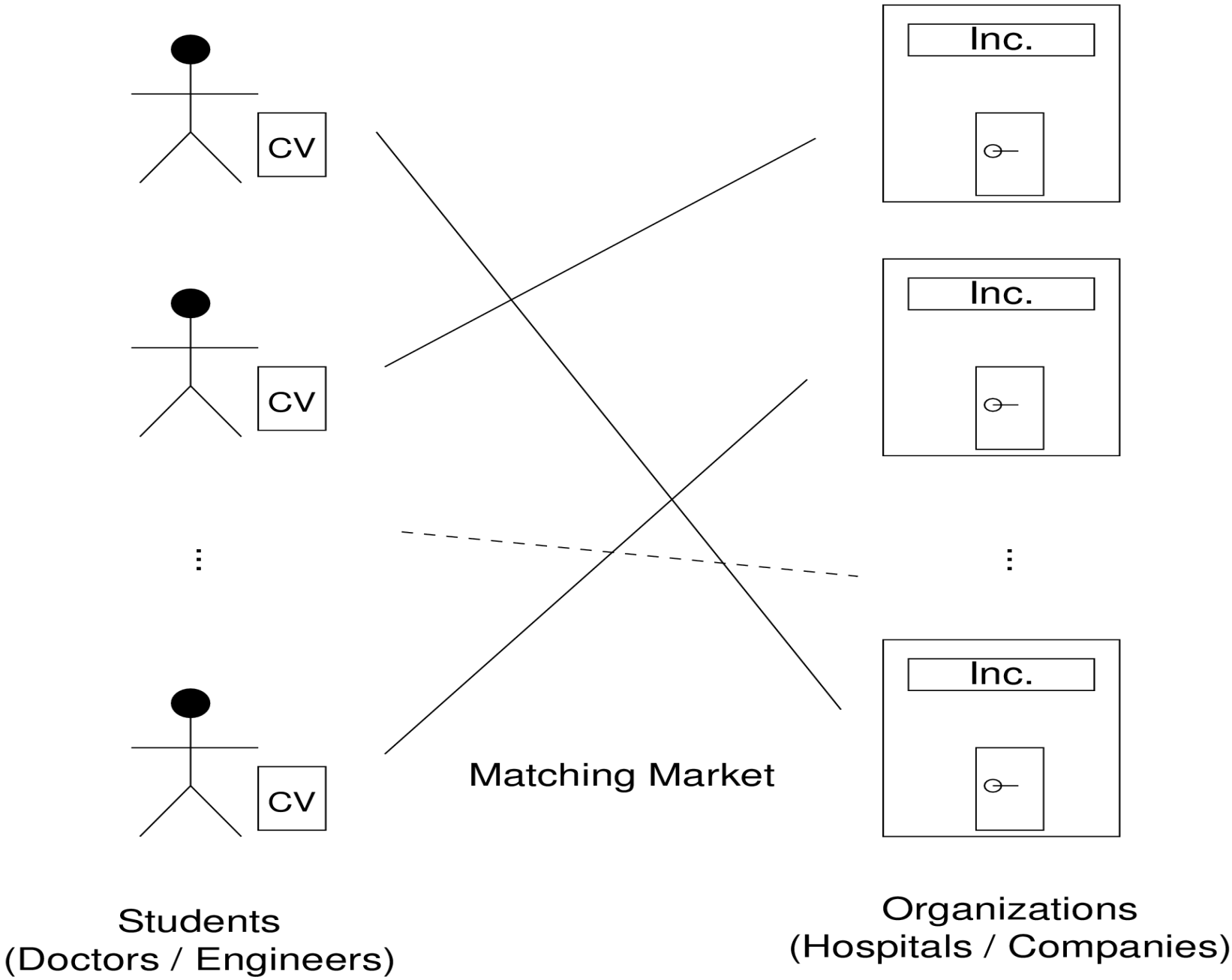


Sponsored Search Auction (Contd.)

Google asks for a sealed bid from the advertisers

- Run an auction on those bids
- The auction is *Generalized Second Price Auction*
- This mechanism is efficient for a single slot
  - ➔ Slot goes to the bidder who values it most
- It is also truthful
- Bidders participate voluntarily in this auction

Stable Matching



Stable Matching (Contd.)

- Each player has a order of preferences among the alternatives on the other side of the market
- Goal: finding a stable match
- Stable match: no agent can improve their current match
- A stable match always exists (Gale – Shapley 1962)

Stable Matching (Contd.)

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Nobel Prize in Economics, 2012

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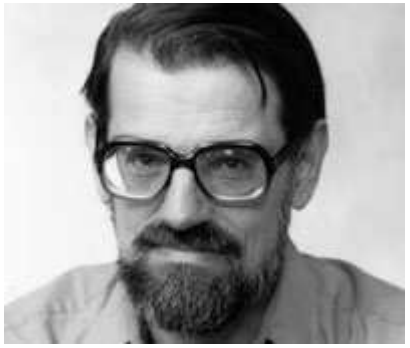


Lloyd S. Shapley

"for the theory of stable allocations and the practice of market design"

Alvin E. Roth

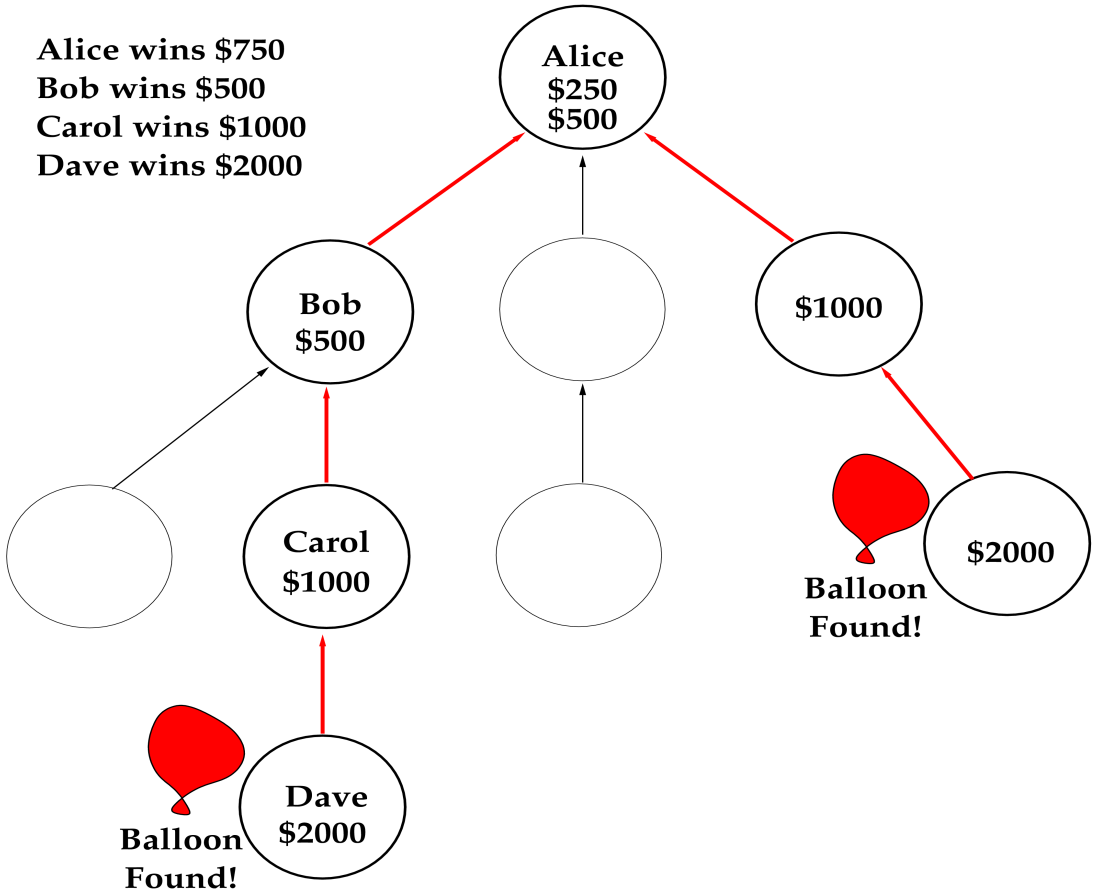
DARPA Red Balloon Challenge, 2009

Reward:
$40,000 for locating all 10 balloons

DARPA Network Challenge Project Report. In http://archive.darpa.mil/networkchallenge/, 2010.

MIT winning team's strategy

- The team crowdsource the information about the balloon
- Reward the chain that finds the balloon
- The payment scheme is geometric



G. Pickard,W. Pan, I. Rahwan, M. Cebrian, R. Crane, A. Madan, and A. Pentland.
Time-critical Social Mobilization. Science, 334:509–512, 2011.

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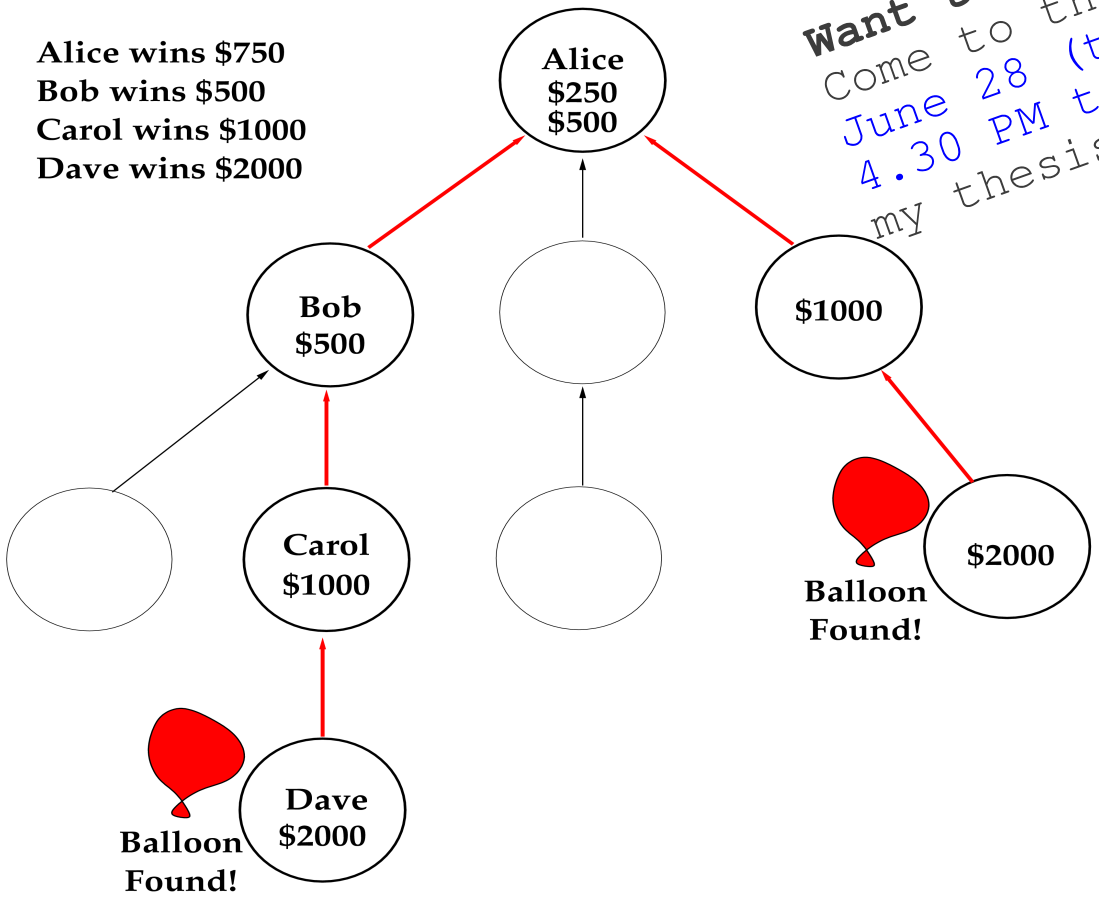


Want to know more?
Come to the talk on June 28 (this Fri) at 4.30 PM to CSA 252 for my thesis colloquium

G. Pickard,W. Pan, I. Rahwan, M. Cebrian, R. Crane, A. Madan, and A. Pentland. Time-critical Social Mobilization. Science, 334:509–512, 2011.

Reviewing Game Theory

Tools from Microeconomics

Game Theory

Mathematical study of conflict and cooperation among rational and intelligent agents.

- *Rational* agents maximize their (expected) utilities
- *Intelligent* players make optimal moves given a game

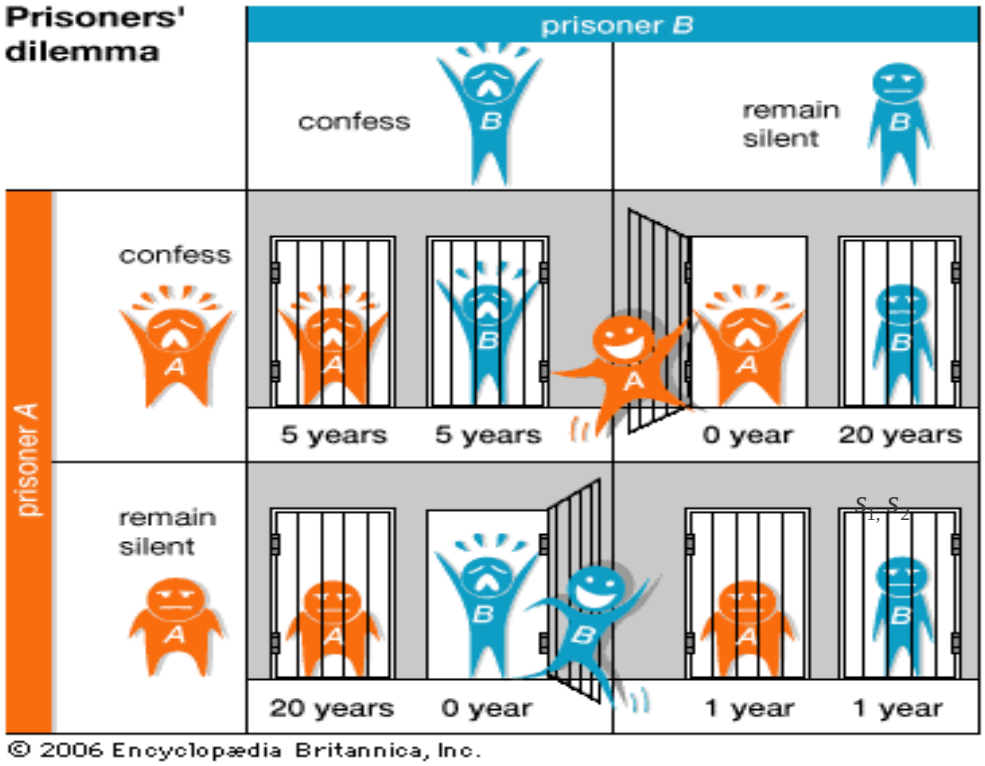
➔ This helps in understanding the moves of an institution
➔ Predictive approach

Mechanism Design

"Engineering" approach to Economic Theory

➔ Start with a goal or social objective
➔ Design institutions (mechanisms) to achieve these goals
➔ Prescriptive approach

The Prisoner's Dilemma Game



Dominant Strategy:
Player's payoff is always at least as high as any other strategy *irrespective* of what other player(s) play

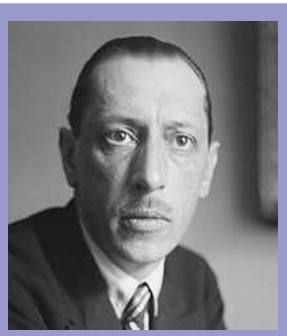
A strategy profile (s_1, s_2) is Dominant Strategy Equilibrium, if both s_1 and s_2 are Dominant

	Confess	Remain Silent
Confess	-5 , -5	0 , -20
Remain Silent	-20 , 0	-1 , -1

Neighboring Country's Dilemma

	Tension, Tension	Capture, Devastation
	Devastation, Capture	Prosper, Prosper

Bach or Stravinsky Game

		
	2,1	0,0
	0,0	1,2

Matching Pennies Game

	1,-1	-1,1
	-1,1	1,-1

Mechanism Design

Example 1: Fair Division

Mother
Social Planner
Mechanism Designer

Kid 1
Rational and
Intelligent

Kid 2
Rational and
Intelligent

Example 1: Fair Division

Mother
Social Planner
Mechanism Designer

Kid 1
Rational and
Intelligent

Kid 2
Rational and
Intelligent

Question: how to divide the cake so that each kid is happy with his portion?

Fair Division Problem (Contd.)

Kid 1 thinks he got at least half
Kid 2 thinks he got at least half

This is called a *fair division*

Notions of fairness is subjective

If the mother knows that the kids see the division the same way as she does, the solution is simple

She can divide it and give to the children

Fair Division Problem (Contd.)

What if Kid 1 has a different notion of equality than that of the mother

Mother thinks she has divided it equally
Kid 1 thinks his piece is smaller than Kid 2's

Difficulty:
Mother wants to achieve a fair division
But does not have enough information to do this on her own
Does not know which division is fair

Question:
Can she design a mechanism under the incomplete knowledge that achieves fair division?

Fair Division Problem (Contd.)

Solution:

Ask Kid 1 to divide the cake into two pieces
Ask Kid 2 to pick his piece

Why does this work?

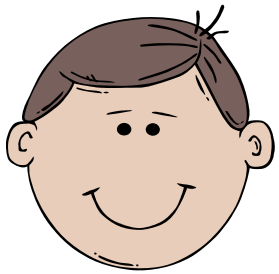
- Kid 1 will divide it into two pieces which are equal in his eyes
  - Because if he does not, Kid 2 will pick the bigger piece
  - So, he is indifferent among the pieces
  - HAPPY

- Kid 2 will pick the piece that is bigger in his eyes
  - HAPPY

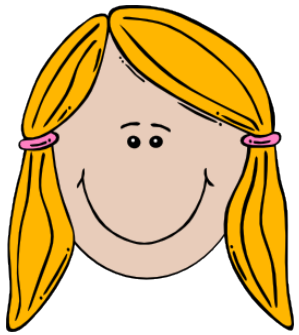
Example 2: Voting

Four candidates compete in a vote

Alice



Bob



Carol

Dave

Voting (Contd.)

Four candidates compete in a vote

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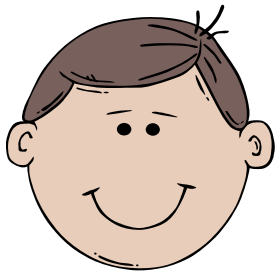
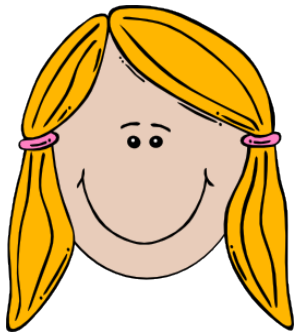


Bob



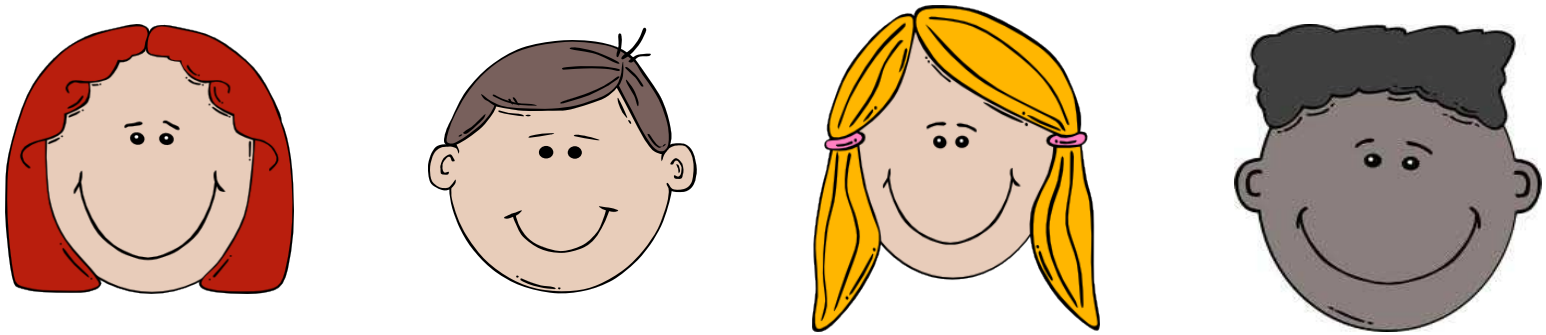
Carol

Dave

7 Voters

Voting (Contd.)

Four candidates compete in a vote



Alice Bob Carol Dave

7 Voters

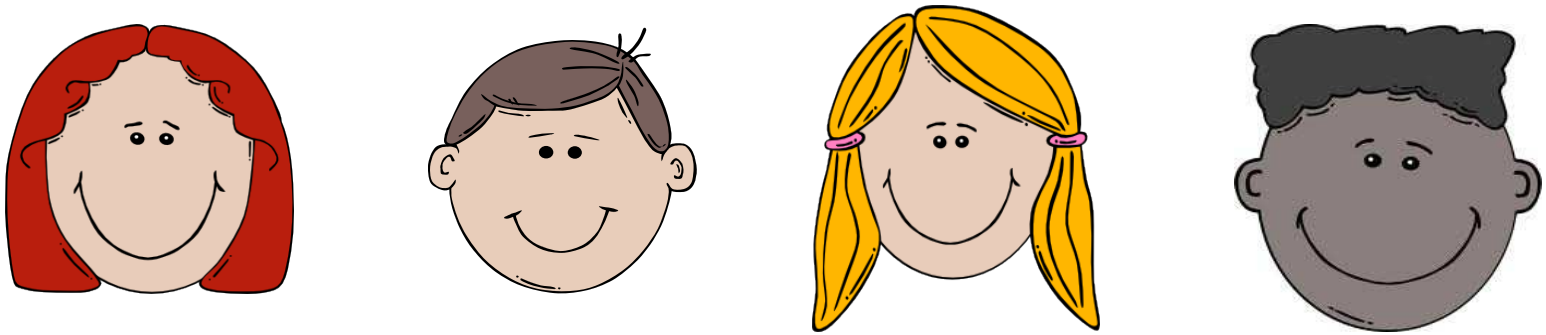
3 Voters
A > D > B > C

2 Voters
B > A > C > D

2 Voters
C > D > B > A

Voting (Contd.)

Four candidates compete in a vote



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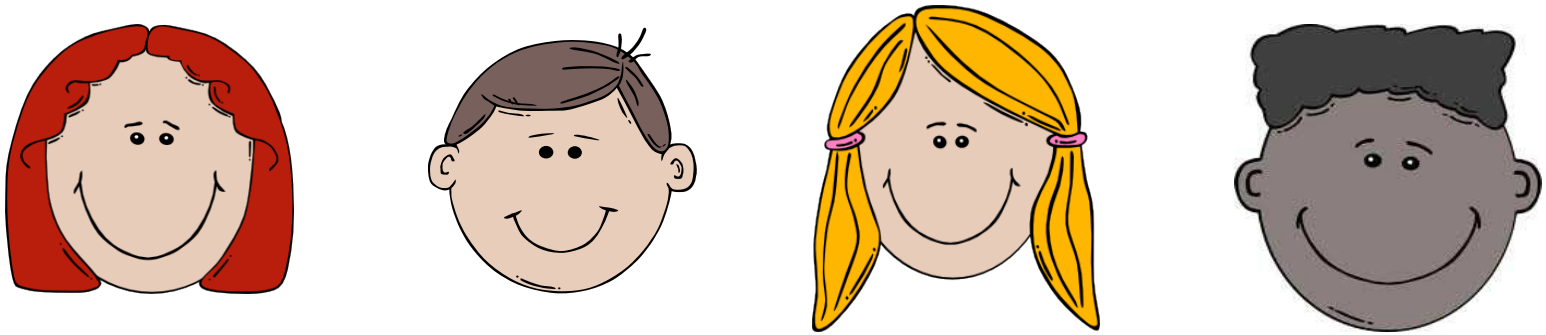
Who should win?

Voting (Contd.)

Four candidates compete in a vote



Alice Bob Carol Dave

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Alice (plurality rule!)

Voting (Contd.)

3 Voters:	A > D > B > C
2 Voters:	B > A > C > D
2 Voters:	C > D > B > A

- Give each of the voters a ballot
- Ask to pick one candidate
- Run the Plurality Rule

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- Alice wins!

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- Run the Plurality Rule
- Alice wins!
- But voters are strategic
- Notice the preferences of the last 2 voters
- They prefer B over A

Voting (Contd.)

3 Voters:	A > D > B > C
2 Voters:	B > A > C > D
2 Voters:	B > C > D > A

- Give each of the voters a ballot
- Ask to pick one candidate
- Run the Plurality Rule
- Alice wins!
- But voters are strategic
- Notice the preferences of the last 2 voters
- They prefer B over A
- Can manipulate to make Bob the winner

Maybe the voting rule is flawed?

Voting (Contd.)

3 Voters:	A > D > B > C
2 Voters:	B > A > C > D
2 Voters:	C > D > B > A

- How about a different voting rule
- Ask the voters to submit the whole preference profile
- Give scores to the ranks:
  - n-1 for top, n-2 for the next, … , 0 to the last
  - Here n = 4

Voting (Contd.)

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- Borda voting (1770)

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- Borda voting (1770)
- A = 13, B = 11, C = 8, D = 10
- Alice wins!

Voting (Contd.)

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Is it manipulable?

Voting (Contd.)

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2 Voters:	B > A > C > D
2 Voters:	C > D > B > A

- How about a different voting rule
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  - Here n = 4
- Borda voting (1770)

Yes

Voting (Contd.)

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2 Voters:	B > A > C > D
2 Voters:	B > C > D > A

- How about a different voting rule
- Ask the voters to submit the whole preference profile
- Give scores to the ranks:
  - n-1 for top, n-2 for the next, … , 0 to the last
  - Here n = 4
- Borda voting (1770)
- A = 13, B = 15, C = 6, D = 8
- Bob wins!

Voting (Contd.)

3 Voters:	A > D > B > C
2 Voters:	B > A > C > D
2 Voters:	C > D > B > A

Question: Can we design any truthful voting scheme that is socially optimal?

Voting (Contd.)

3 Voters:	A > D > B > C
2 Voters:	B > A > C > D
2 Voters:	C > D > B > A

Question: Can we design any truthful voting scheme that is socially optimal?

Answer: No (unfortunately)!

Allan Gibbard

Mark Satterthwaite

Gibbard (1973) – Satterthwaite (1975) Theorem

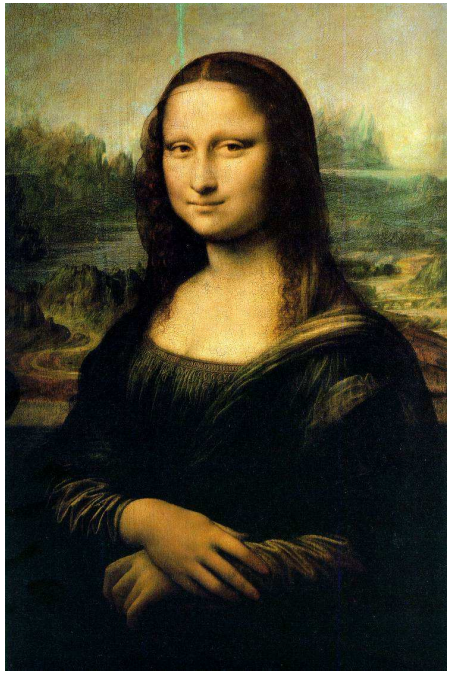
With unrestricted preferences and three or more distinct alternatives, no rank order voting system can be unanimous, truthful, and non-dictatorial

Example 3: Auction

Two art collectors bidding for a painting

Player 1

Metropolitan Museum of Art



Player 2

Musée du Louvre

Auction (Contd.)

Goal of the auctioneer:

- To allocate the painting to the agent who values it the most
- But does not know how much each agent values it
- Solving an optimization problem with private information

The auctioneer can ask the agents to bid for the painting

Question: what mechanism should be implemented to achieve the auctioneers goal?

i.e., the painting goes to the agent who values it the most

Attempt 1: First Price Auction

Highest bidder gets the painting, pays his/her bid

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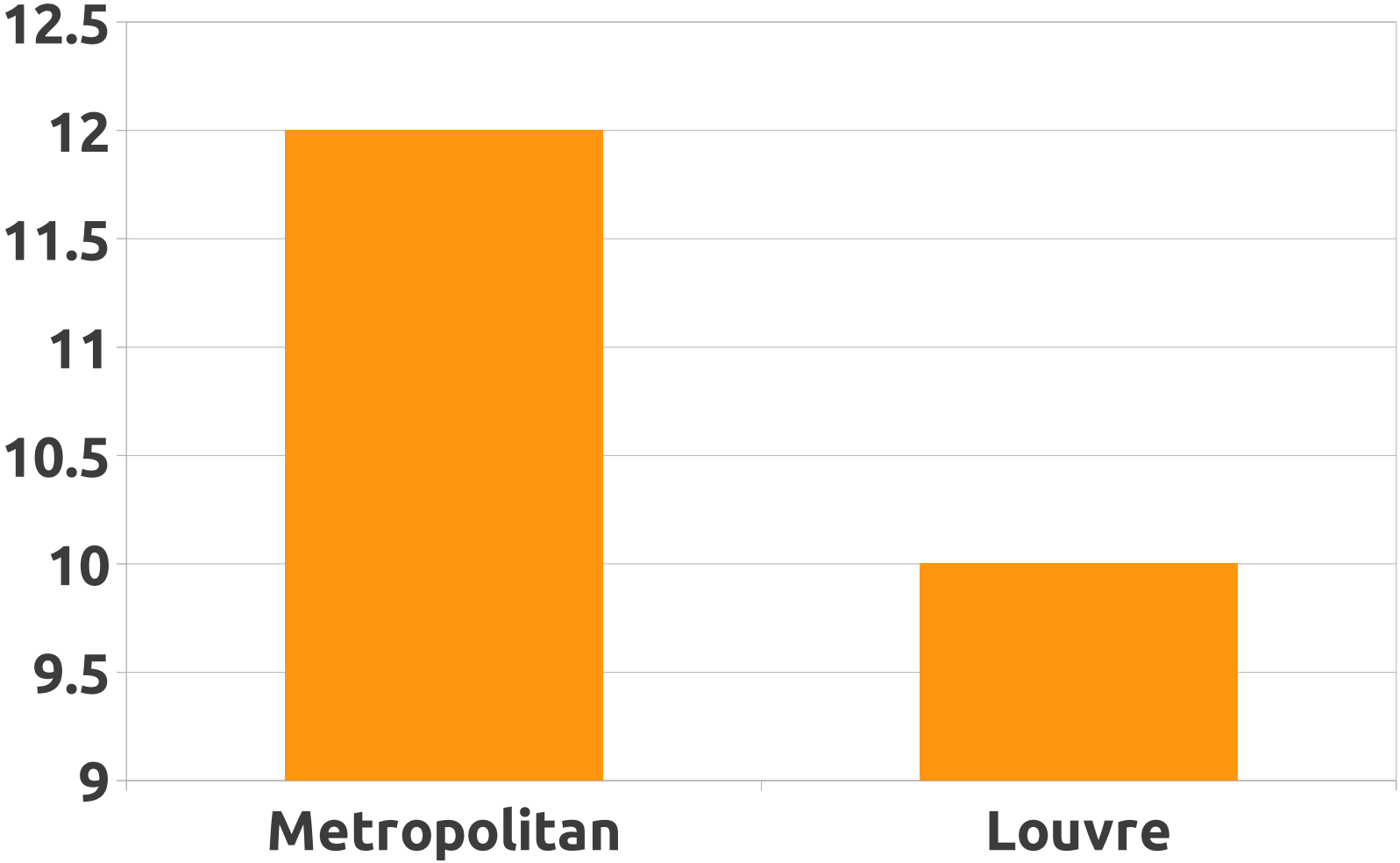

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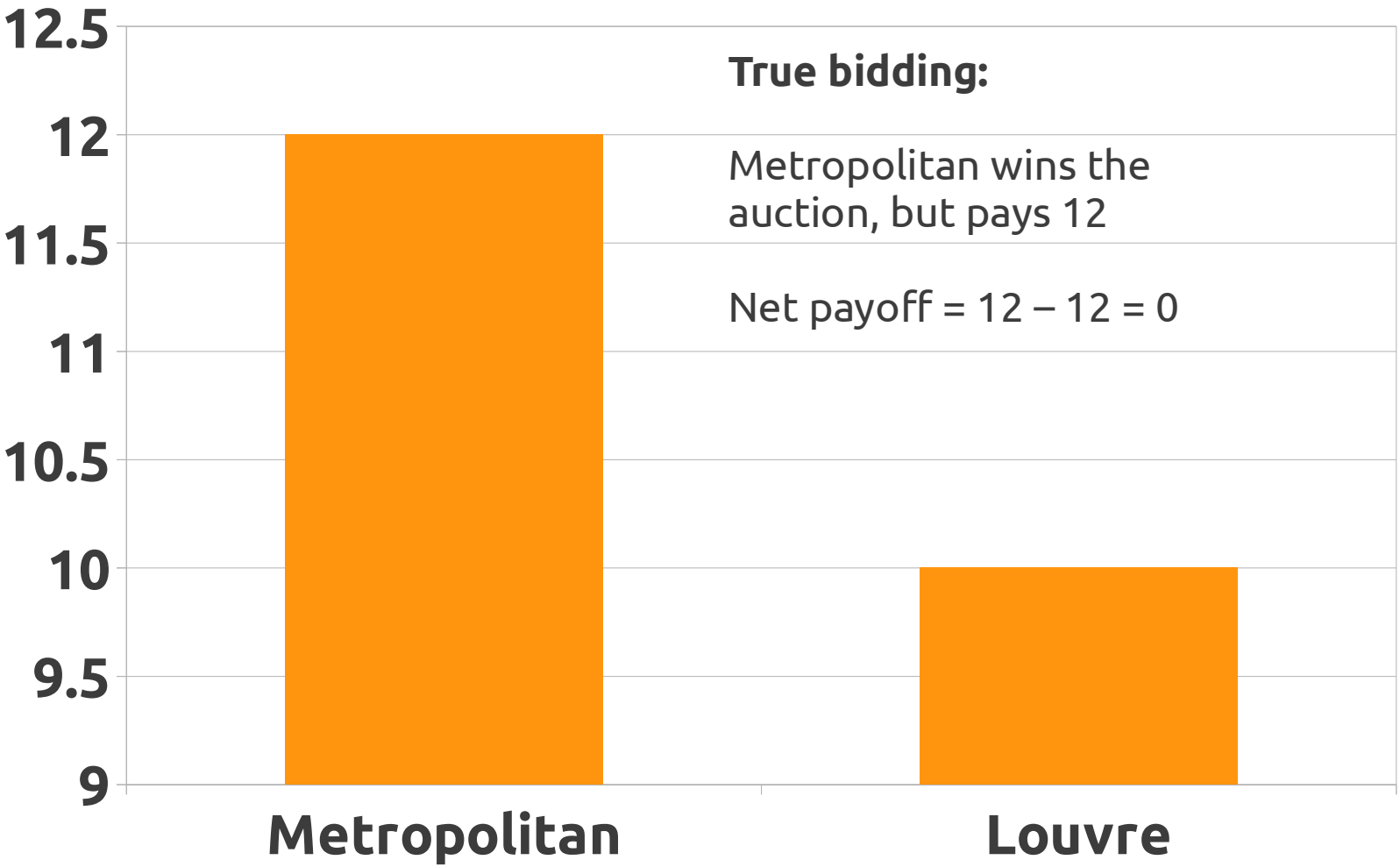


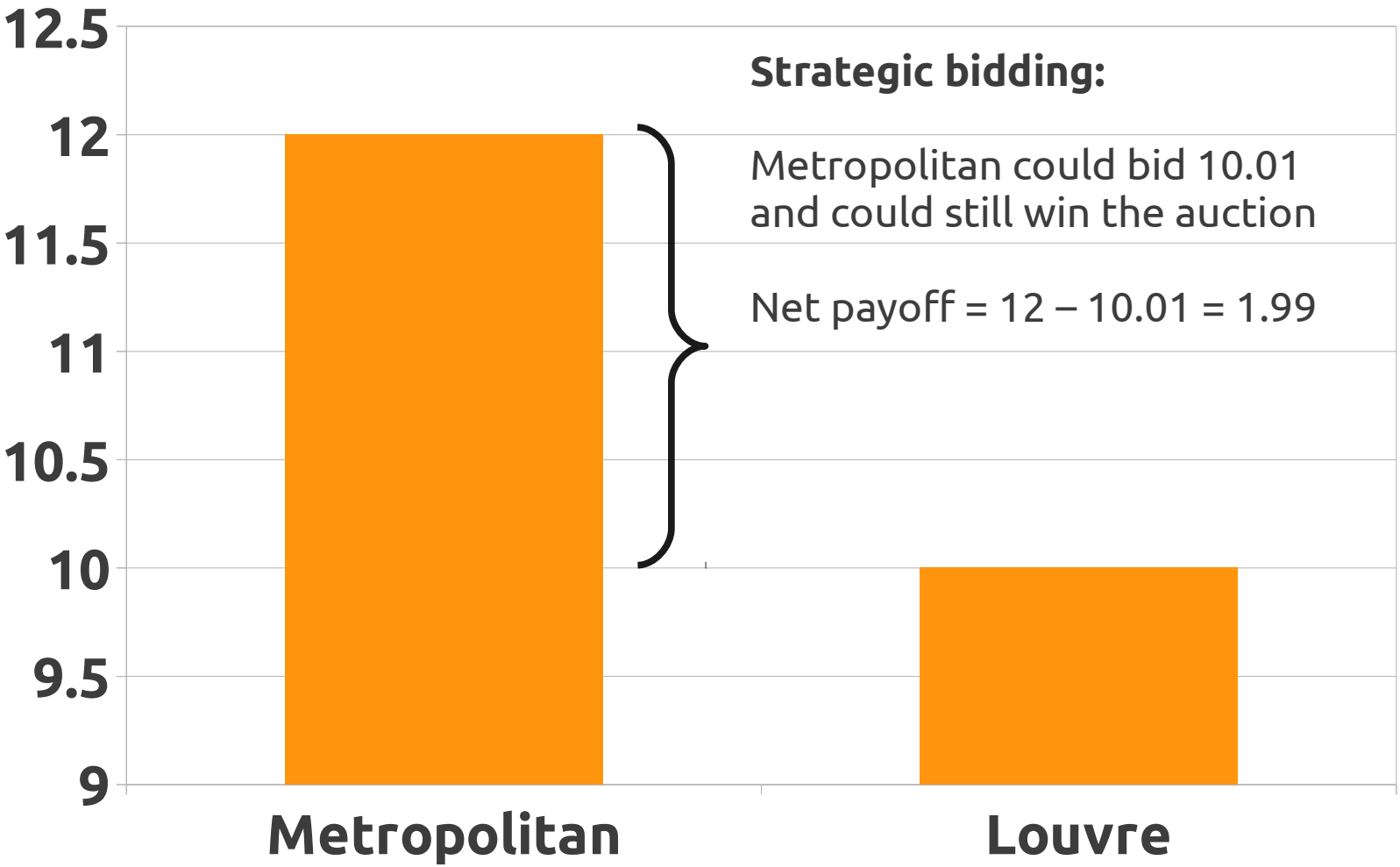
True bidding:

Metropolitan wins the auction, but pays 12

Net payoff = 12 – 12 = 0

Attempt 1: First Price Auction

Highest bidder gets the painting, pays his/her bid



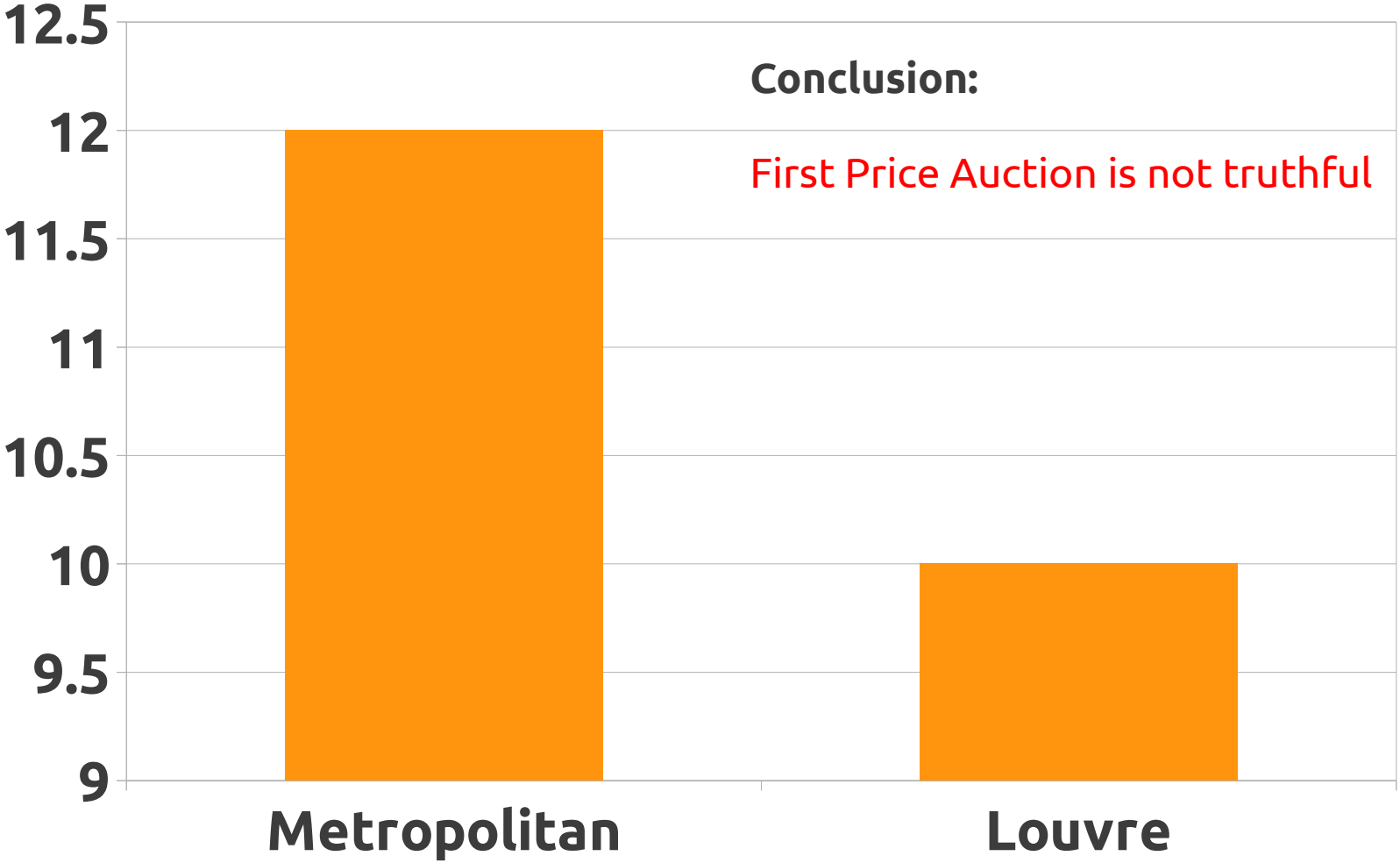
Strategic bidding:

Metropolitan could bid 10.01 and could still win the auction

Net payoff = 12 – 10.01 = 1.99

Attempt 1: First Price Auction

Highest bidder gets the painting, pays his/her bid



Conclusion:

First Price Auction is not truthful

Attempt 2: Second Price Auction

Highest bidder gets the painting, pays the next highest bid

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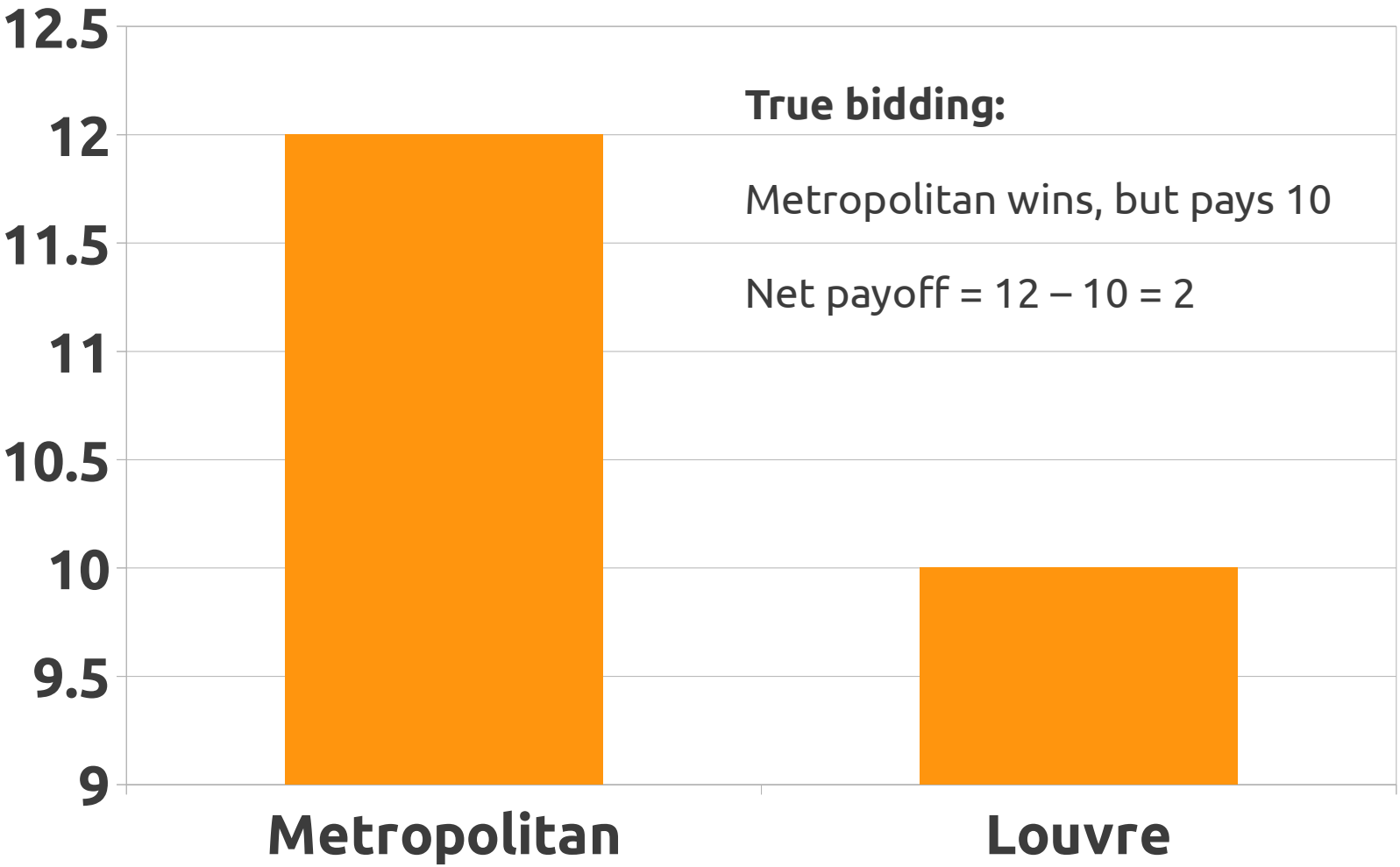
Highest bidder gets the painting, pays the next highest bid



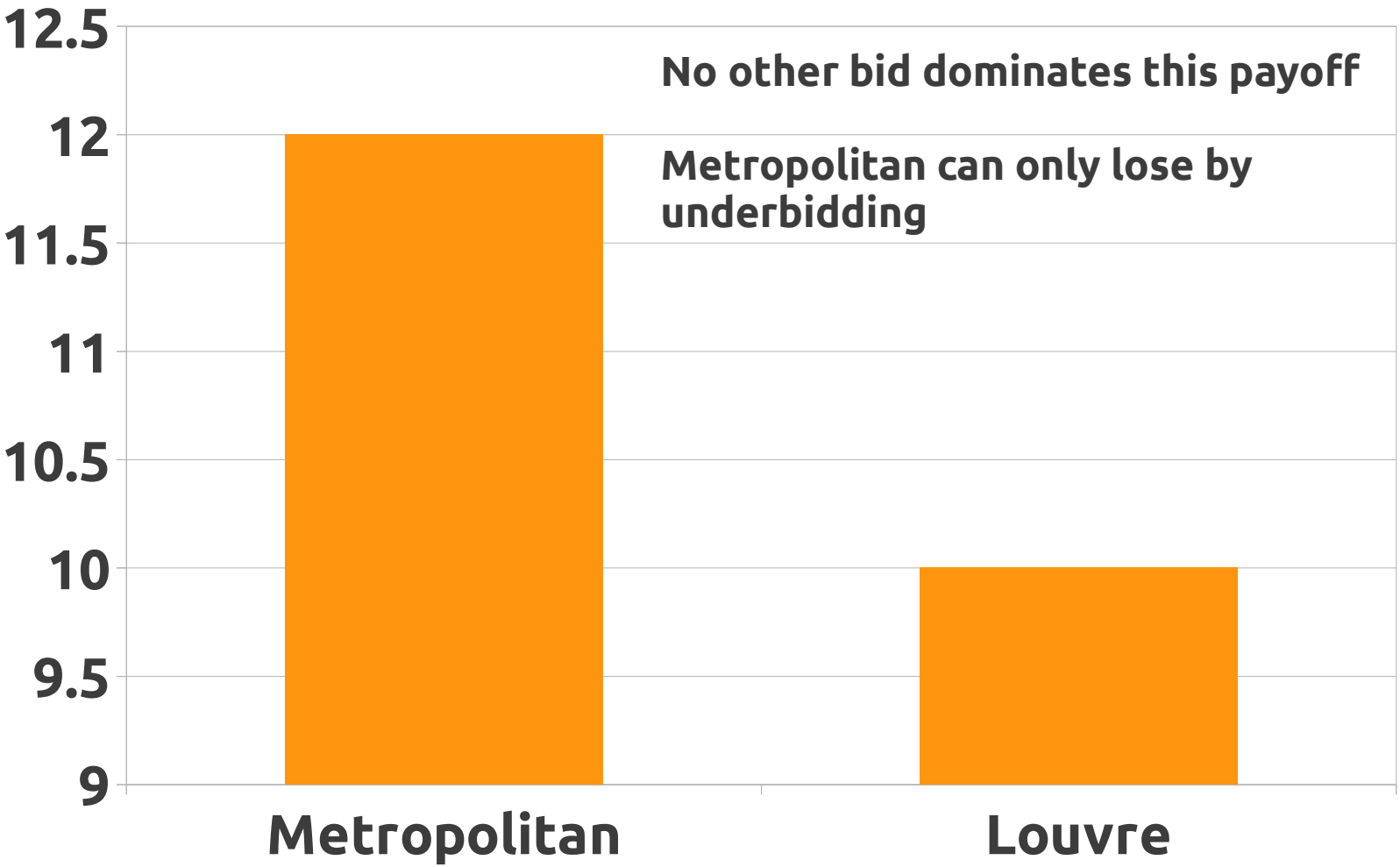
True bidding:

Metropolitan wins, but pays 10

Net payoff = 12 – 10 = 2

Attempt 2: Second Price Auction

Highest bidder gets the painting, pays the next highest bid



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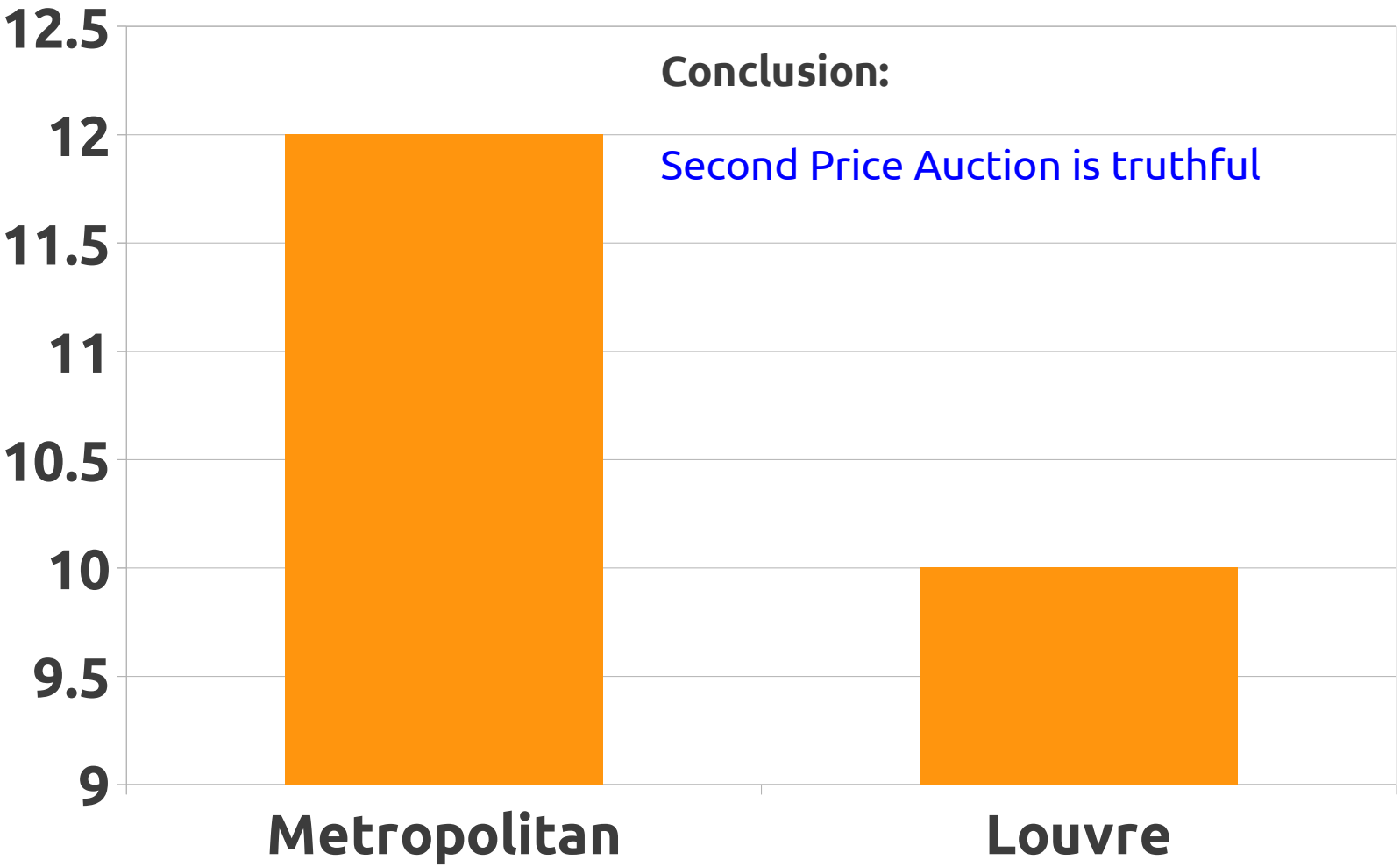


Conclusion:

Second Price Auction is truthful

The Pioneers of Game Theory

John Von Neumann

Founded Game Theory with Oskar Morgenstern (1928-44)

Pioneered the Concept of a Digital Computer and Algorithms

60 years later (2000), there is a convergence

Introduced the concept of Nash equilibrium and its existence

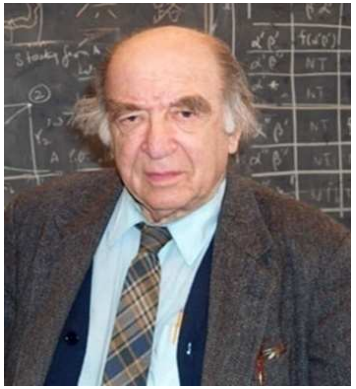
Also famous for his work on cooperative games and Nash bargaining

Nobel prize in Economics: 1994

Biographical movie: A Beautiful Mind

John F. Nash

The Pioneers of Mechanism Design



Leonid Hurwicz

Eric Maskin

Roger Myerson

Jointly awarded the Nobel prize in Economics, 2007

For laying the foundation of Mechanism Design Theory

To Probe Further

- Y. Narahari, Dinesh Garg, Ramasuri Narayanam, and Hastagiri Prakash. *Game Theoretic Problems in Network Economics and Mechanism Design Solutions*. Springer-Verlag, London, 2009.

- Yoav Shoham, Kevin Leyton-Brown. *Multiagent Systems Algorithmic, Game-Theoretic, and Logical Foundations*. Cambridge University Press, 2009. E-book freely downloadable from www.masfoundations.org

Thank You!

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