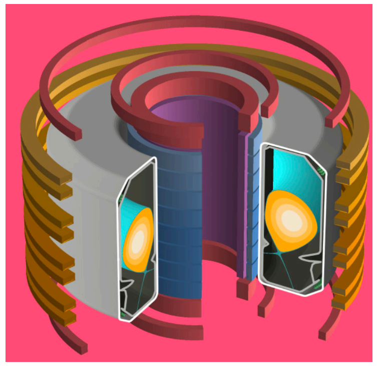
Rethinking the theoretical foundation of reinforcement learning

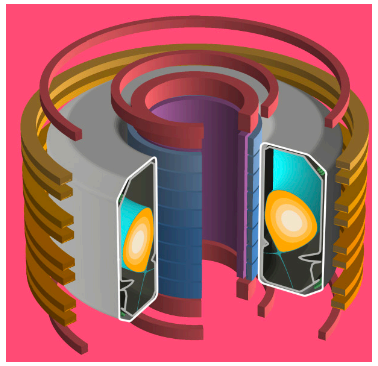
Nan Jiang
University of Illinois at Urbana-Champaign
Feb 24, 2024

@IISc Workshop
Reinforcement Learning: Recent Advances and Challenges Ahead


Google DeepMind
Challenge Match
LEE SEDOL
00:28:28


The international journal of science / 10 February 2022
nature
DRIVING FORCE
AI algorithm outcompetes human champions in Gran Turismo racing game





Key ingredient: **simulator**

- Unlimited data
- Decision w/o real consequences
- Can easily evaluate new strategy

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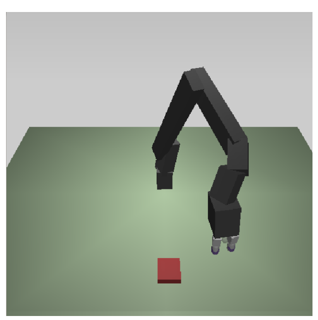
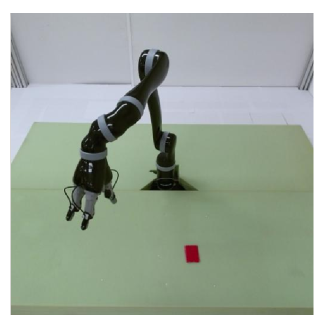

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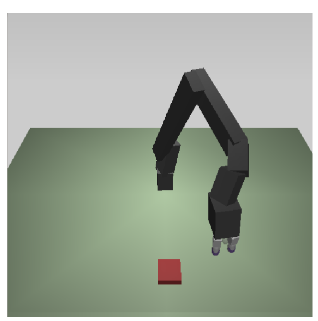

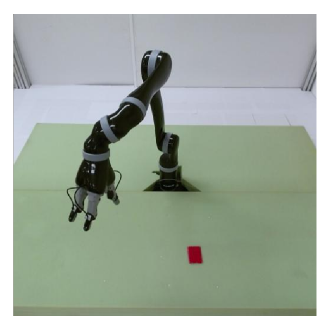

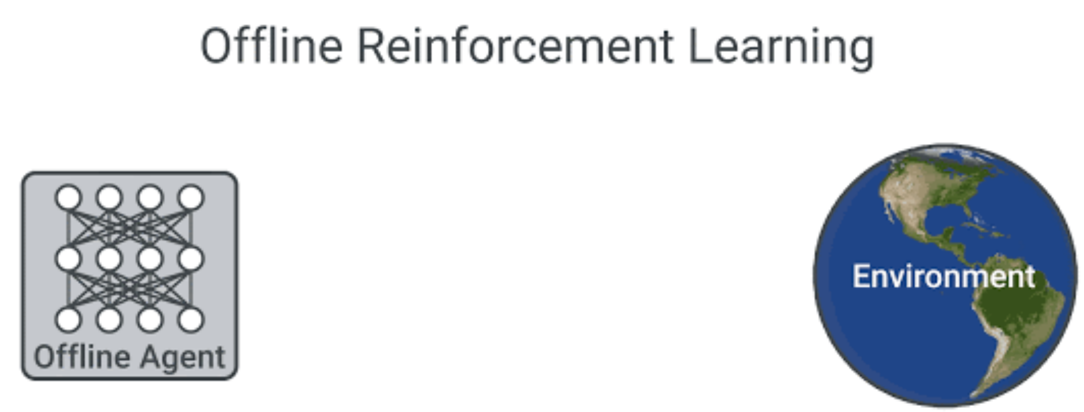

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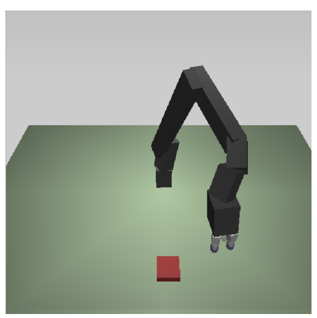
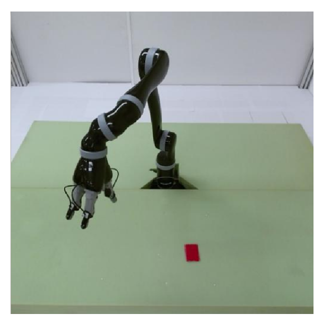
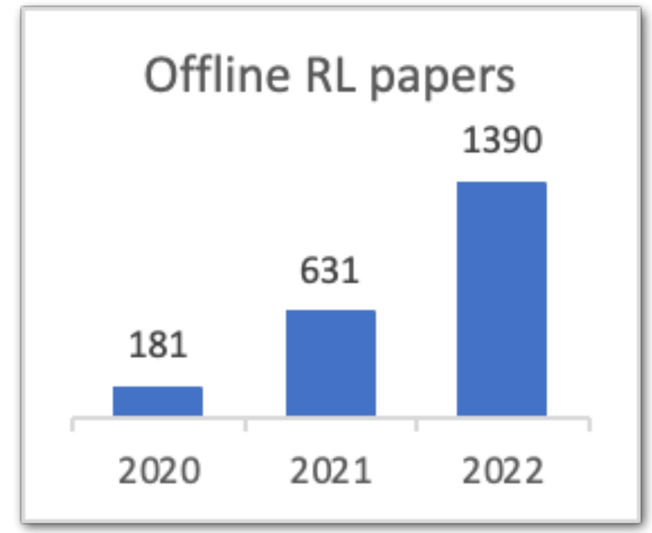
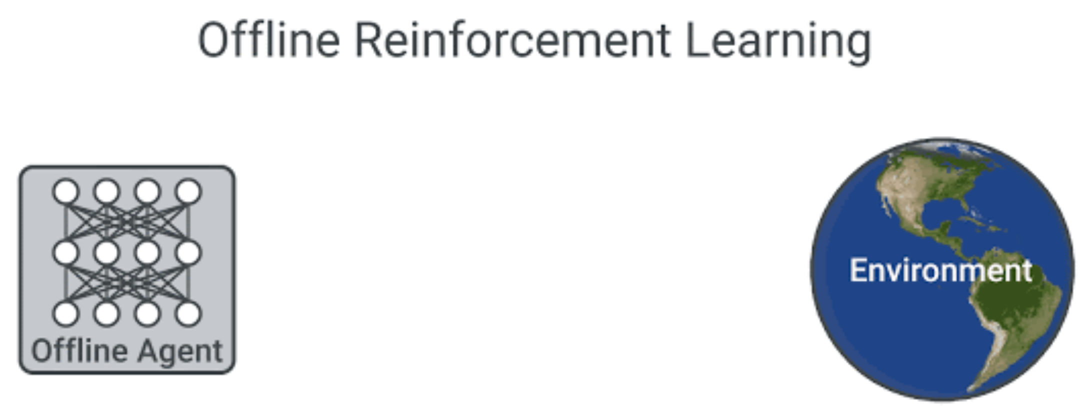
- (Offline) RL in real life









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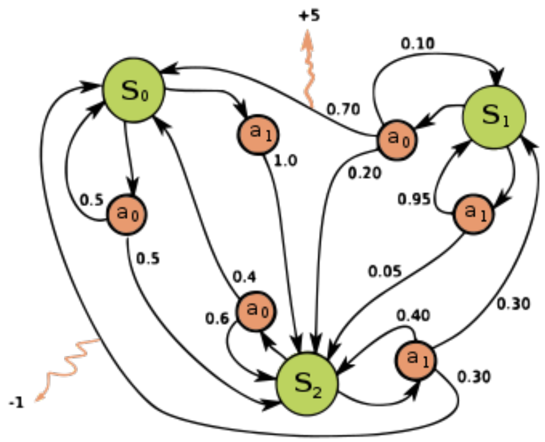
Why are we not seeing (offline) RL deployed everywhere already?

- (Offline) RL in real life
- Role of theory in modern RL

~2015



$\sqrt{HSAT}$ *regret,*
SAH^2/ϵ^2 *sample complexity, …*

~2000

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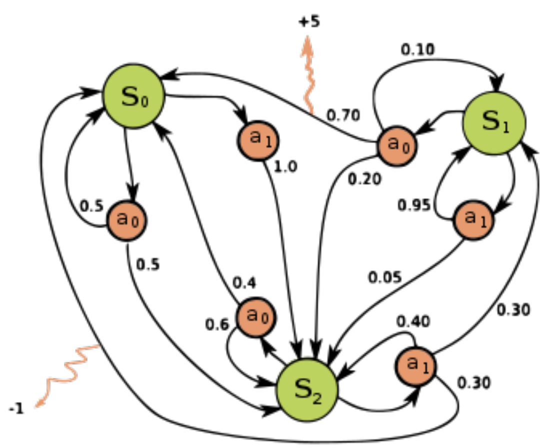

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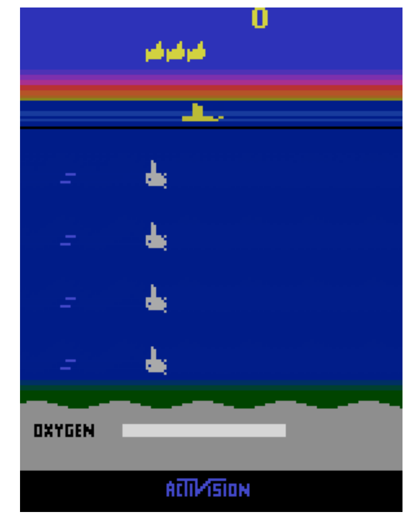

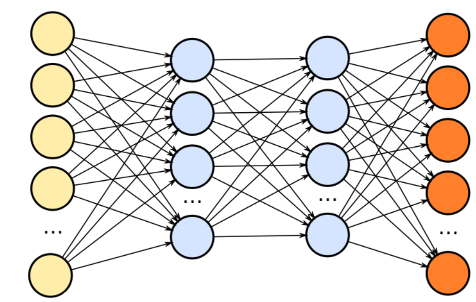

Empirical: *Atari, Mujoco, OpenAI Gym, target network, architecture, …*

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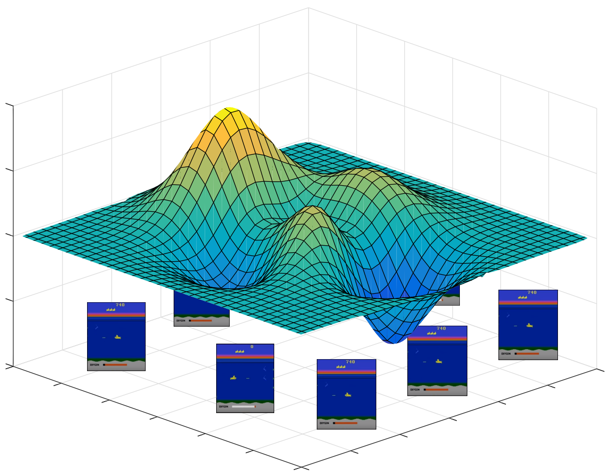
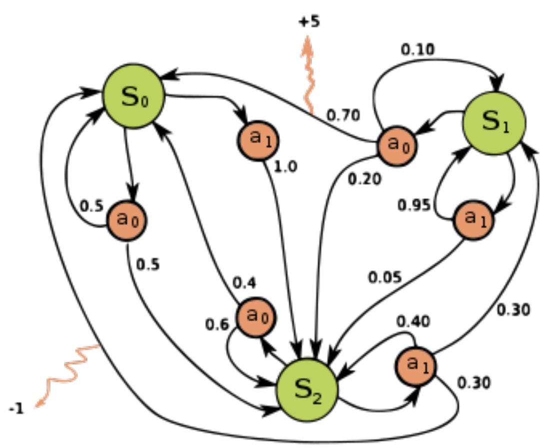


Bellman rank, Eluder dimension, Concentrability, …



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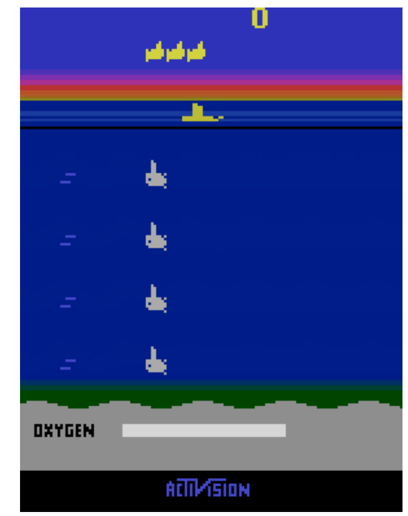

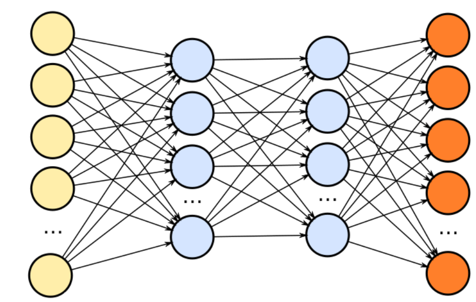

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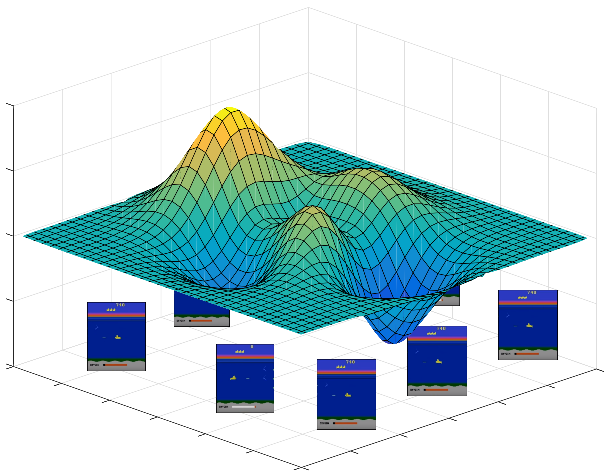

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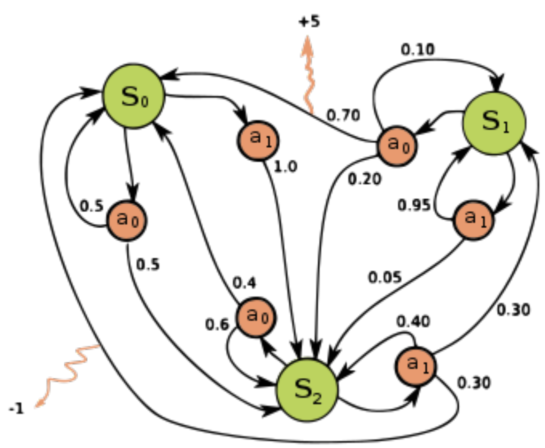

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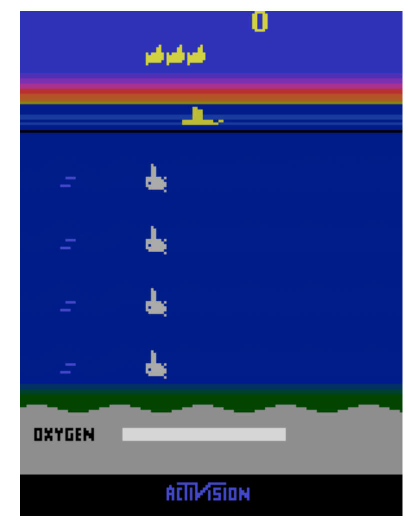

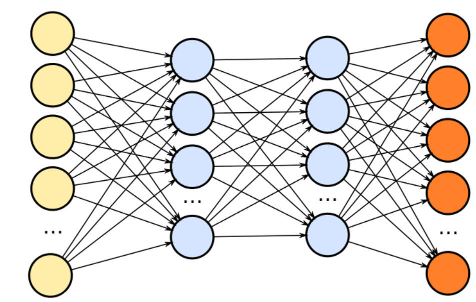

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* finite-sample analysis of ADP & MCTS 00~10

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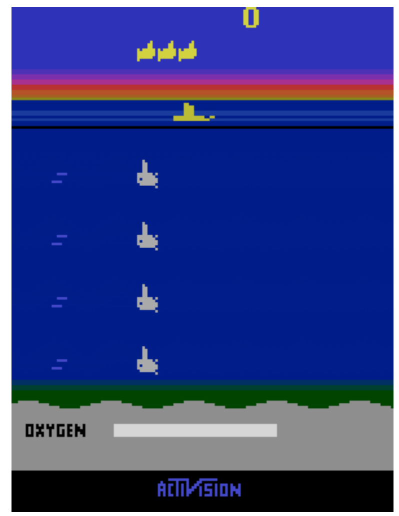
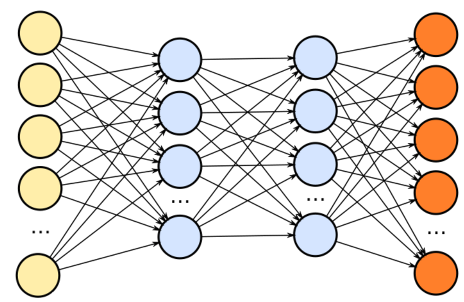
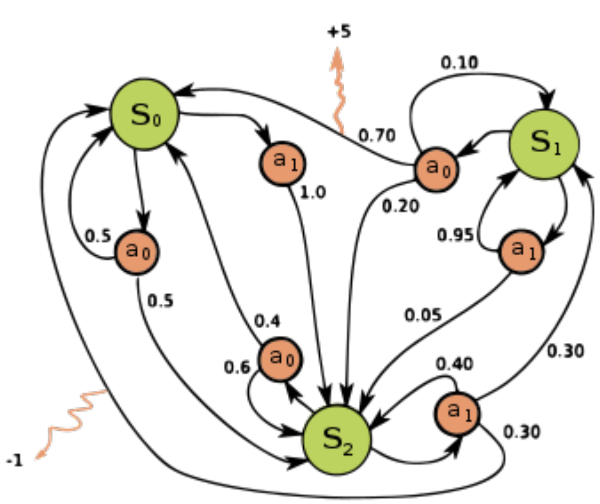




simplify



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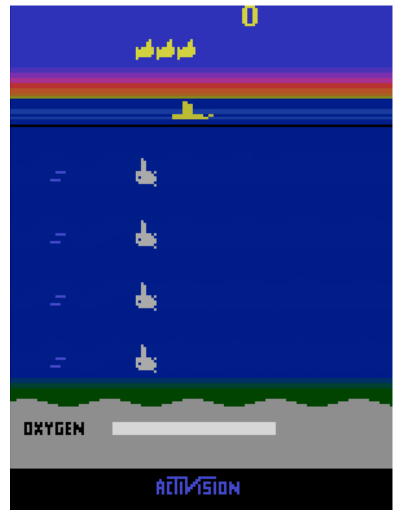
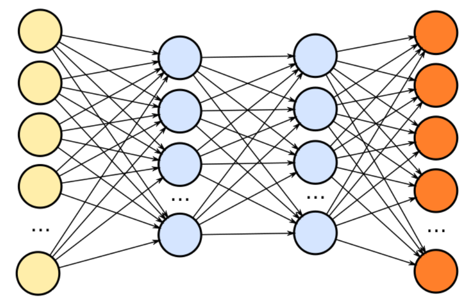
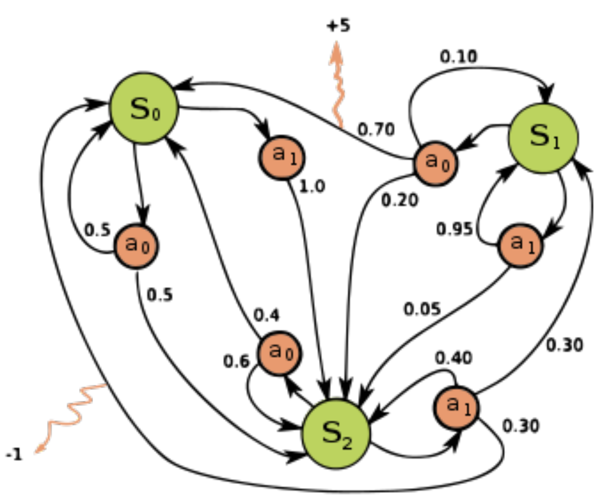




simplify

extend



	$Q^*(s, a)$
(s_1, a_1)	$\dots$
(s_1, a_2)	$\dots$
(s_2, a_1)	$\dots$

"tabular" RL

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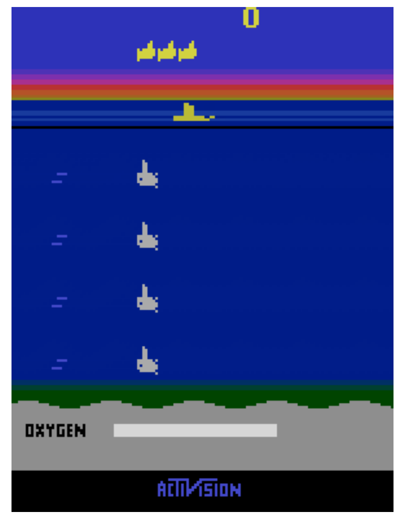

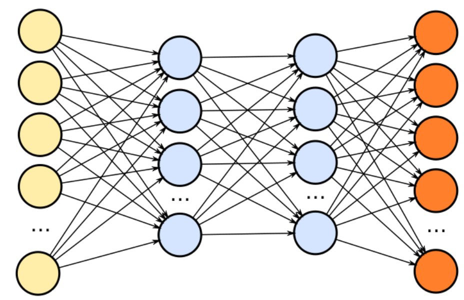

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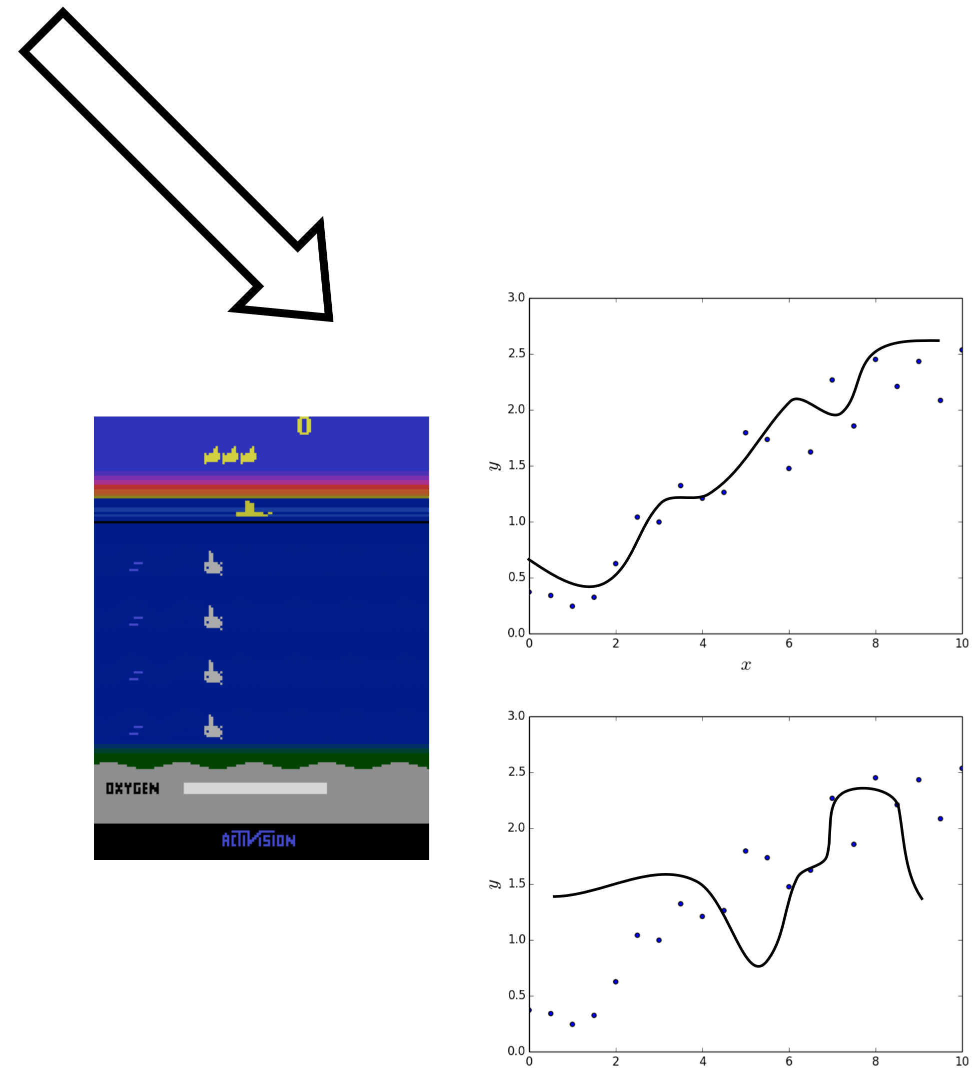

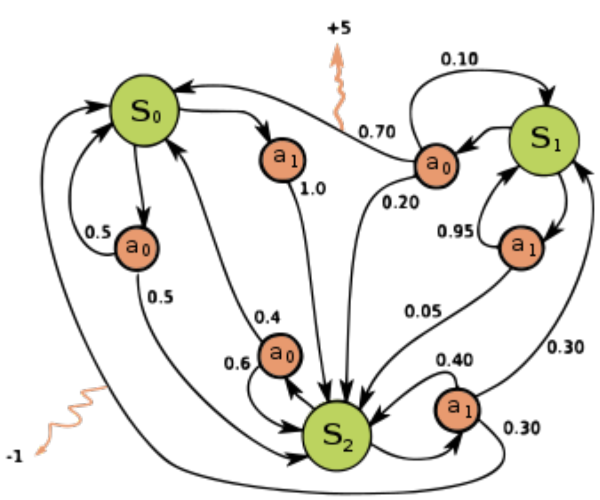

	$Q^*(s, a)$
(s_1, a_1)	...
(s_1, a_2)	...
(s_2, a_1)	...

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modern RL ⇒ real life

Rethinking
theoretical foundation

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modern RL  real life

Rethinking
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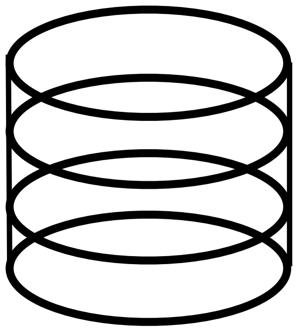
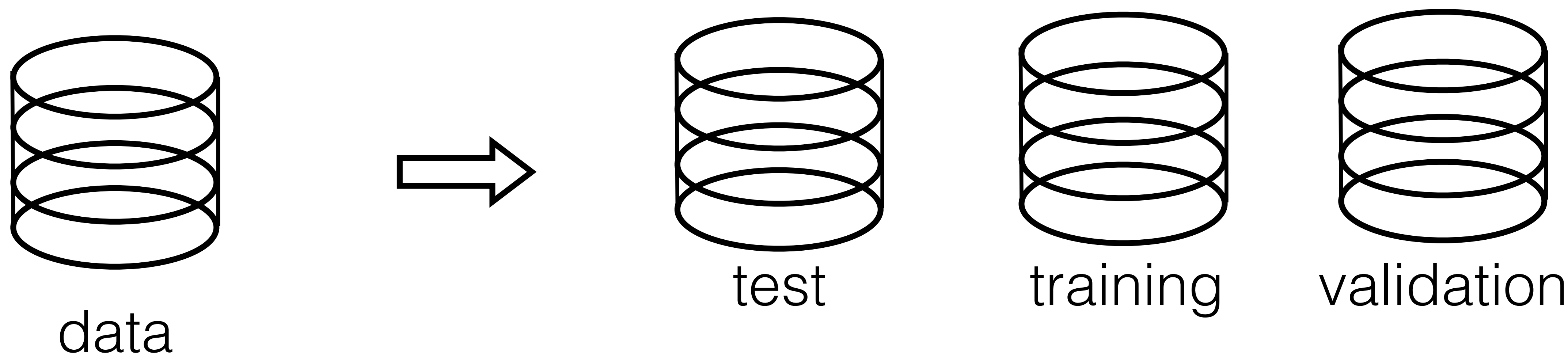
Supervised learning pipeline



data

Supervised learning pipeline



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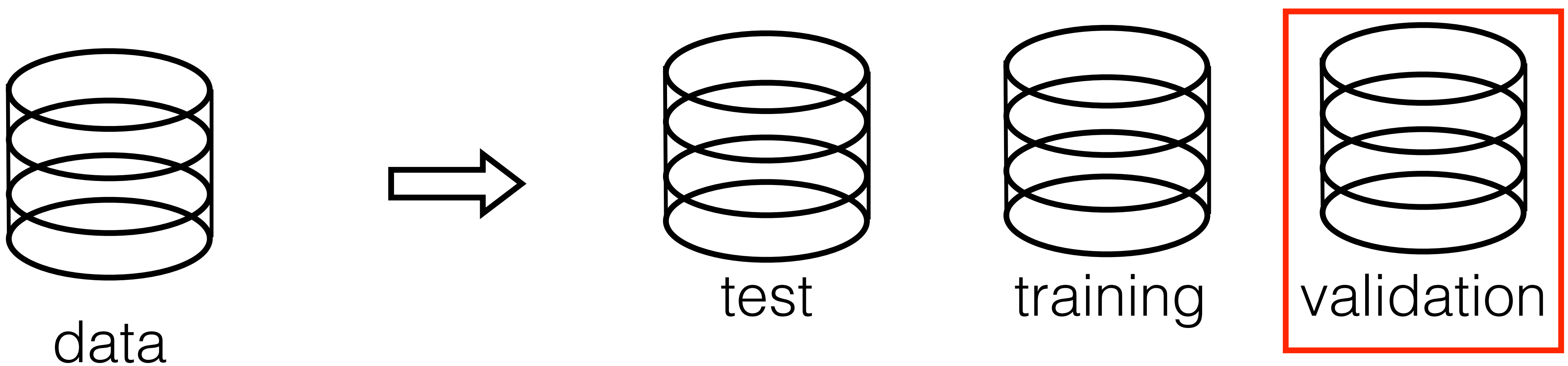
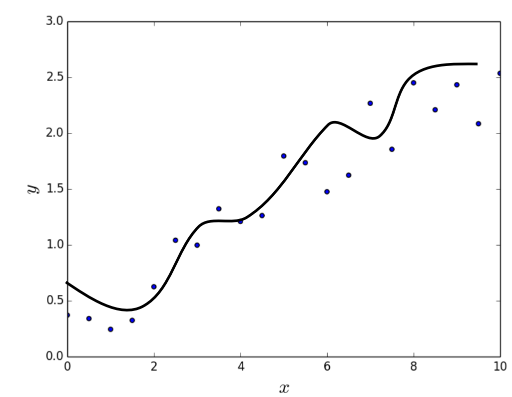
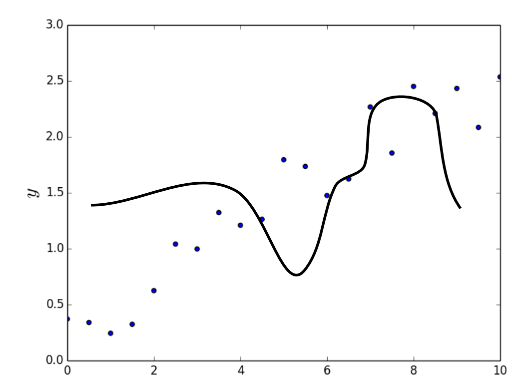
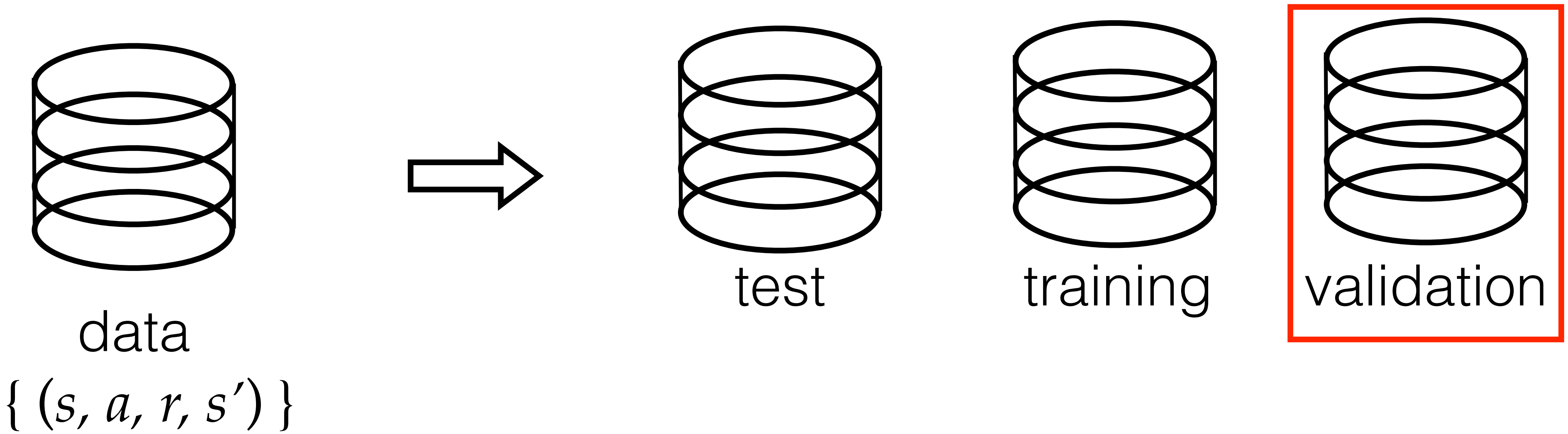
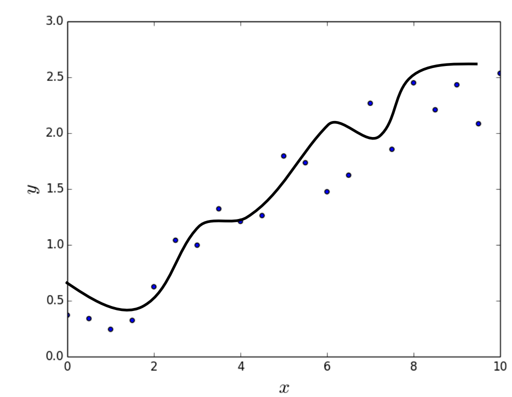


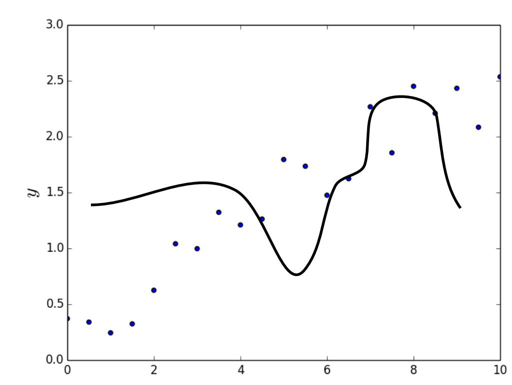




Offline RL pipeline







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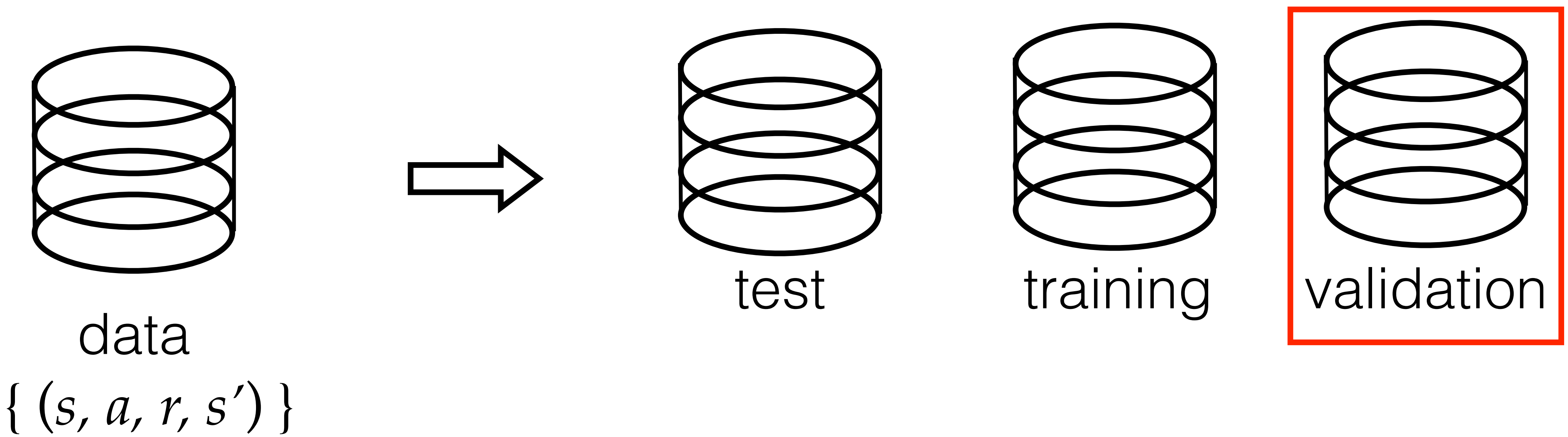

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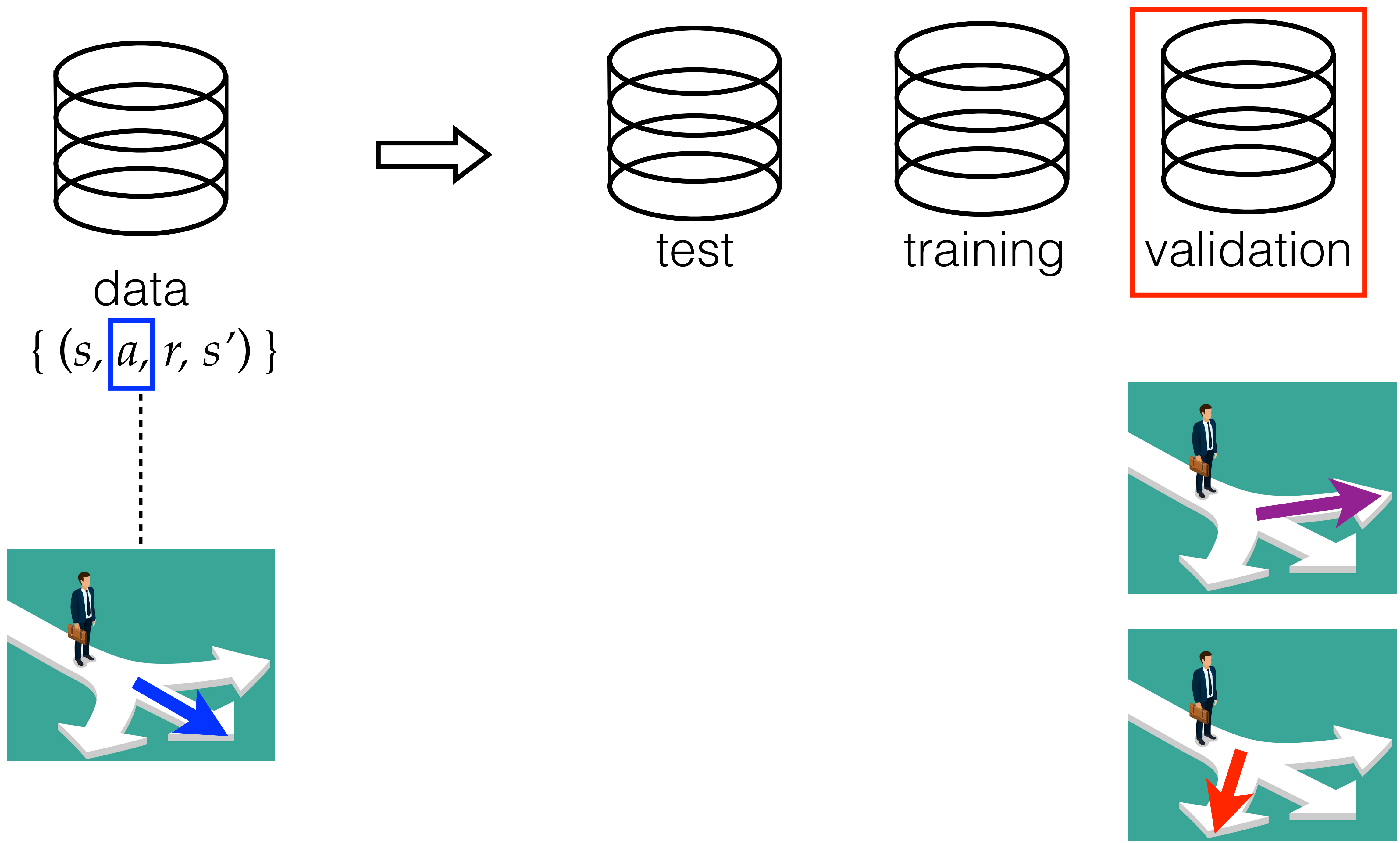

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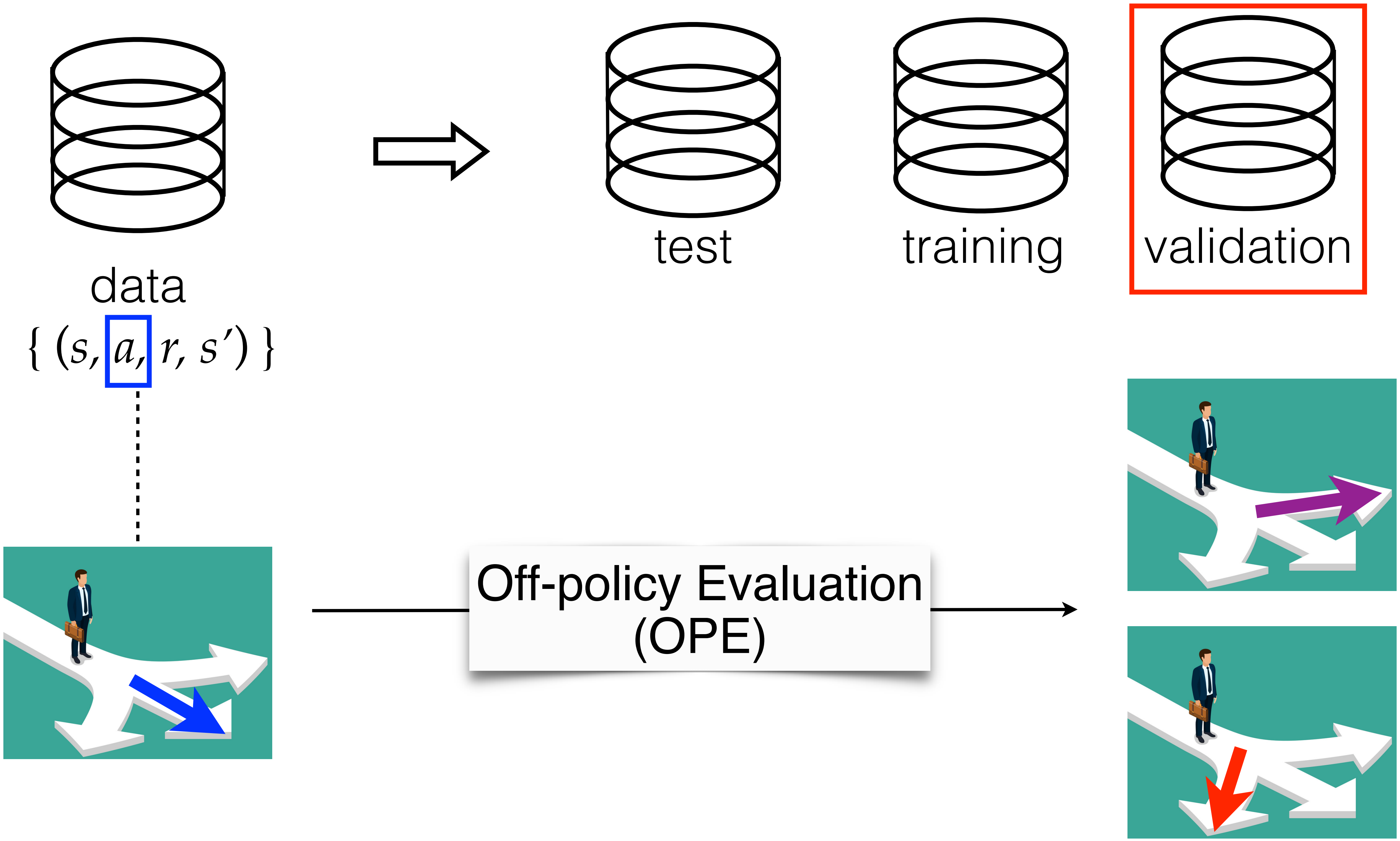
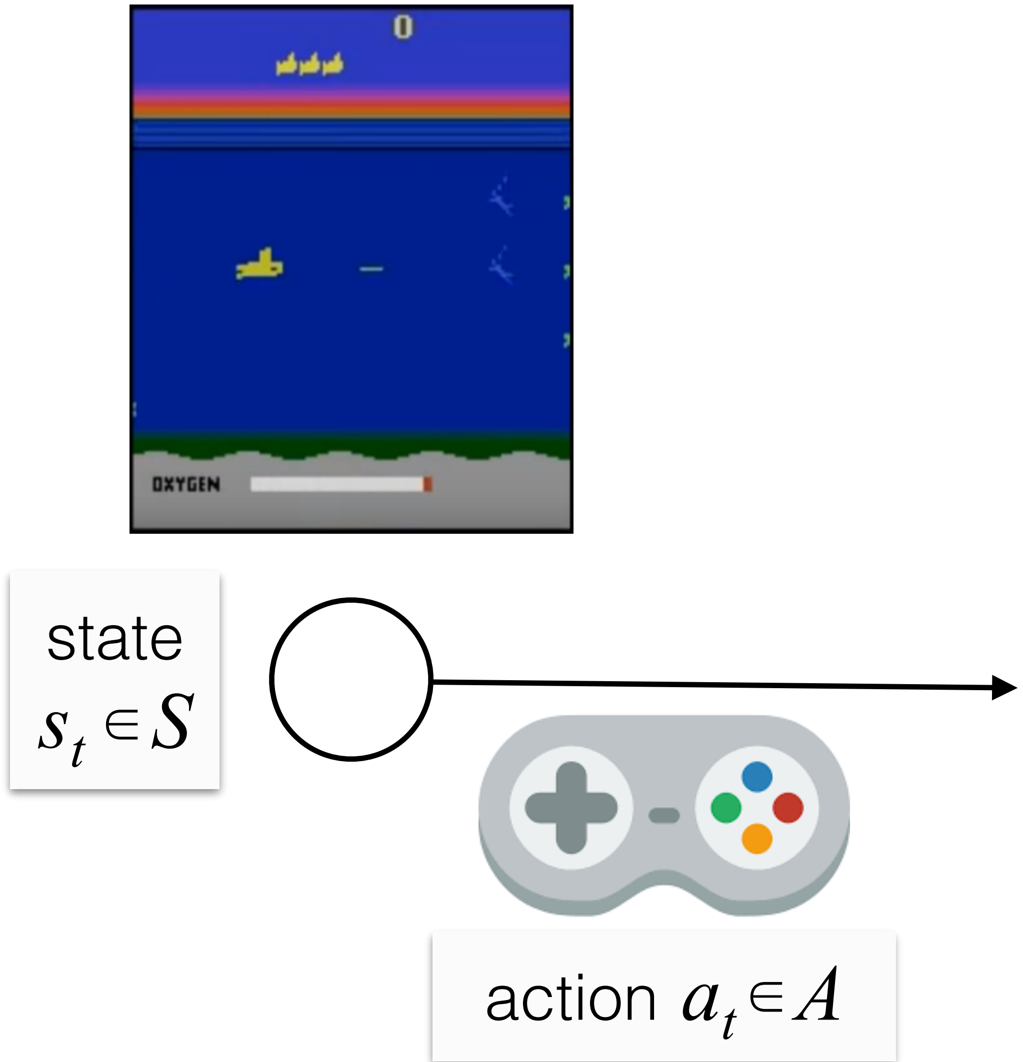
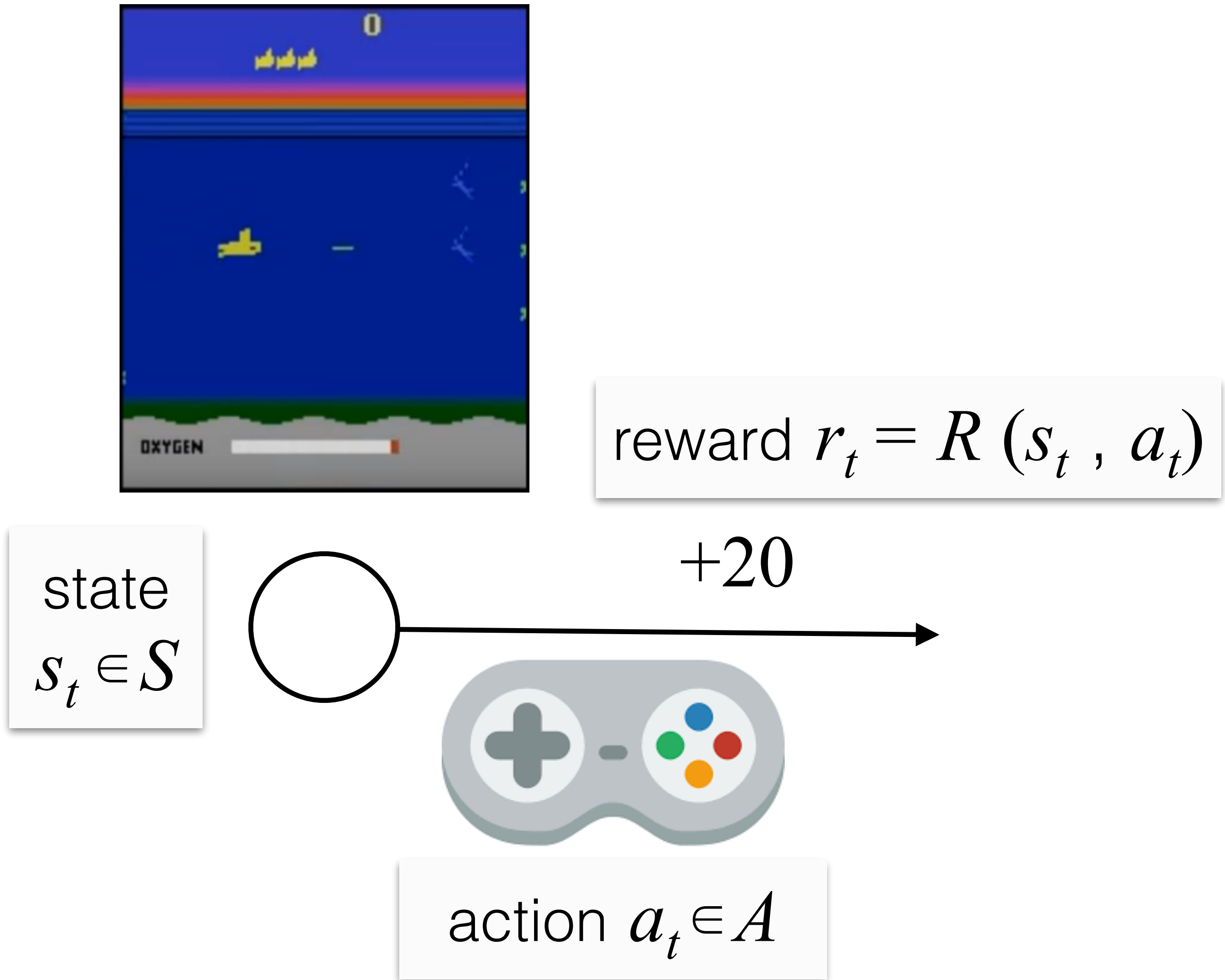
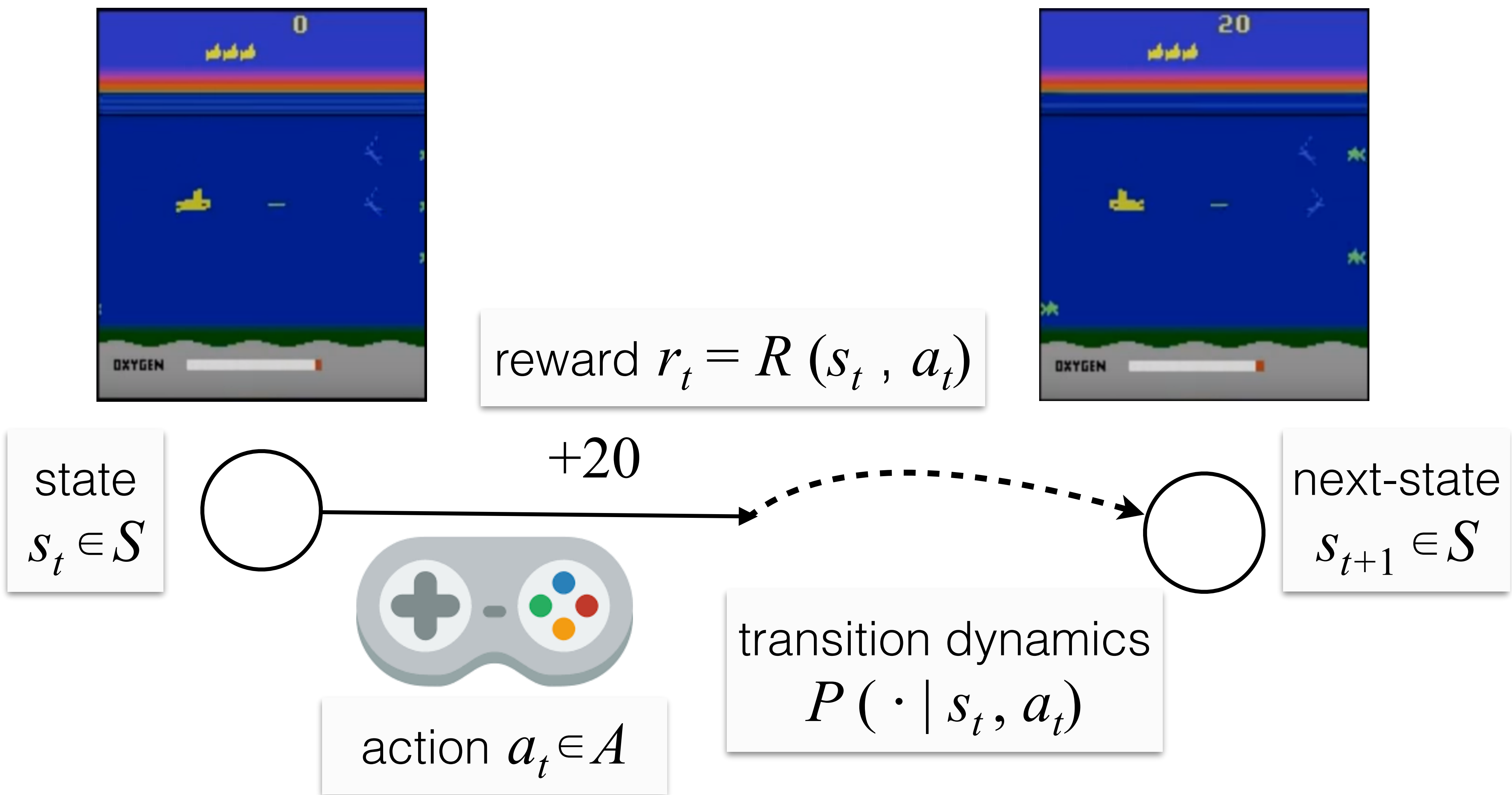
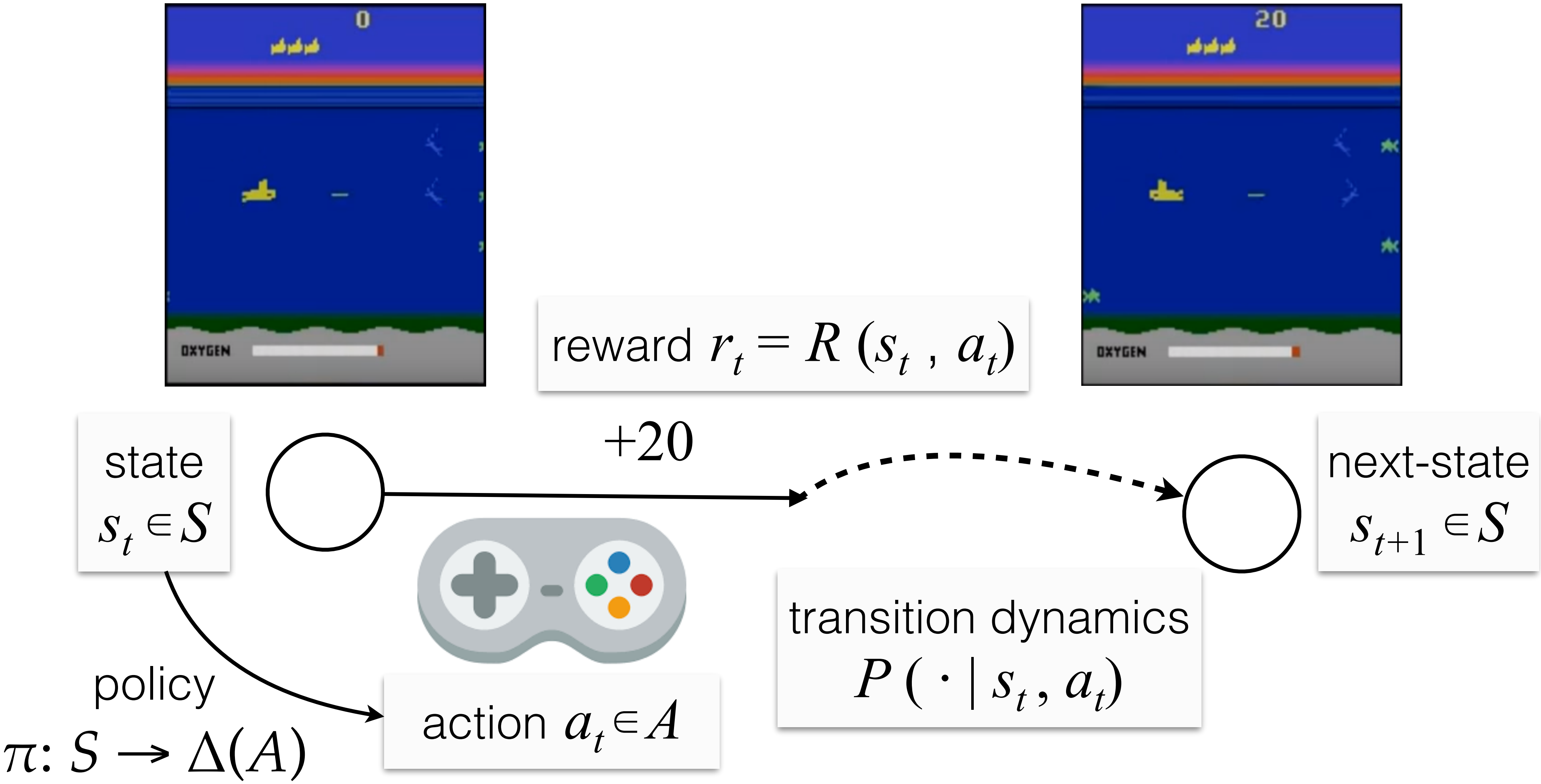


state
$s_t \in S$


OXYGEN
state
$s_t \in S$
action $a_t \in A$


reward $r_t = R(s_t, a_t)$
state
$s_t \in S$
+20
action $a_t \in A$


0
OXYGEN
20
OXYGEN
reward $r_t = R(s_t, a_t)$
+20
state
$s_t \in S$
action $a_t \in A$
transition dynamics
$P(\cdot \mid s_t, a_t)$
next-state
$s_{t+1} \in S$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+20
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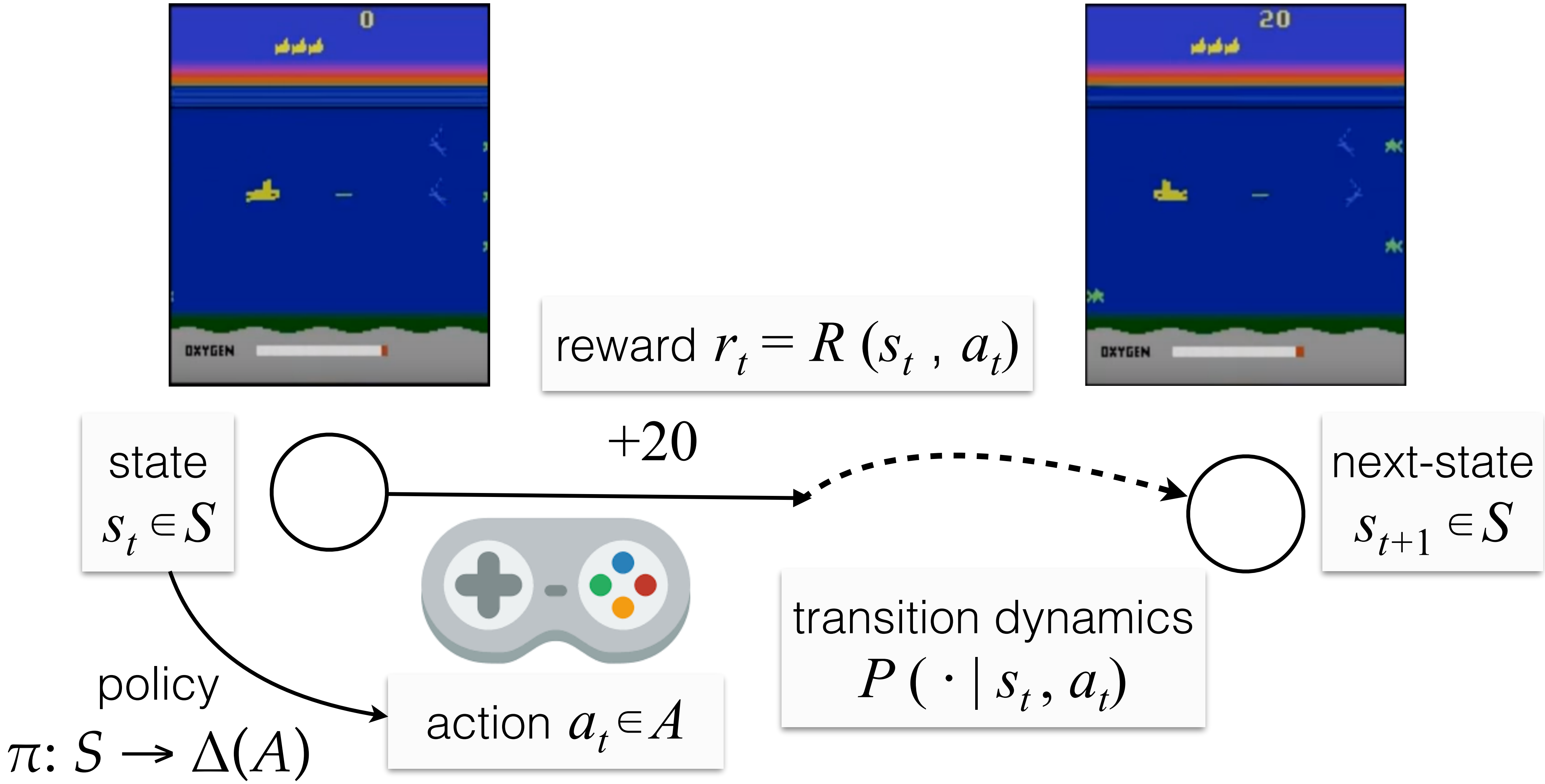
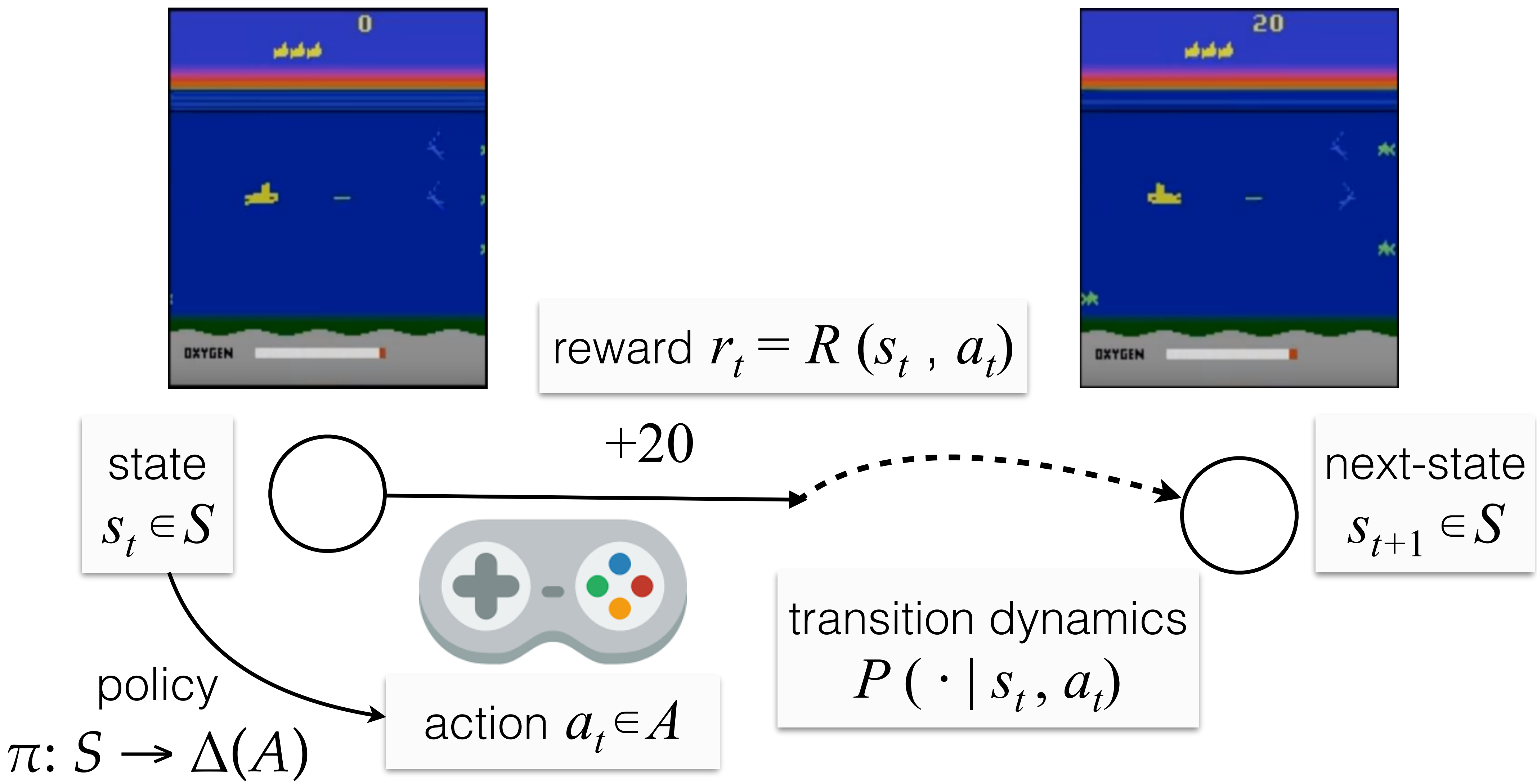
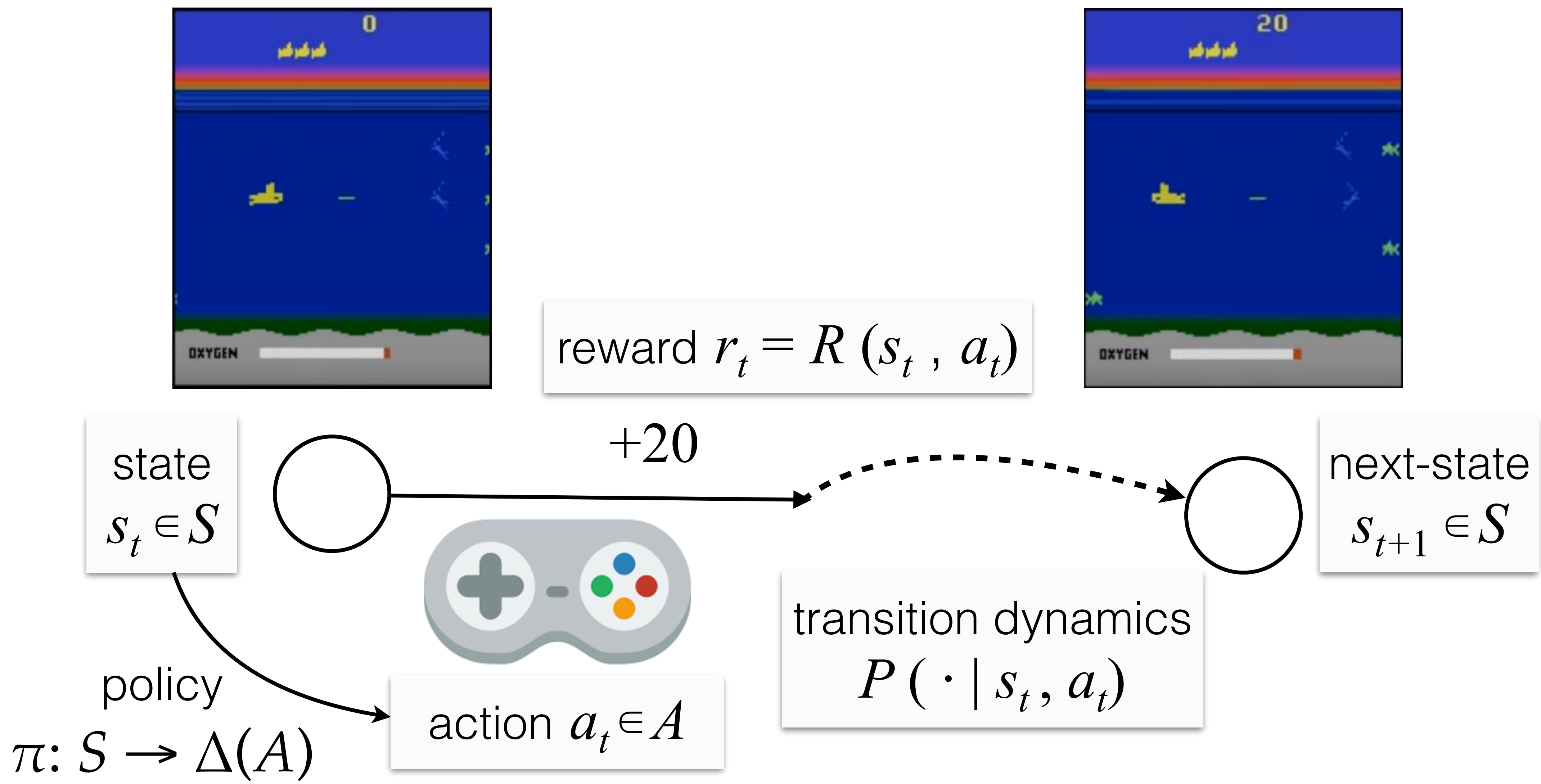
next-state
$s_{t+1} \in S$
transition dynamics
$P(\cdot \mid s_t, a_t)$
policy
$\pi: S \to \Delta(A)$
action $a_t \in A$



$$J(\pi) := \mathbb{E}_\pi [\sum_{t=0}^{\infty} \gamma^t r_t \mid s_0]$$



Policy evaluation: estimate $J(\pi) := \mathbb{E}_\pi[\sum_{t=0}^{\infty} \gamma^t r_t \mid s_0]$ given π



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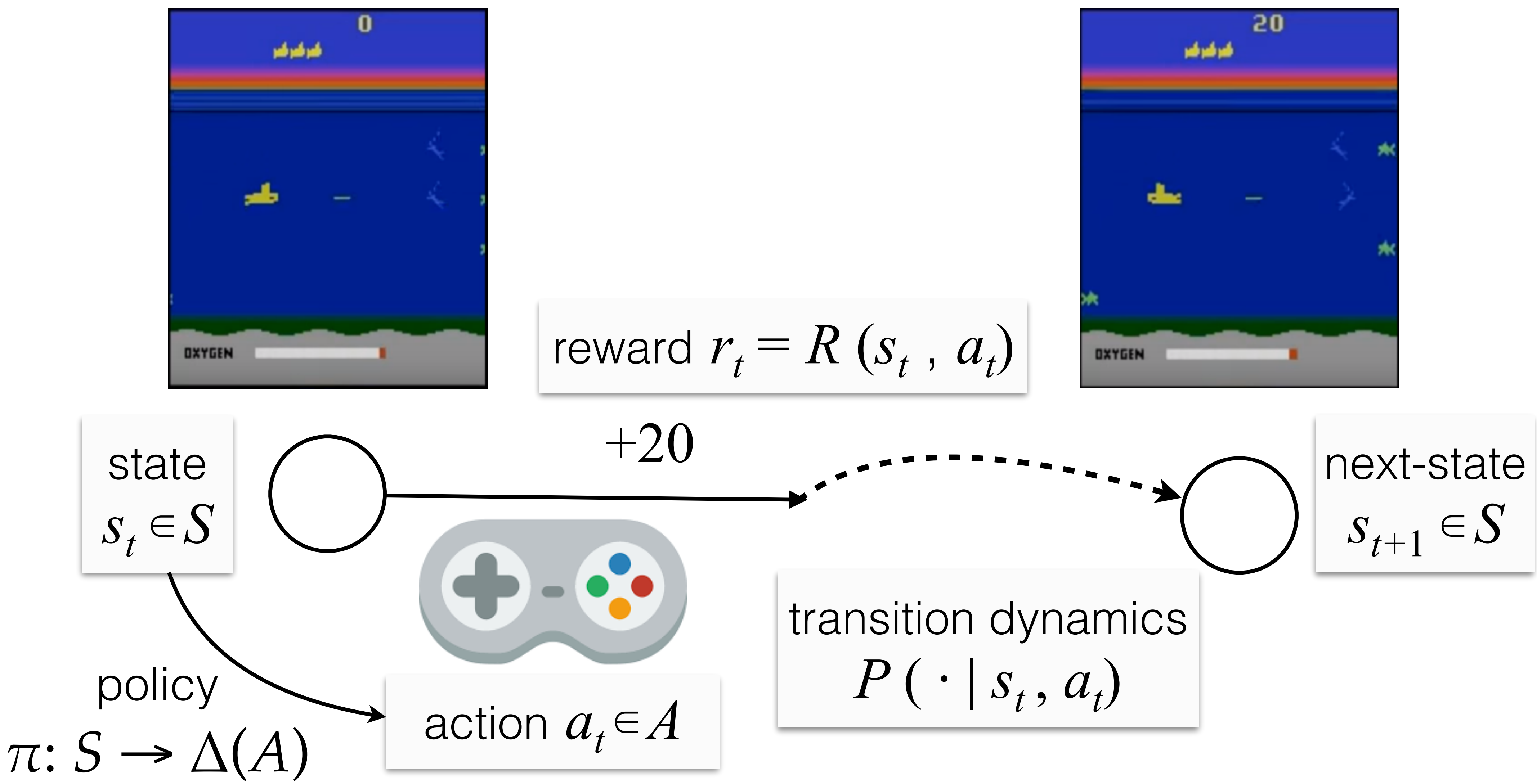
Policy optimization: $\max_\pi J(\pi)$



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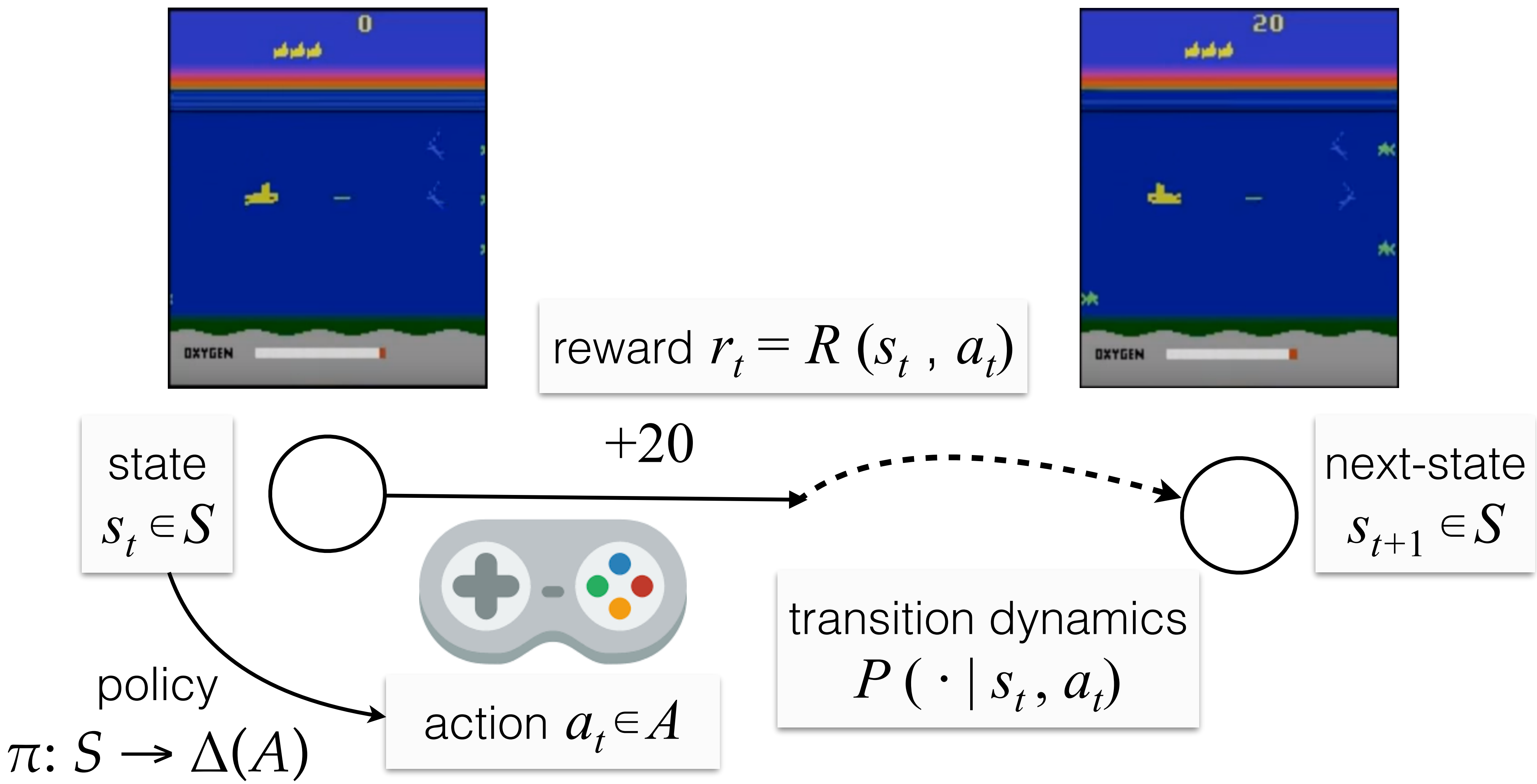
$$= Q^\pi(s_0, \pi)$$



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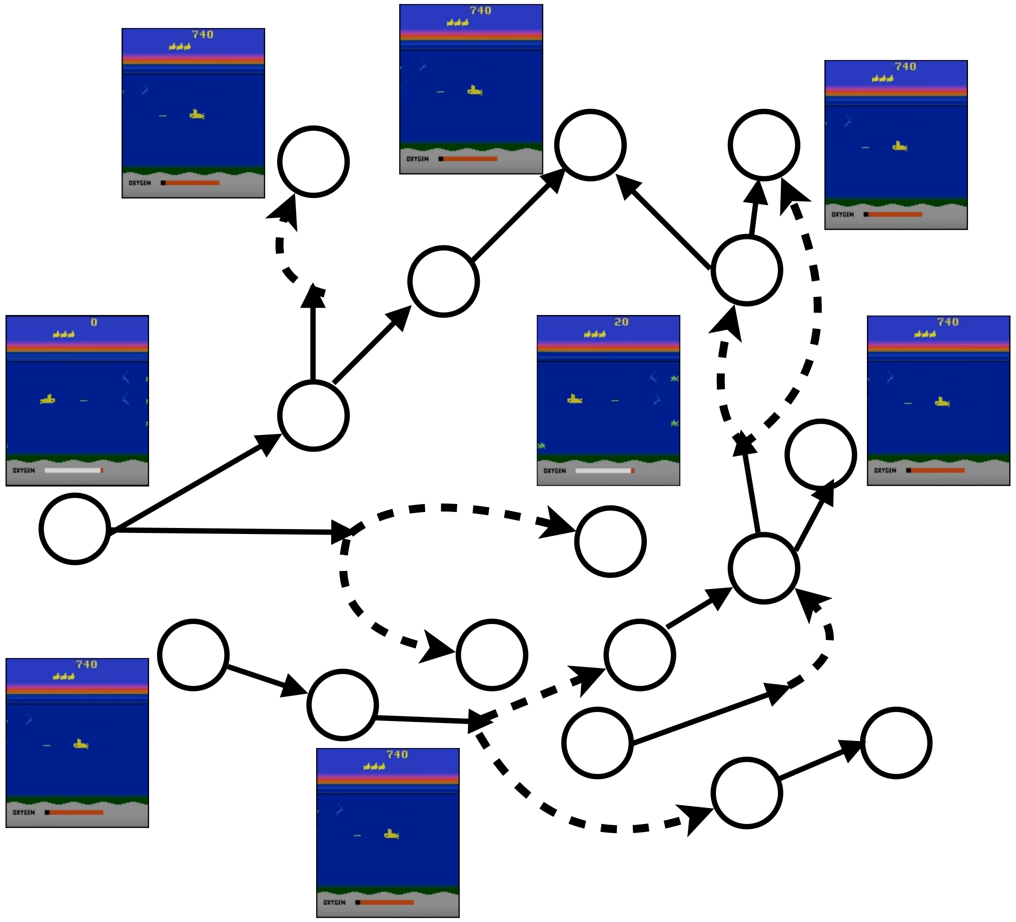
How to find Q^π?

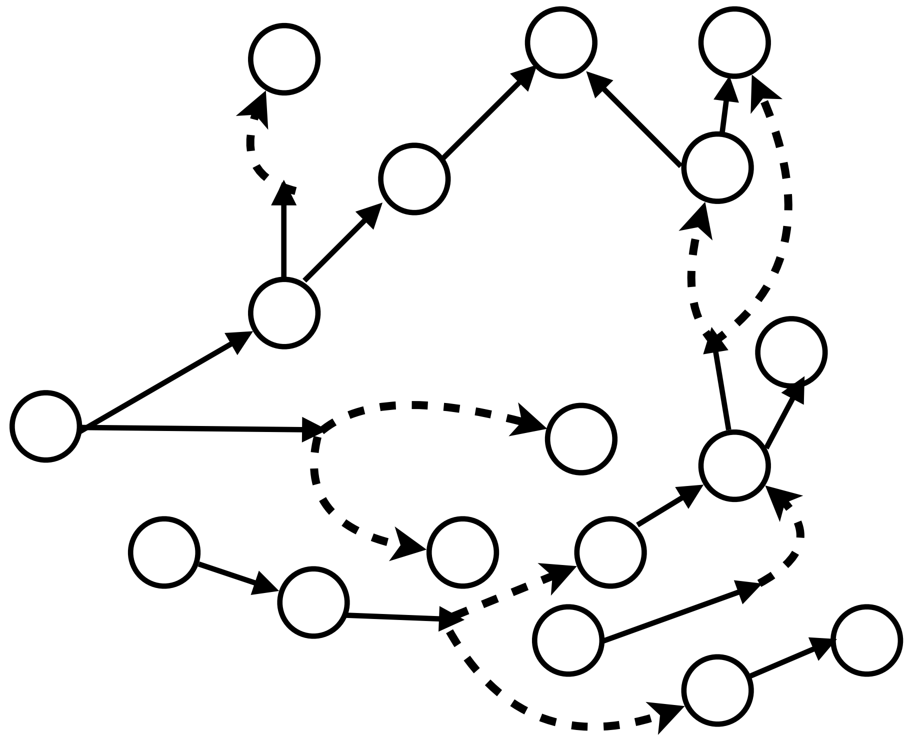
$Q^\pi = \mathcal{T}^\pi Q^\pi \quad \rightarrow \quad |SxA|$ equations

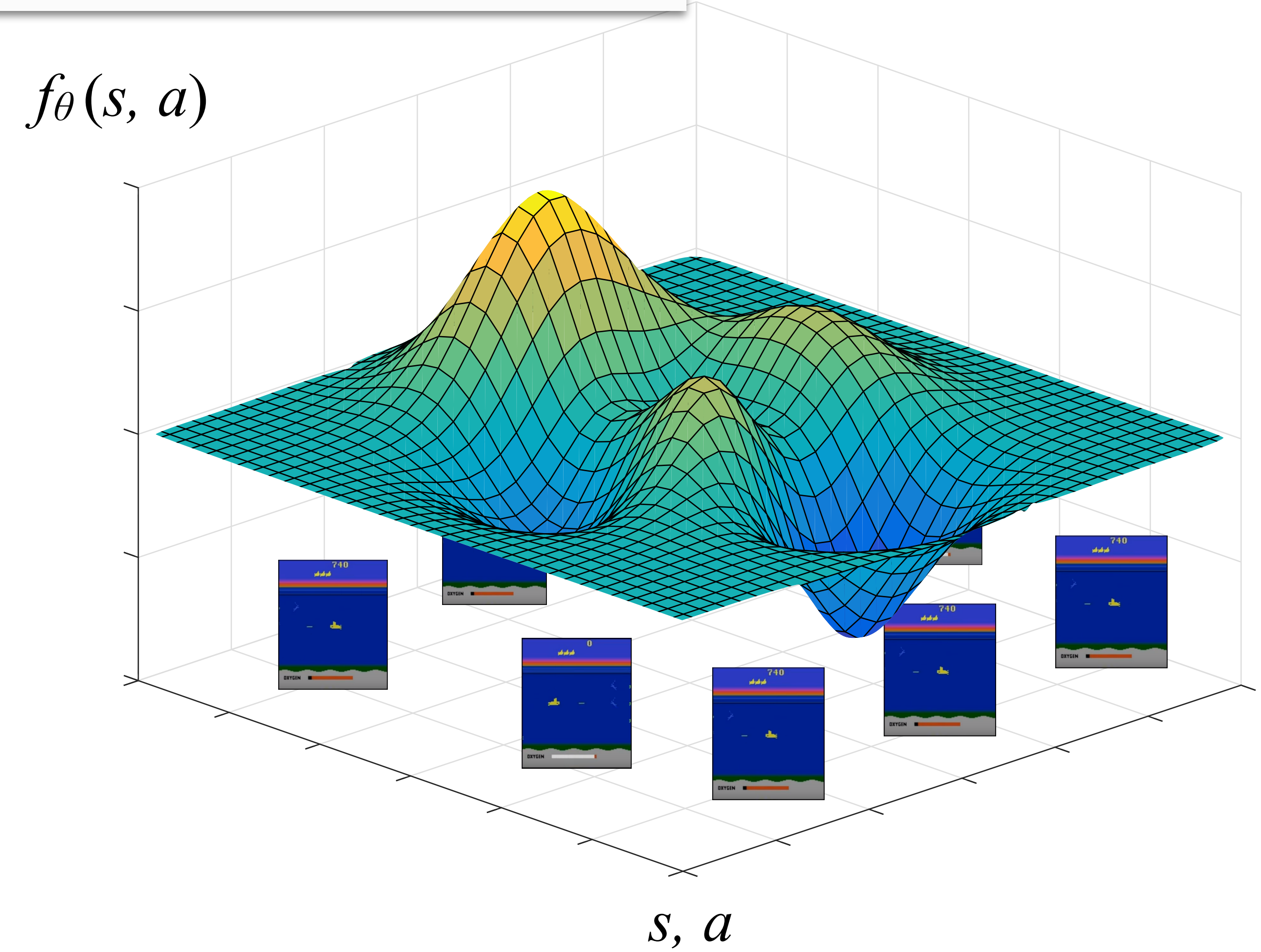


How to find Q^π?

$Q^\pi = \mathcal{T}^\pi Q^\pi \;\; \rightarrow \;\; |S\text{x}A|$ equations **X**

Find θ s.t. $f_\theta \approx Q^\pi$





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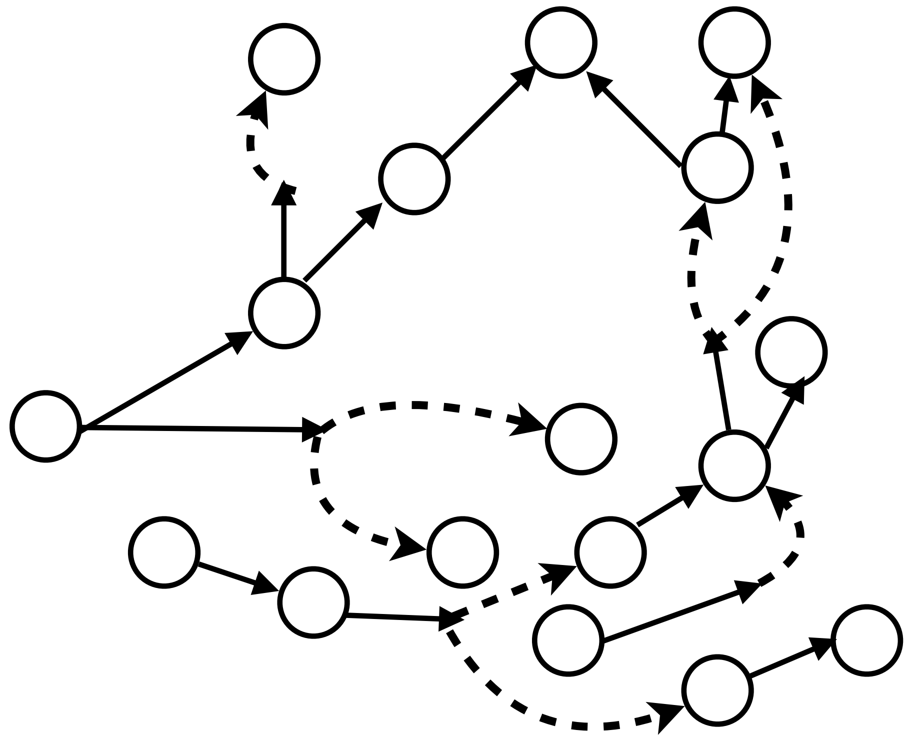
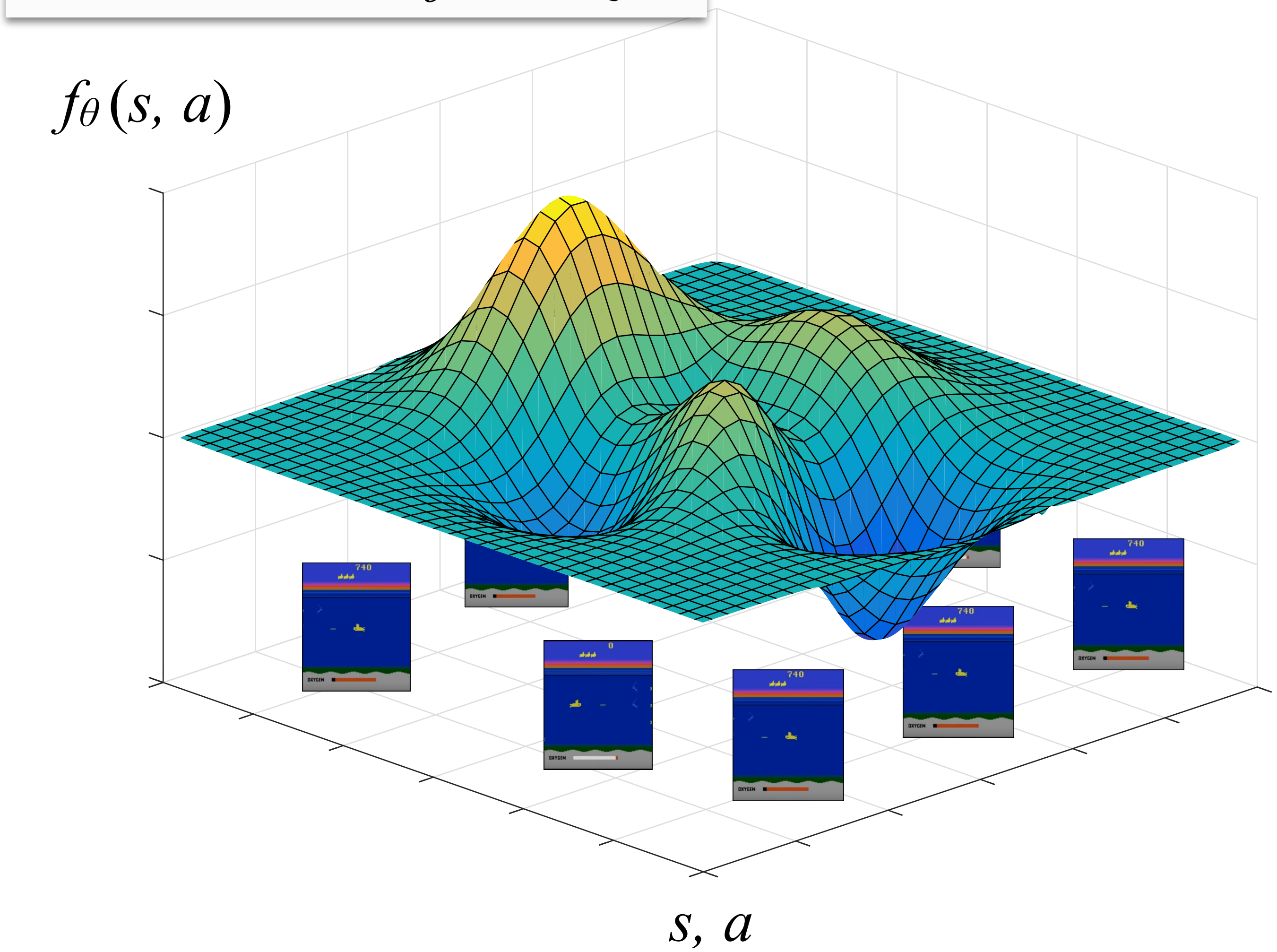
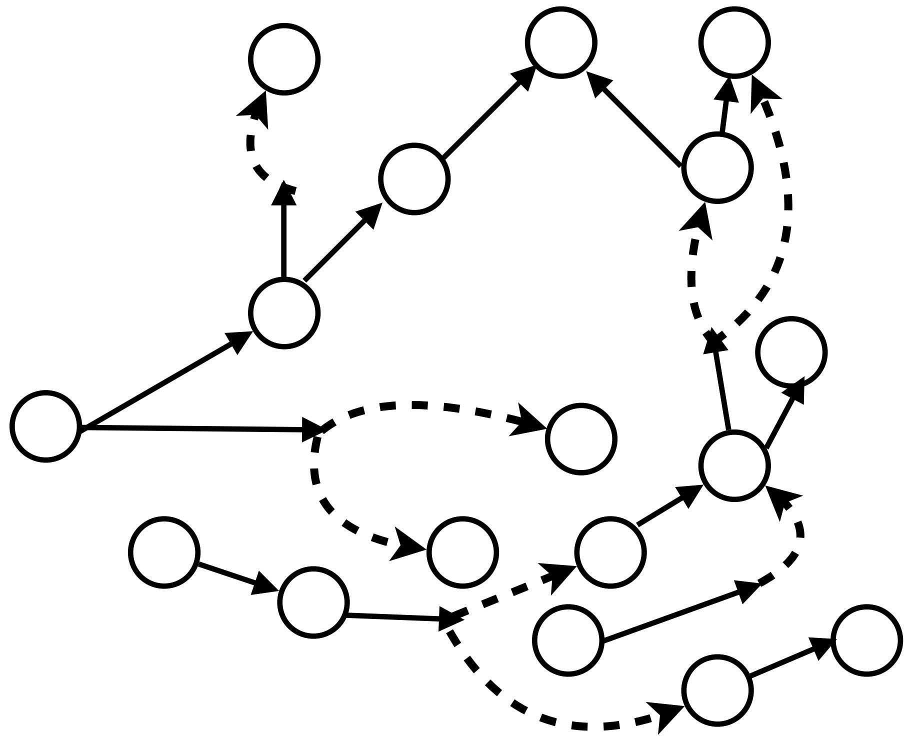




Validation:

(FQE: learn Q^π)

$$f_k \leftarrow \arg\min_{f_\theta} \mathbb{E}_D \left[\left(f_\theta(s,a) - r - \gamma f_{k-1}(s', \pi) \right)^2 \right]$$



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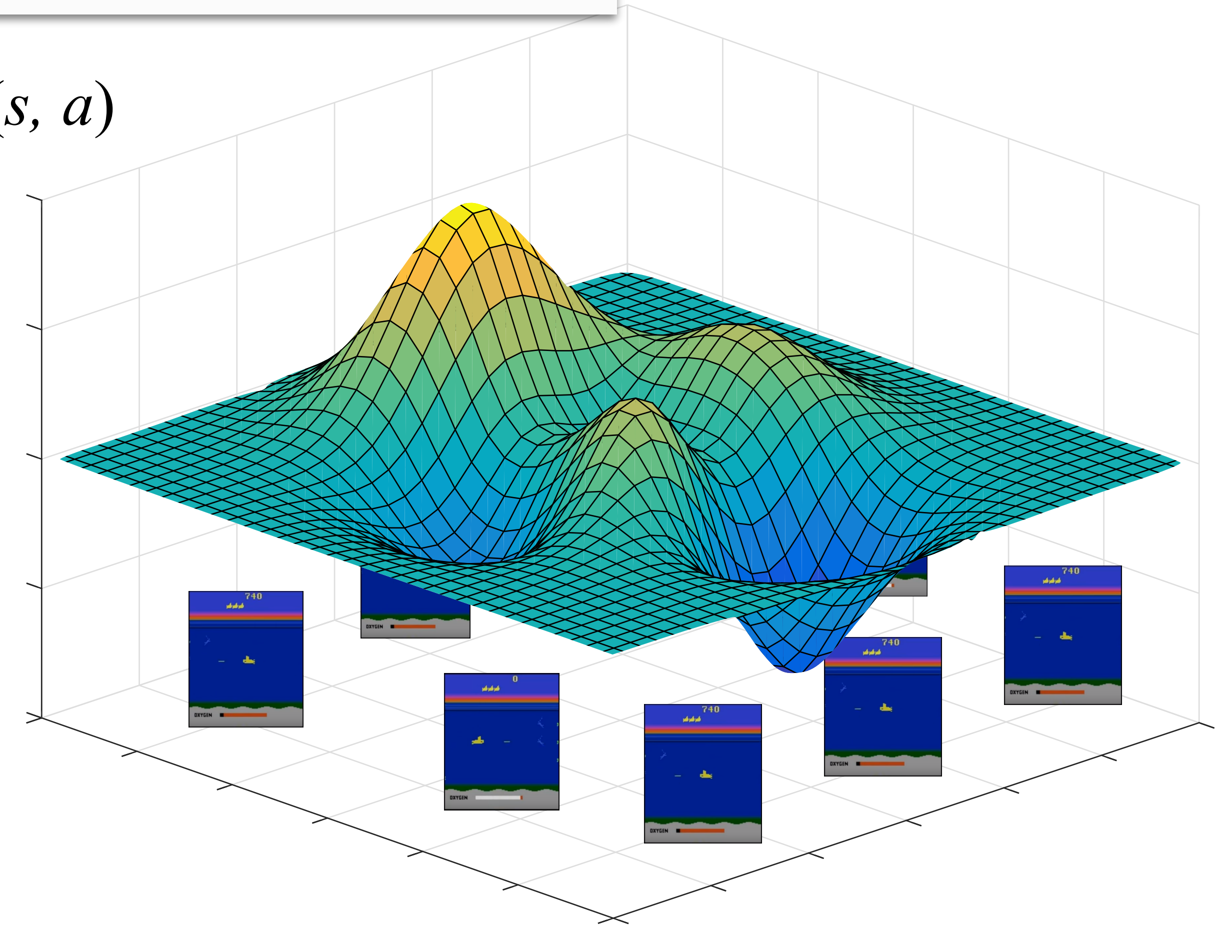

Validation:

(FQE: learn Q^π)

iterative

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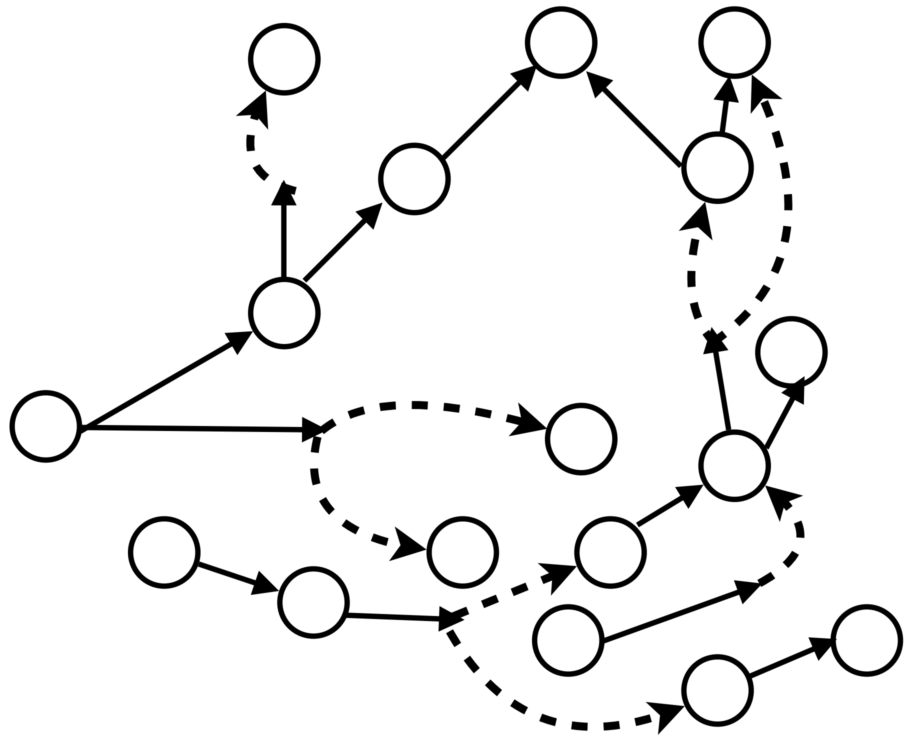

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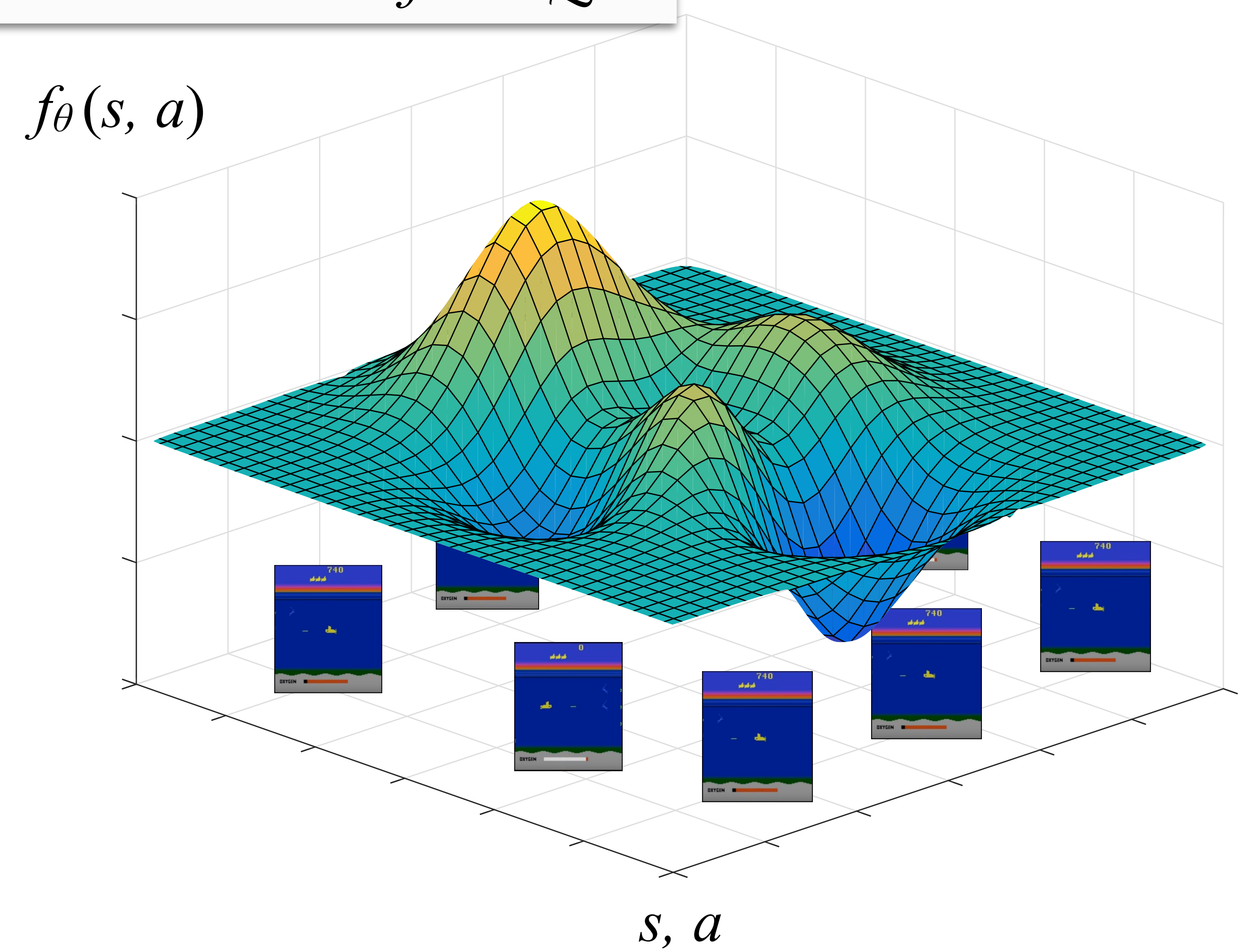

$(s, a, r, s') \sim D$

Validation:

(FQE: learn Q^π)

iterative

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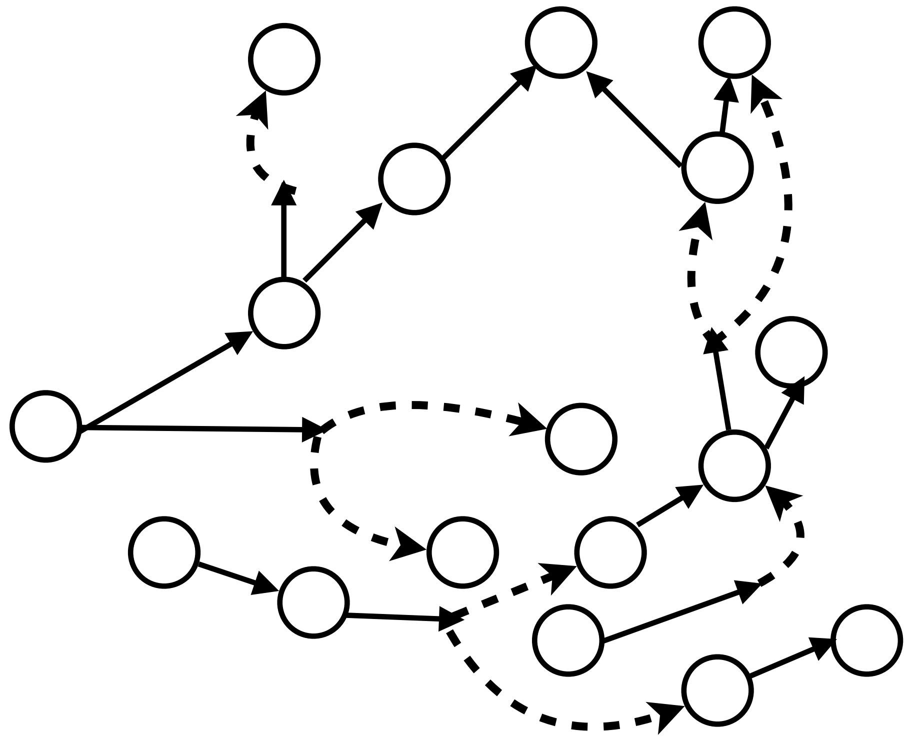

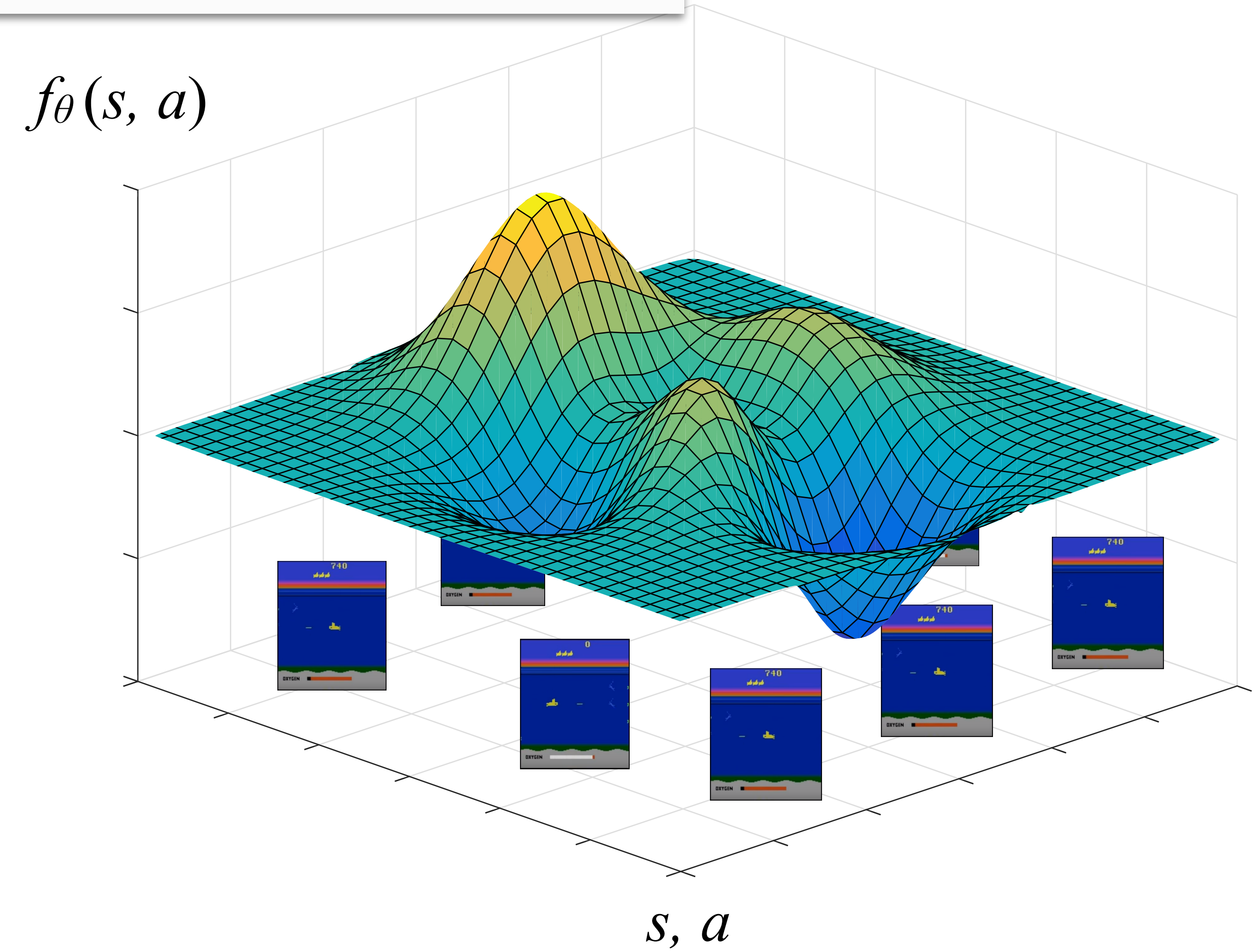

$$s_0, a_0, r_0, s_1, a_1, r_1, s_2, a_2, r_2, s_3, \ldots$$

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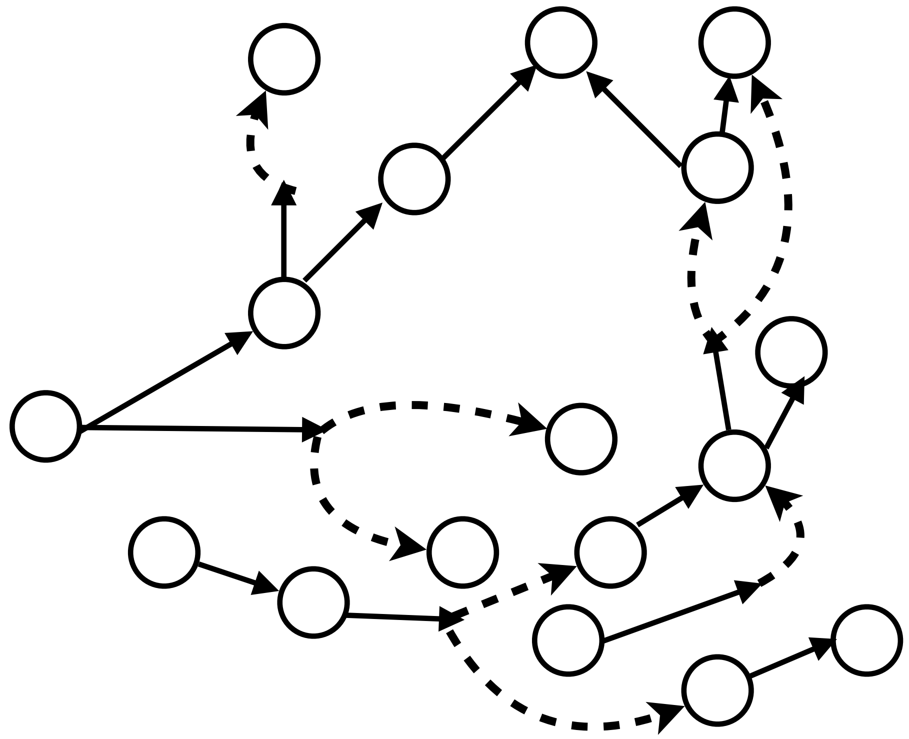

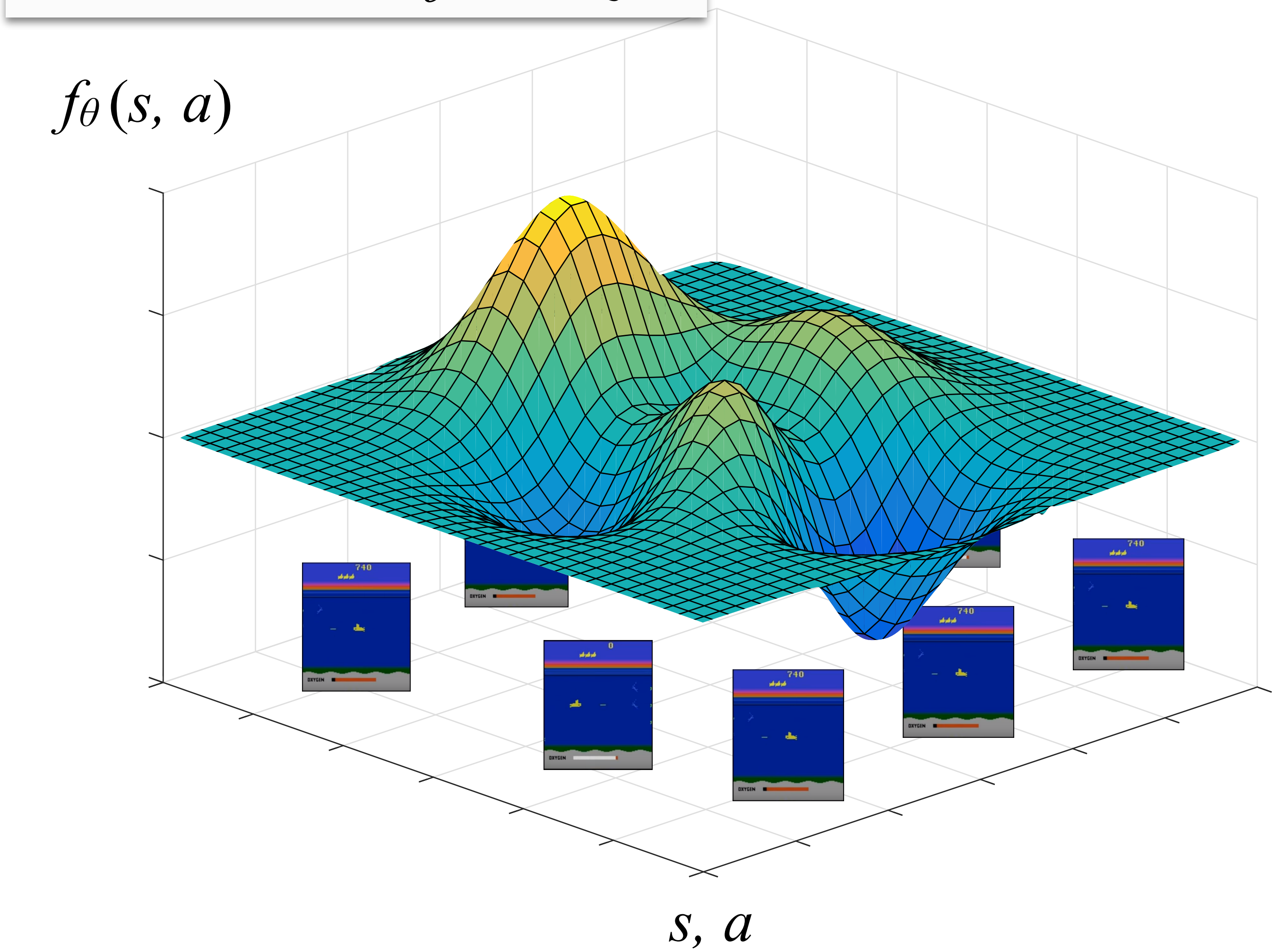

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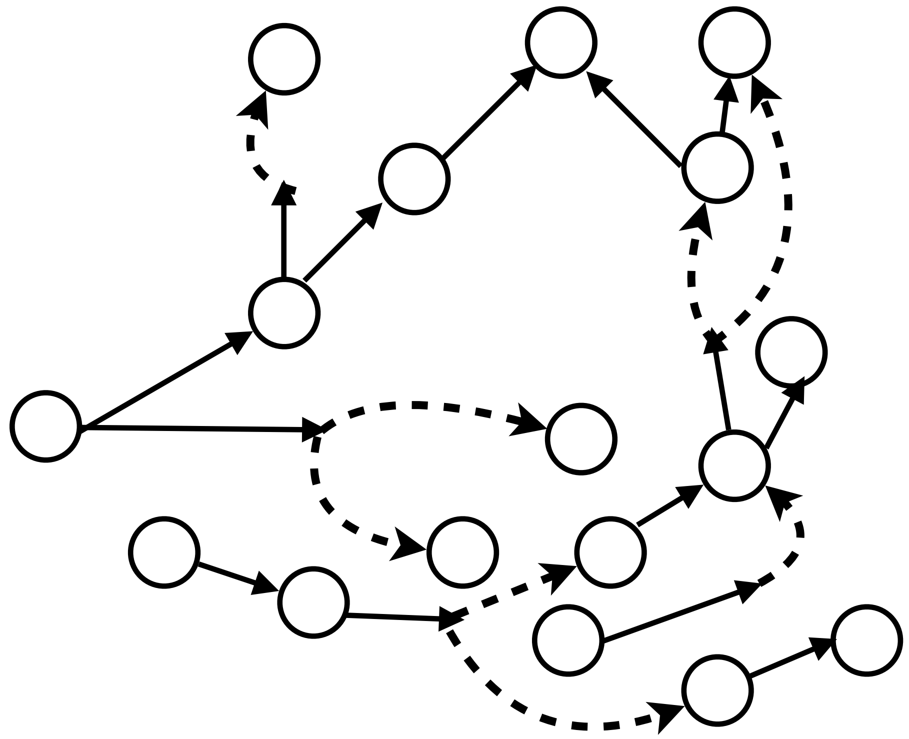

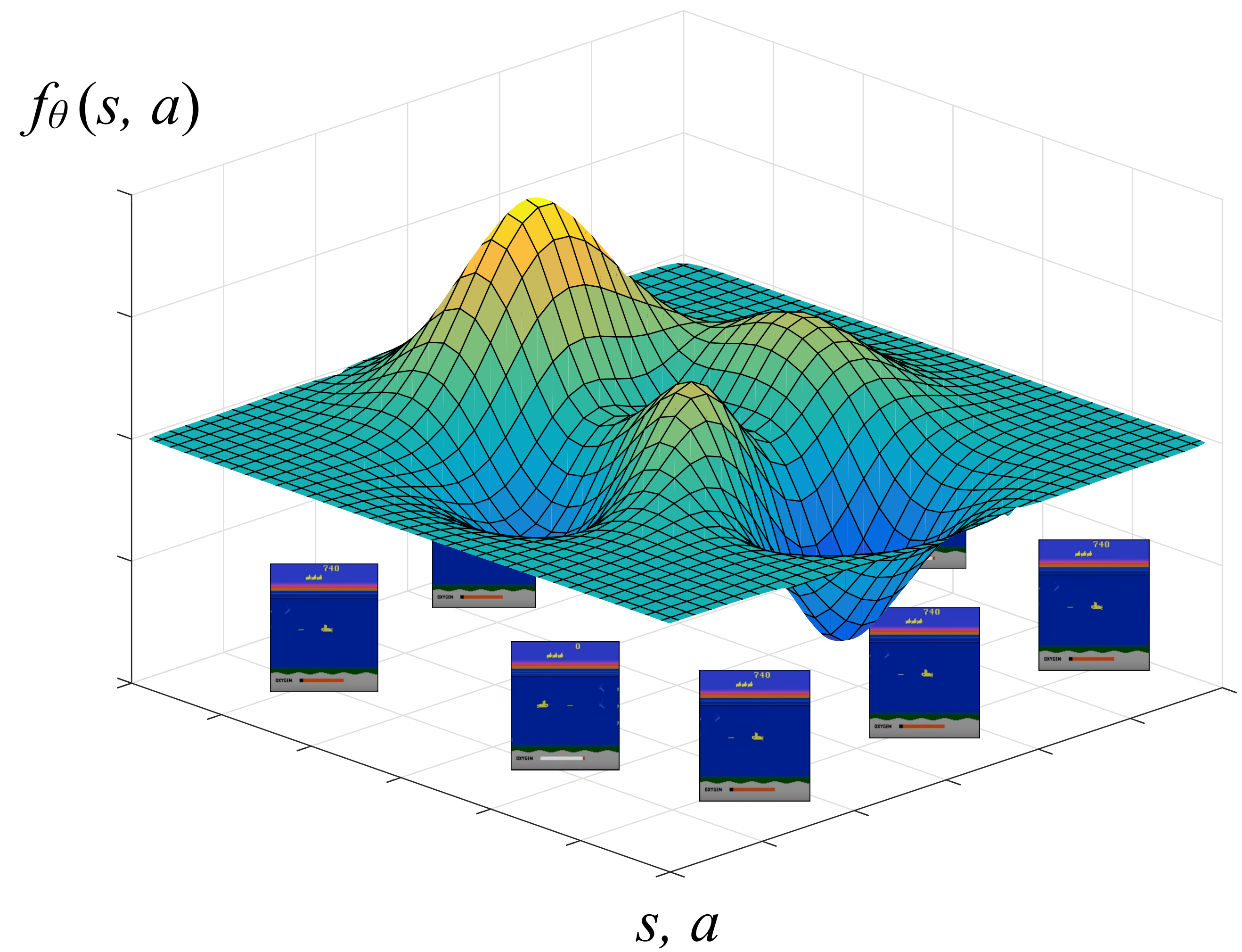

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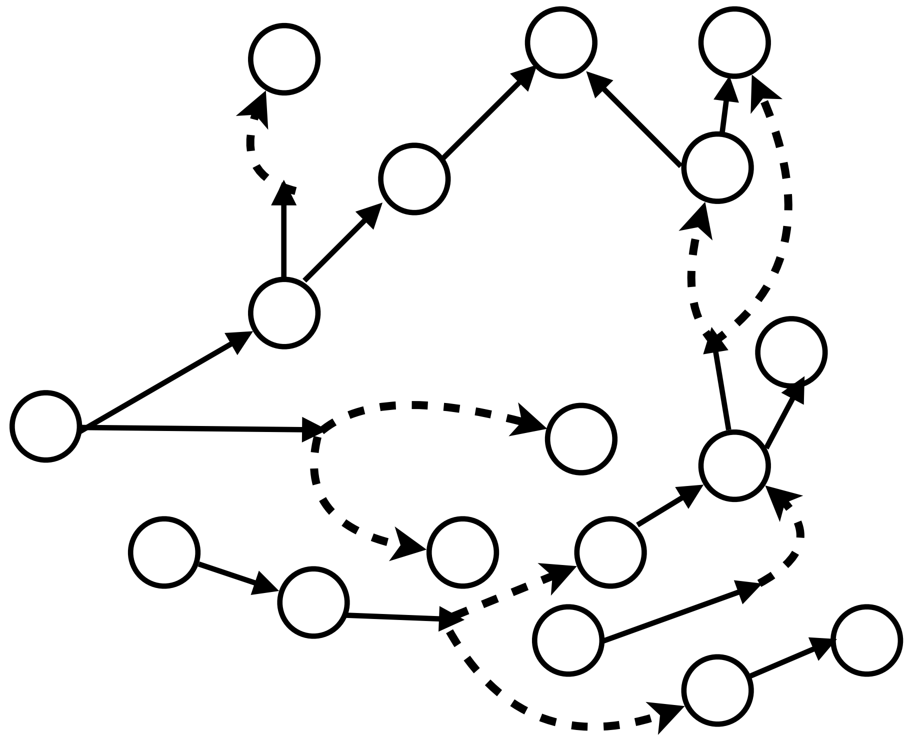

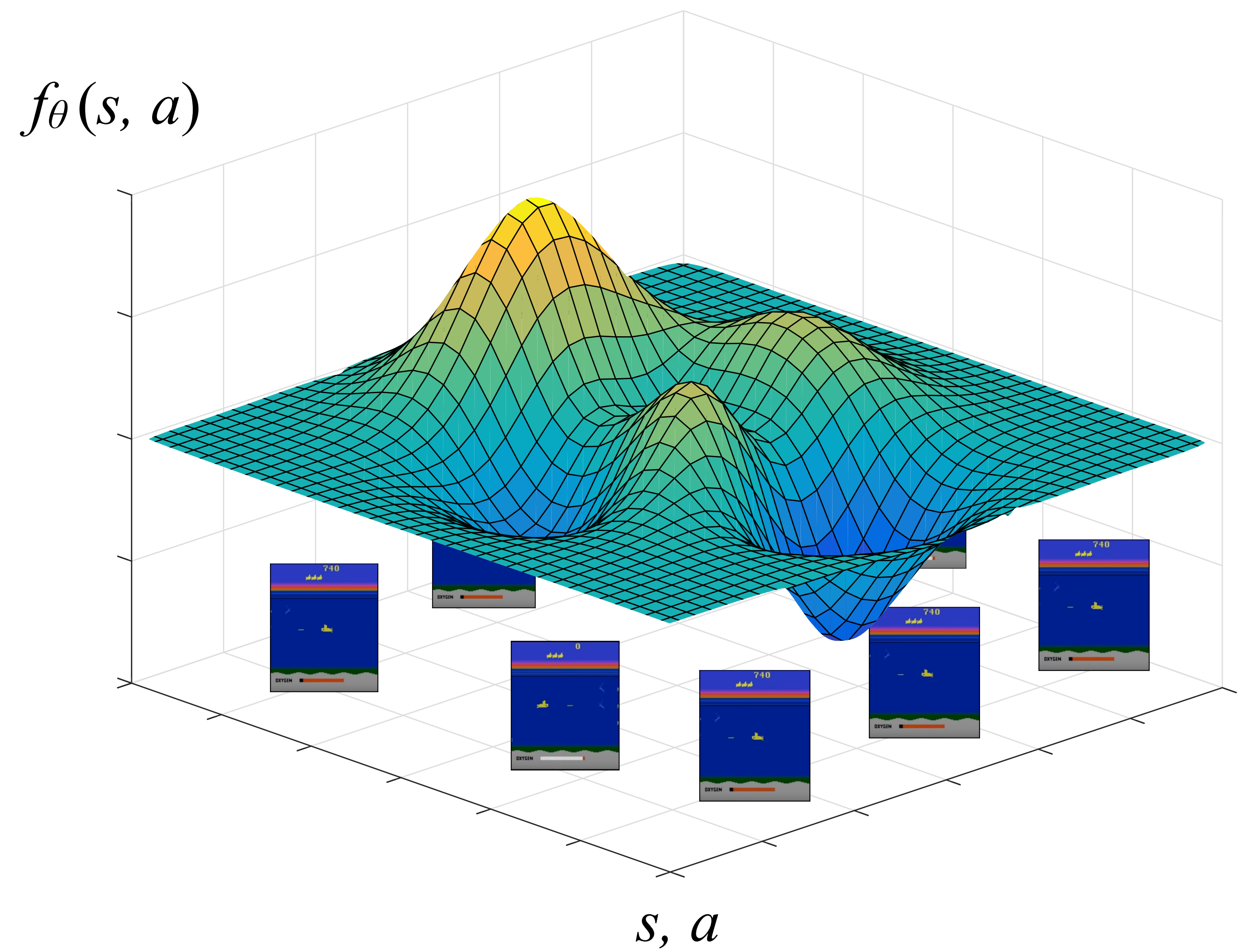

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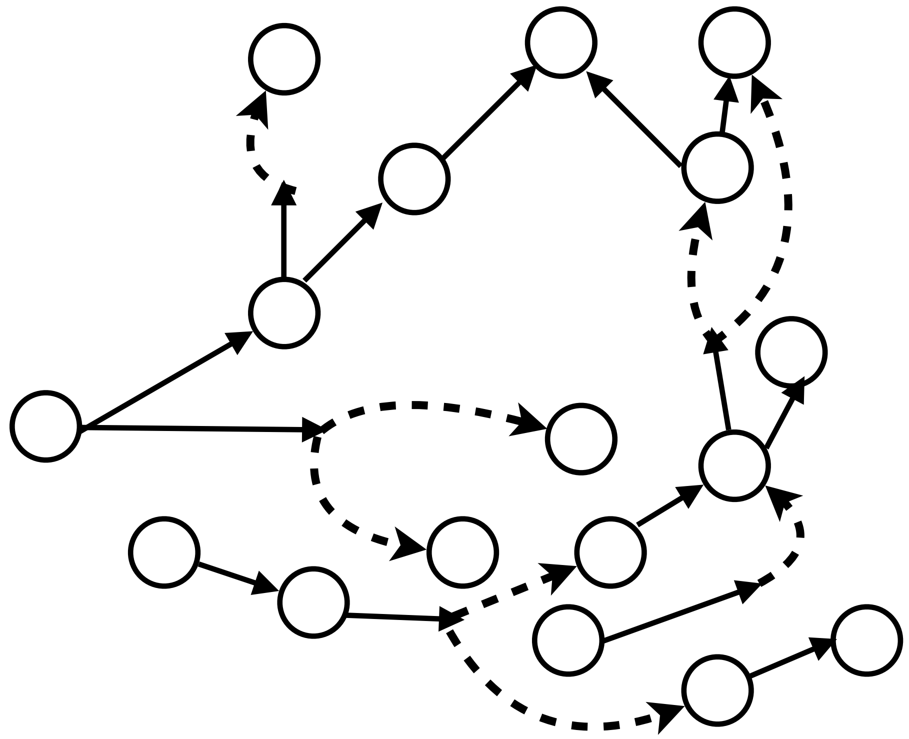

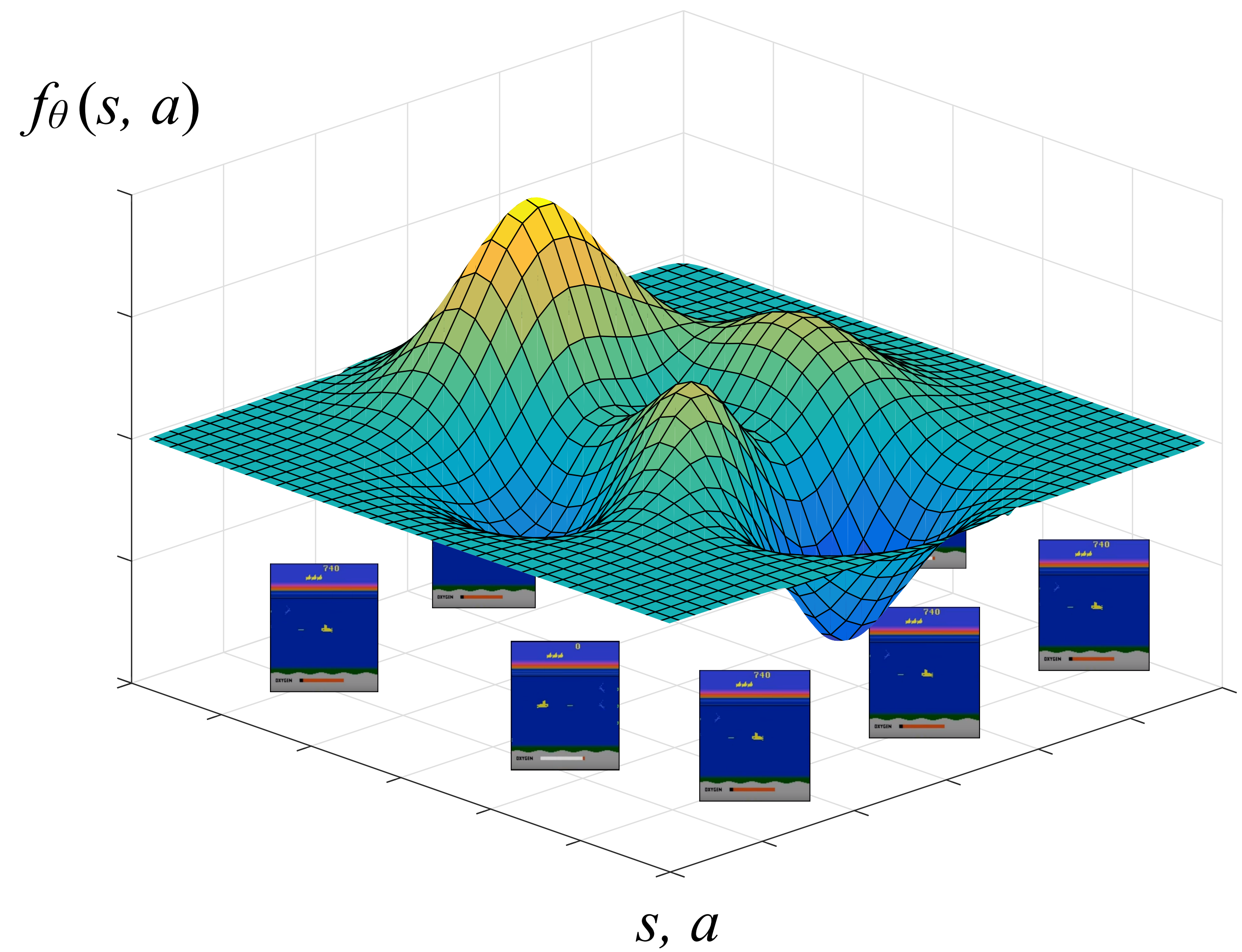

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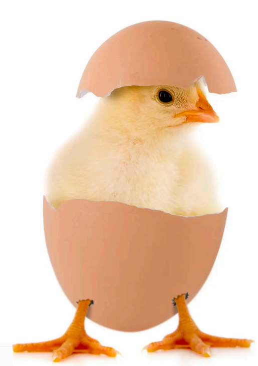

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Importance sampling [Precup'00]

- Hyperparameter-free ✓
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Precup. 2000. Eligibility traces for off-policy policy evaluation.

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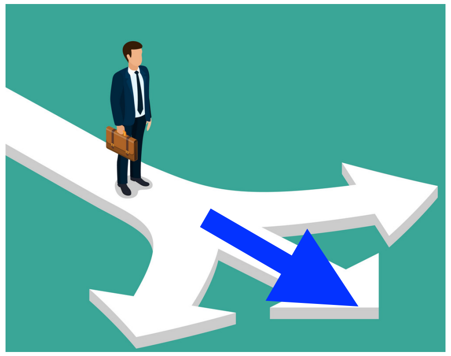

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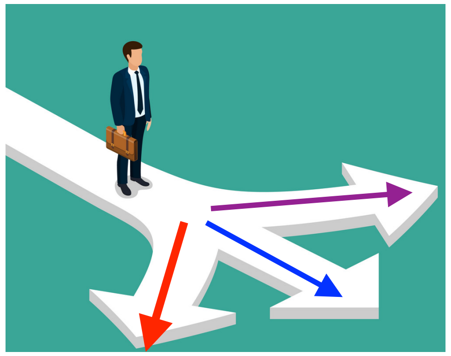

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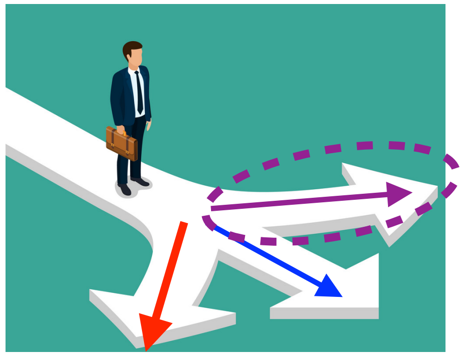

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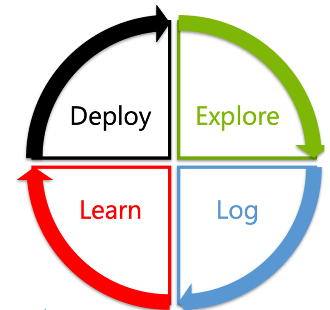

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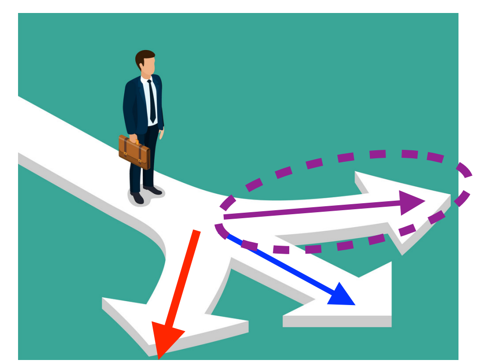

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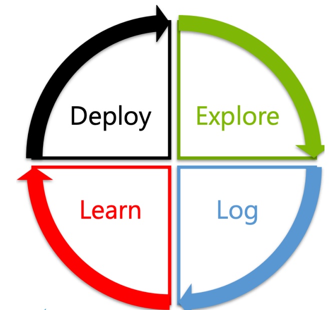

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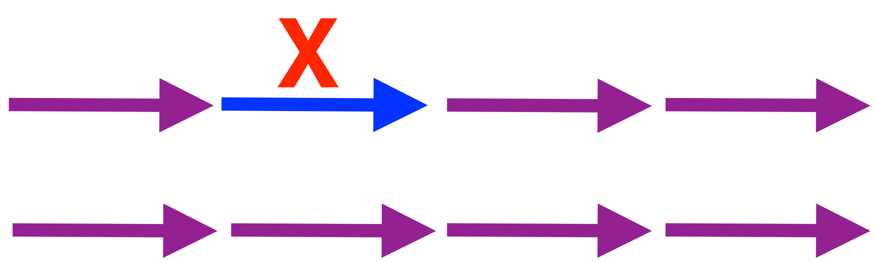

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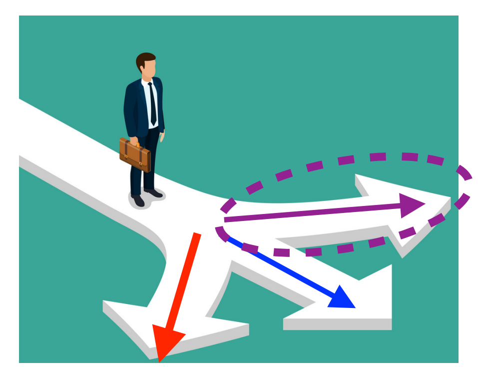

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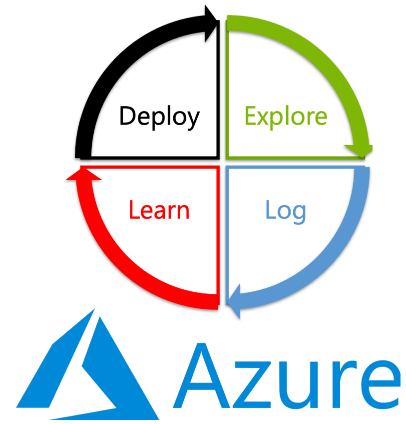

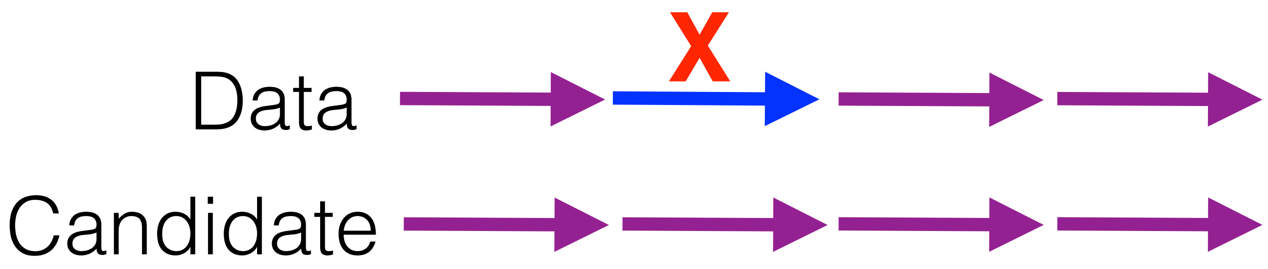

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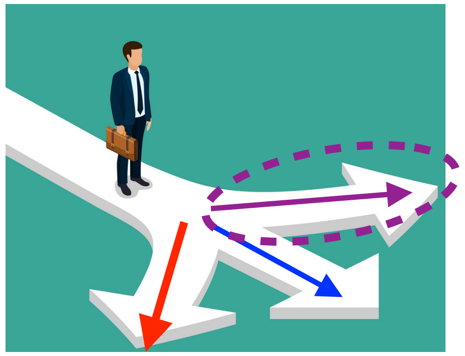

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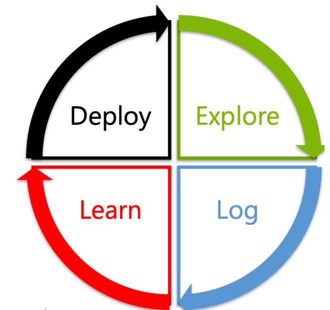

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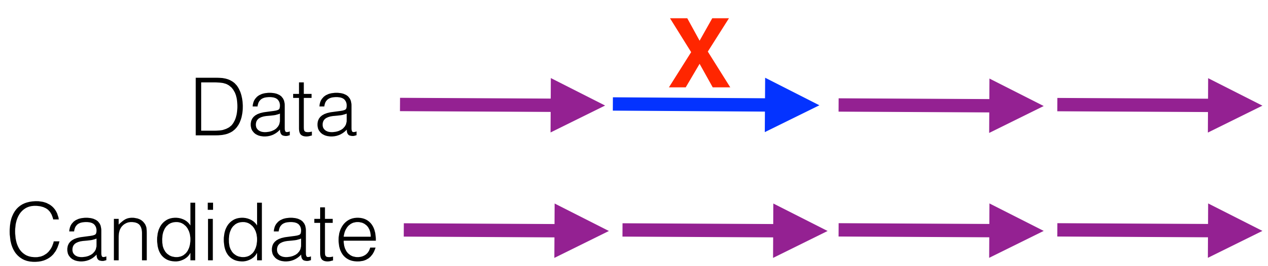

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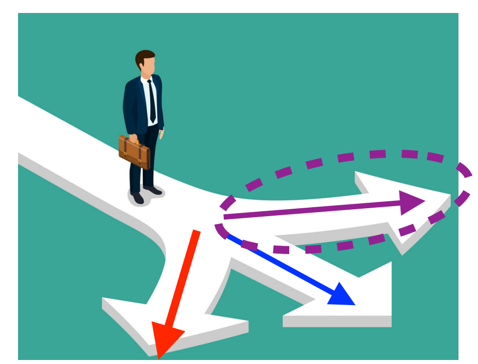
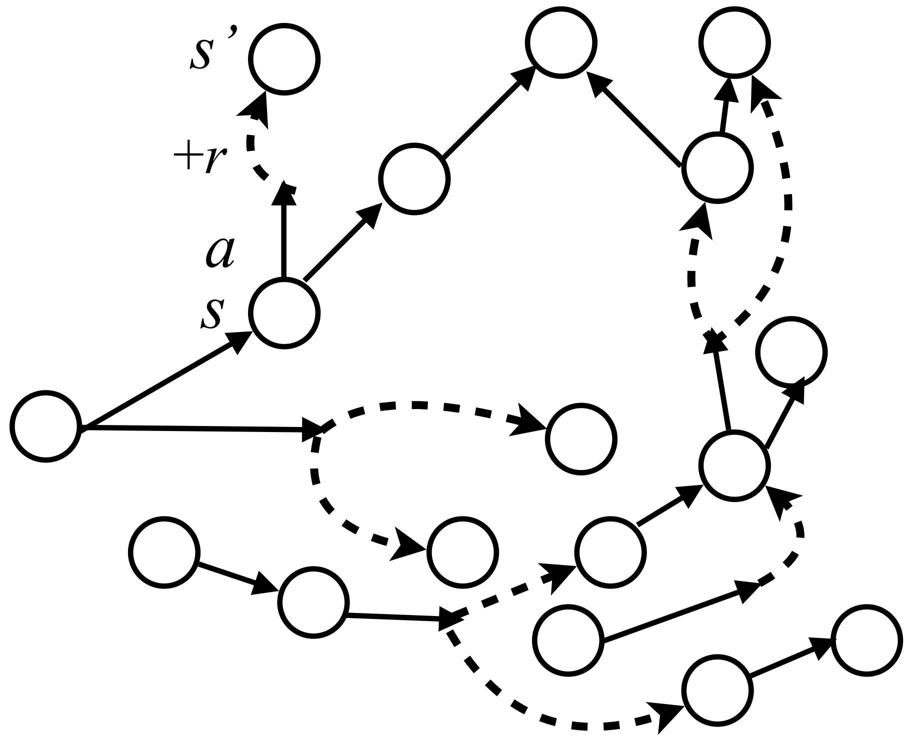


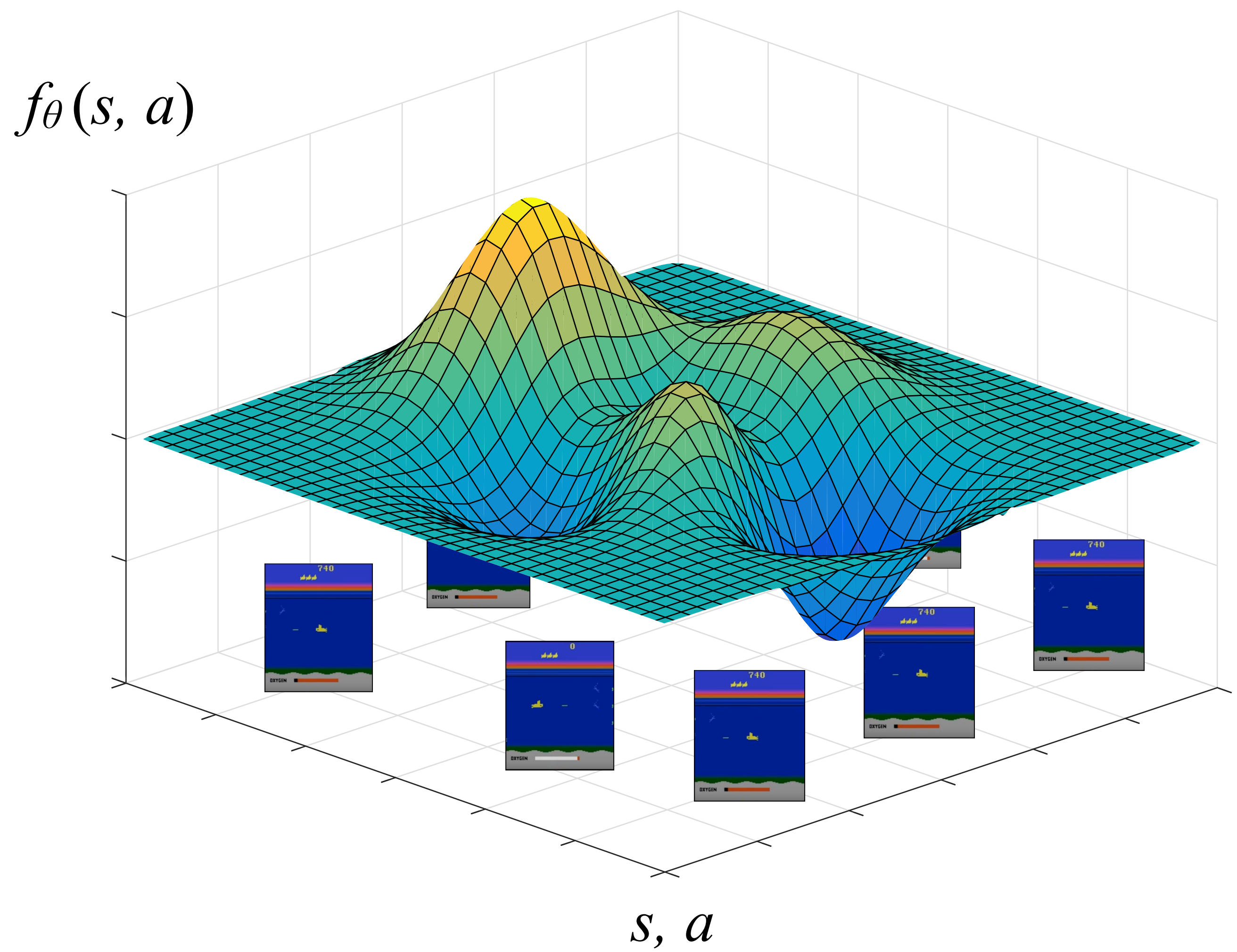
Candidate

Doubly robust [JL'16]

- Even perfect control variate cannot eliminate exponential variance!

Precup. 2000. Eligibility traces for off-policy policy evaluation.
Nan Jiang, Lihong Li. ICML-16. Doubly Robust Off-policy Value Evaluation for Reinforcement Learning.





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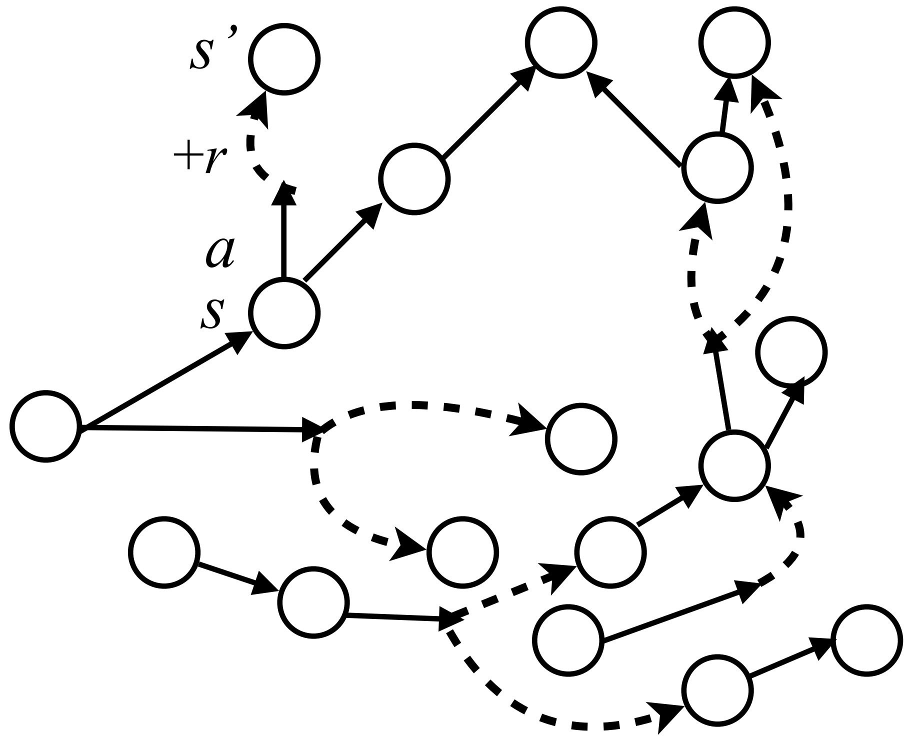

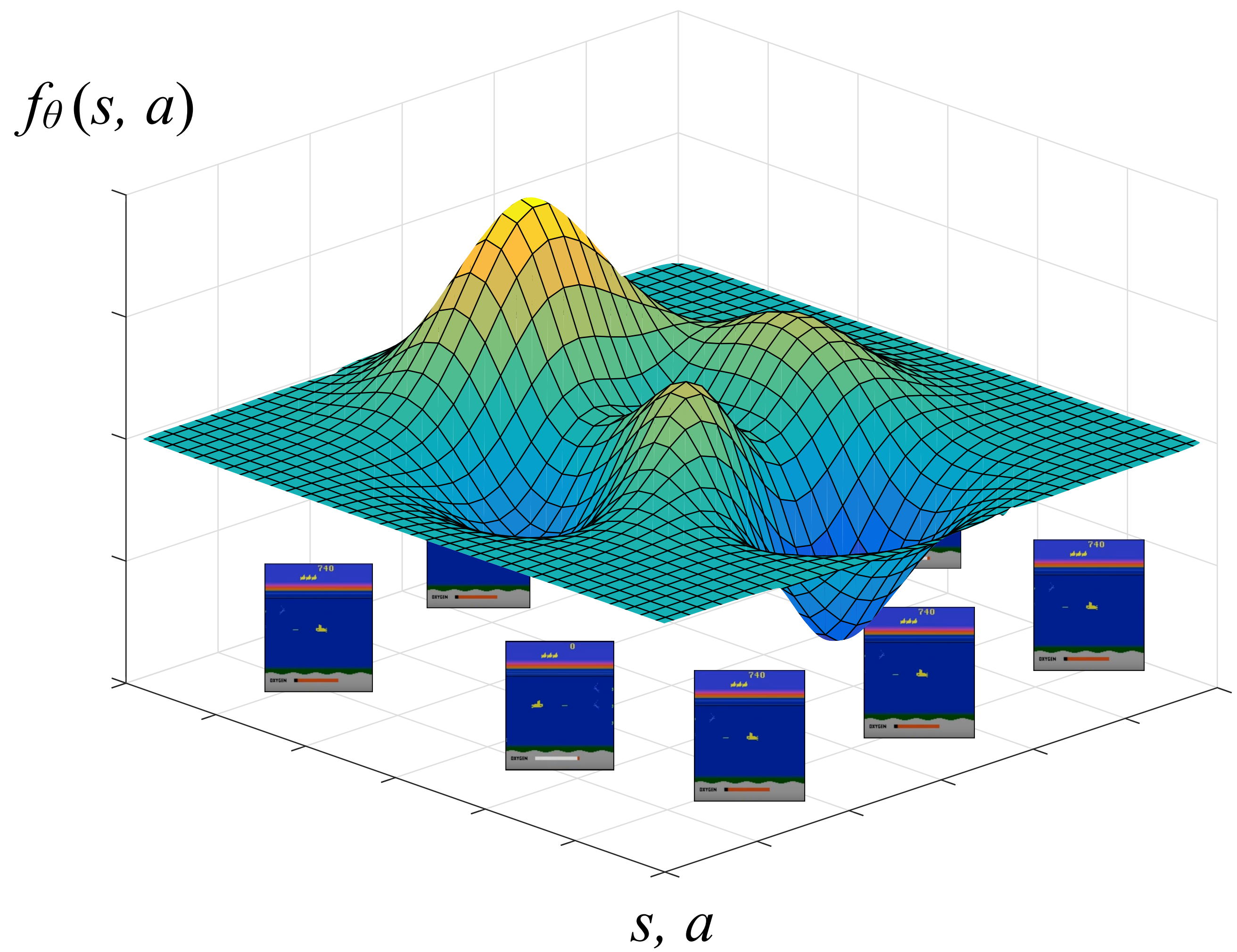

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Simple(?) Problem

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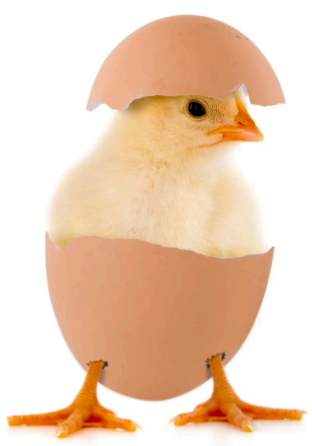

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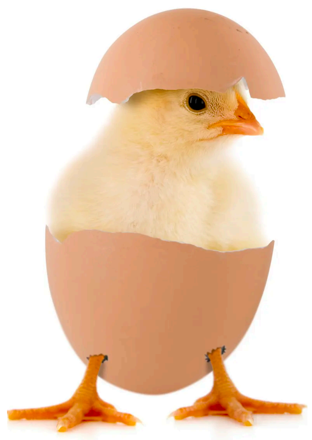

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- Baird'95: design L s.t. $Q^\star \overset{?}{=} \arg\min_{f \in \mathcal{F}} L(f)$

 Baird. 1995. Residual algorithms: Reinforcement learning with function approximation. * original paper concerns V^π

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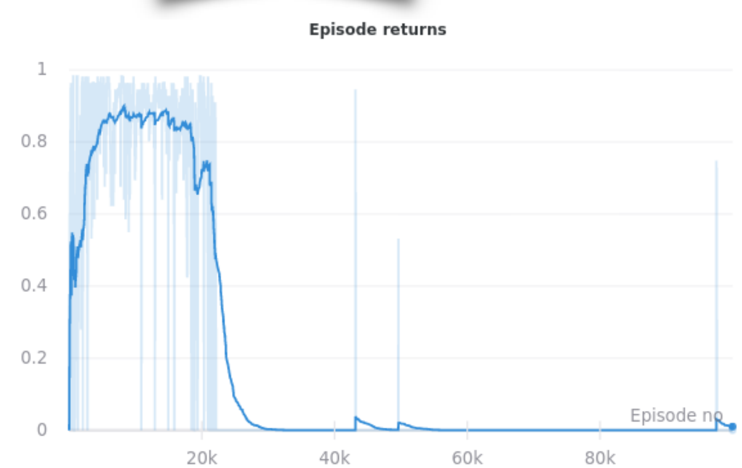

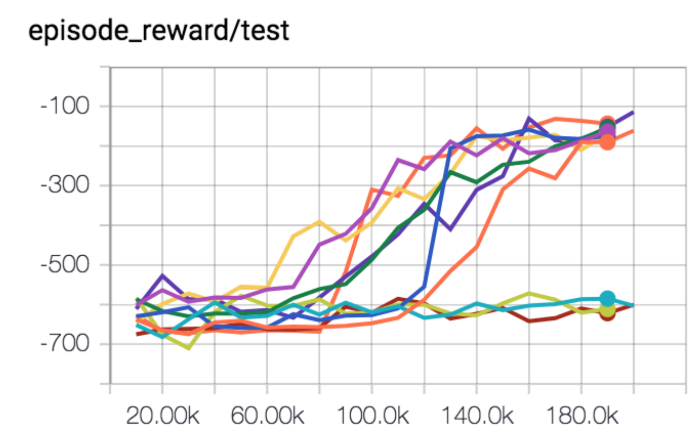

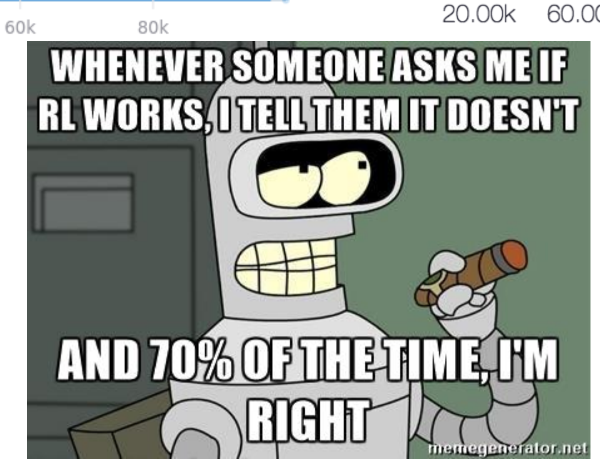

https://www.alexirpan.com/2018/02/14/rl-hard.html

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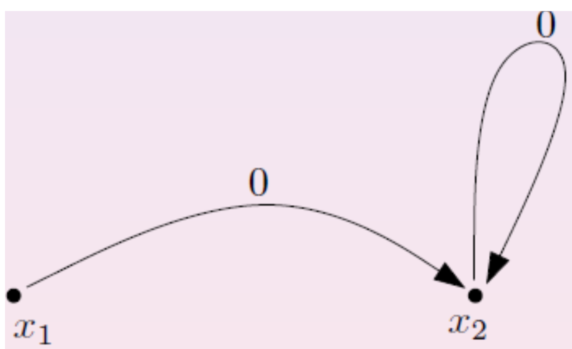
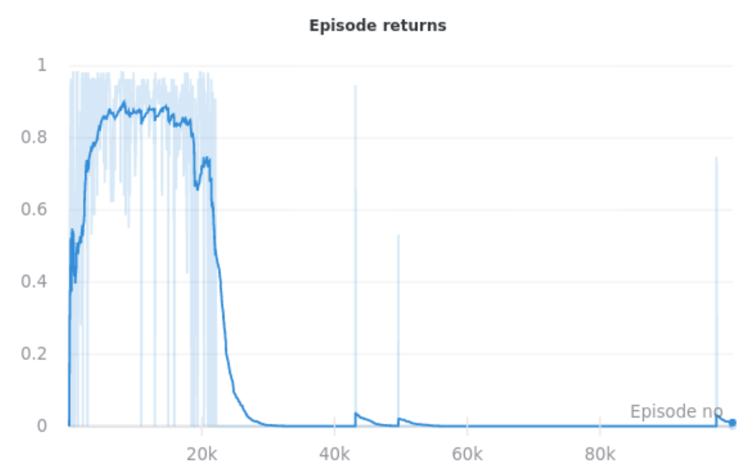
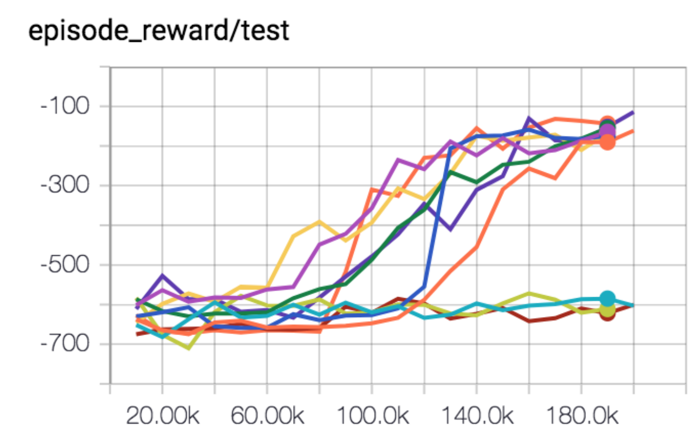
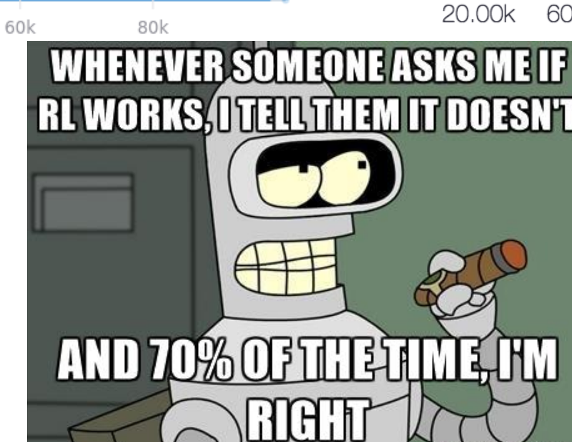


Divergence under 1-d linear
[TvR'96]





"Deadly triad"



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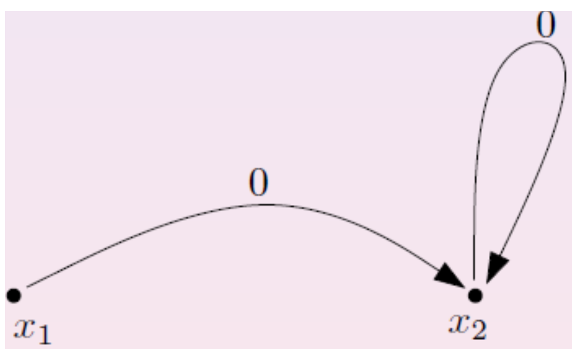

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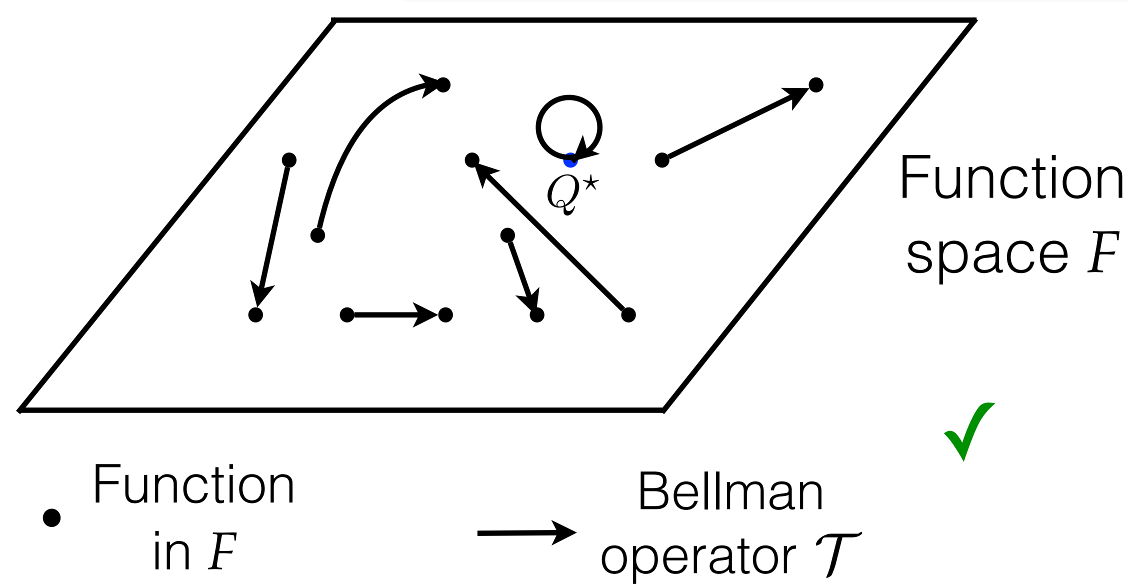

✓

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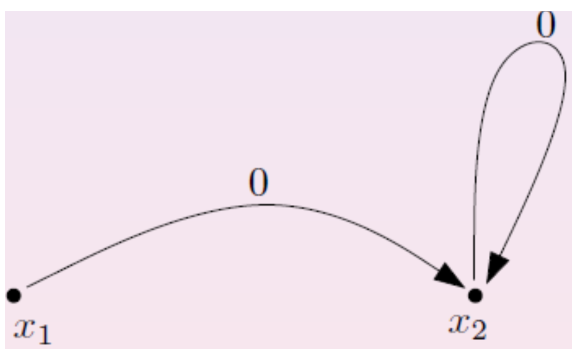
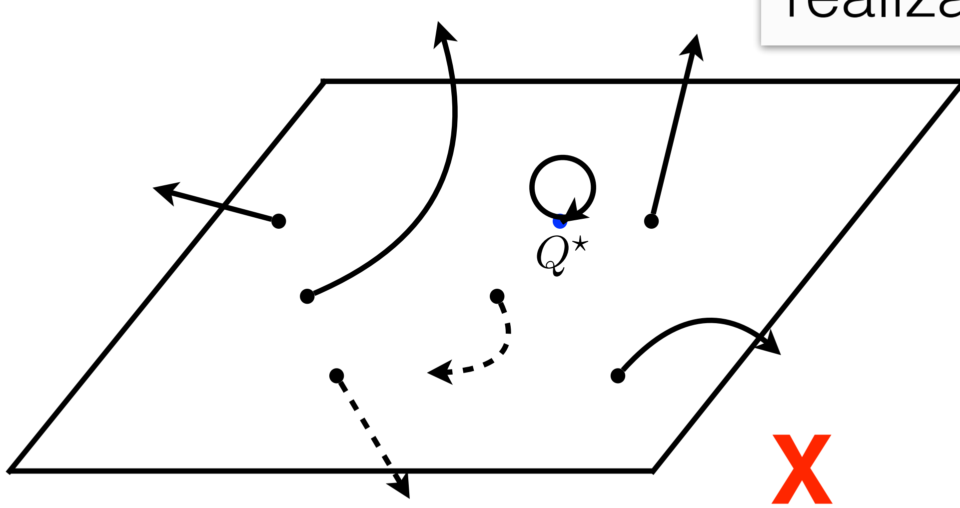


realizability (of Q^*)



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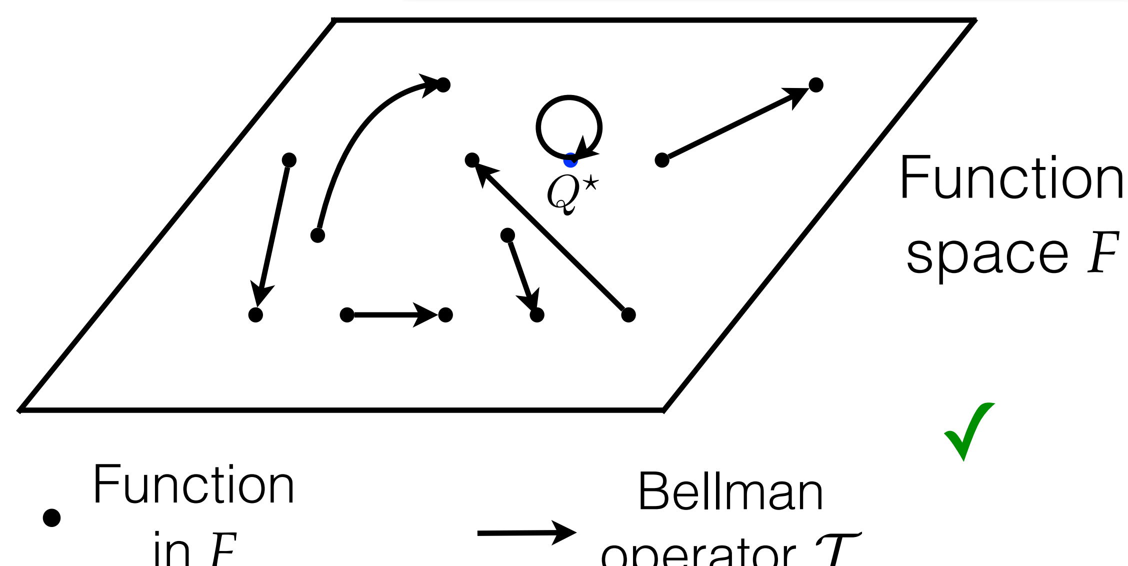

The training perspective

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- Naive "1-sample" estimator is biased

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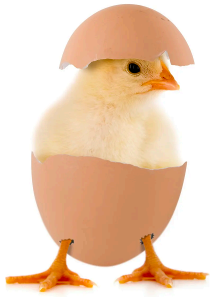
- Naive "1-sample" estimator is biased
  - debasing requires simulator ("*double sampling*" [Baird'95])
  - or, helper class $\mathcal{F}' \ni \mathcal{T}f$ [ASM'08, FS'10]



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Basis of resolution

Our goal

To select b/t f_1, f_2

Gordan. 1995. Stable Function Approximation in Dynamic Programming.

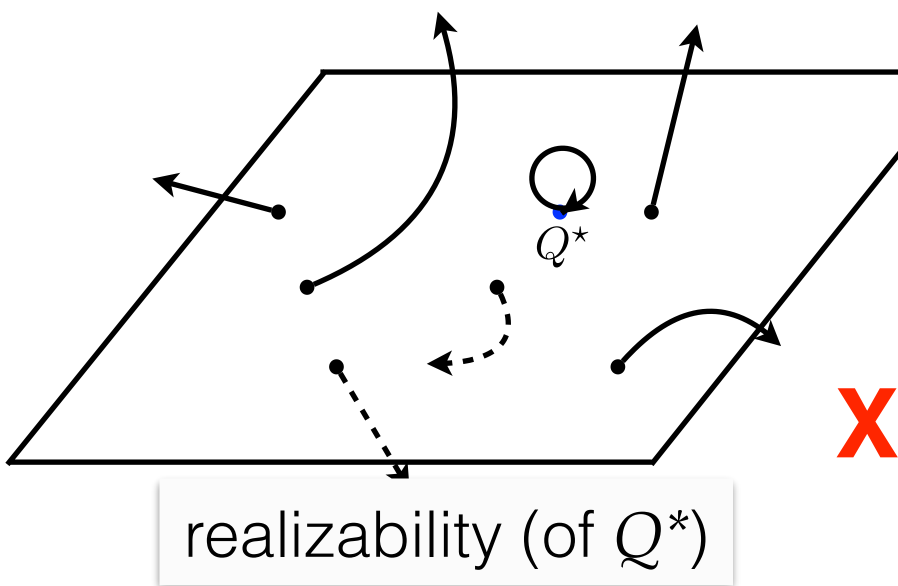
Basis of resolution

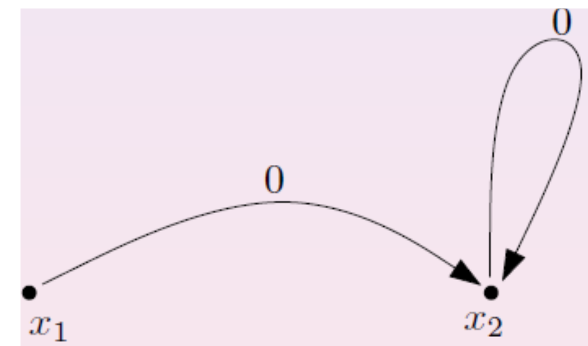
Training: $\hat{f} = f_k$ where

(FQI: learn Q^*)

$$f_k \leftarrow \arg\min_{f_\theta \in \mathcal{F}} \mathbb{E}_D \left[(f_\theta(s,a) - r - \gamma \max_{a'} f_{k-1}(s',a'))^2 \right]$$

iterative





Divergence under 1-d linear $\mathcal{F}$

[TvR'96]

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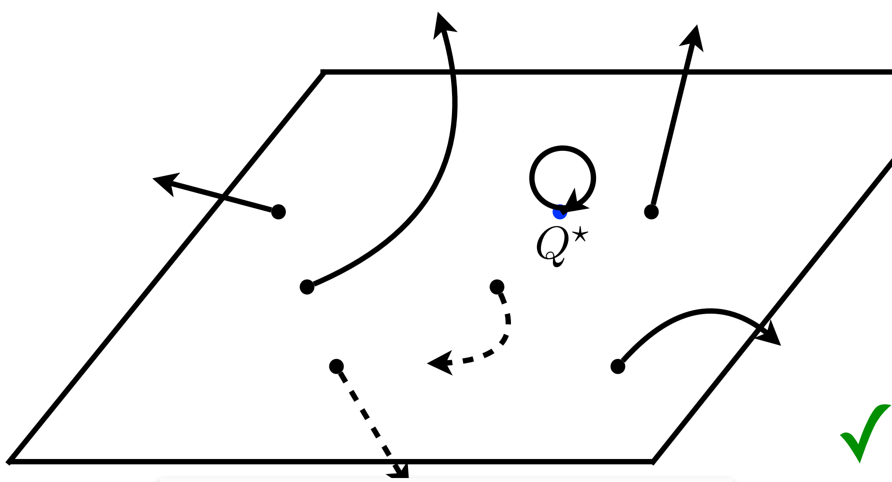
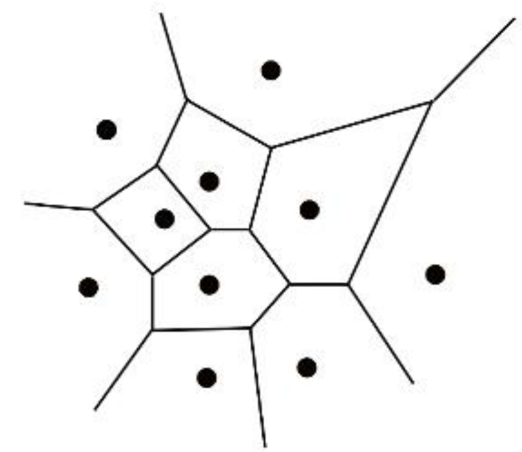


realizability (of Q^*)



Convergence under piecewise constant $\mathcal{F}$! [Gordon'95]

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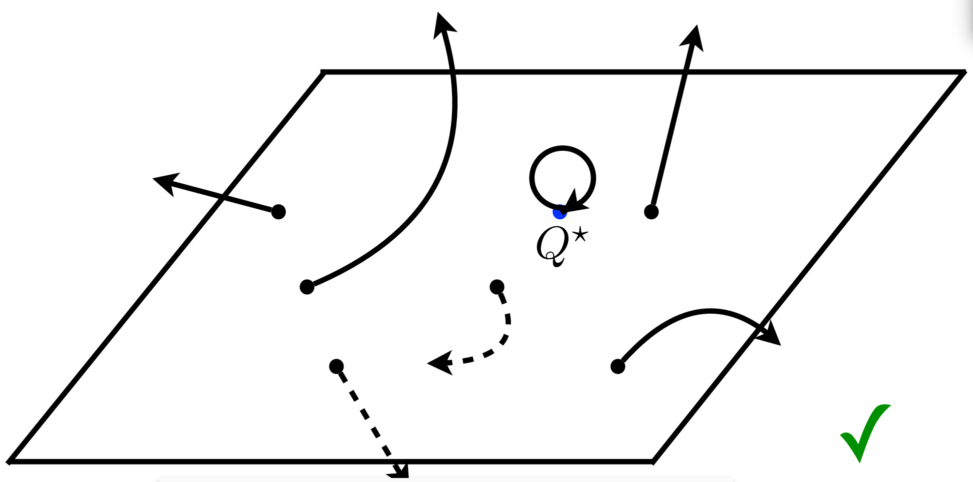

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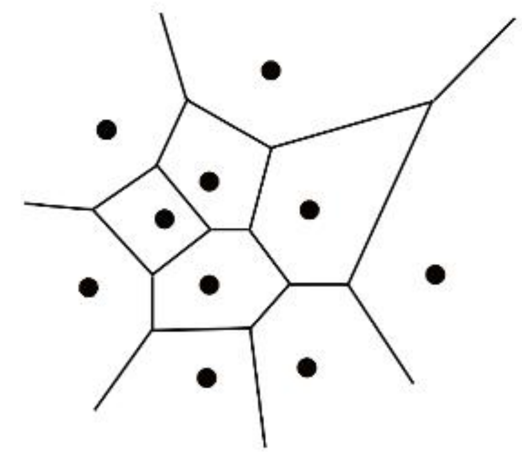

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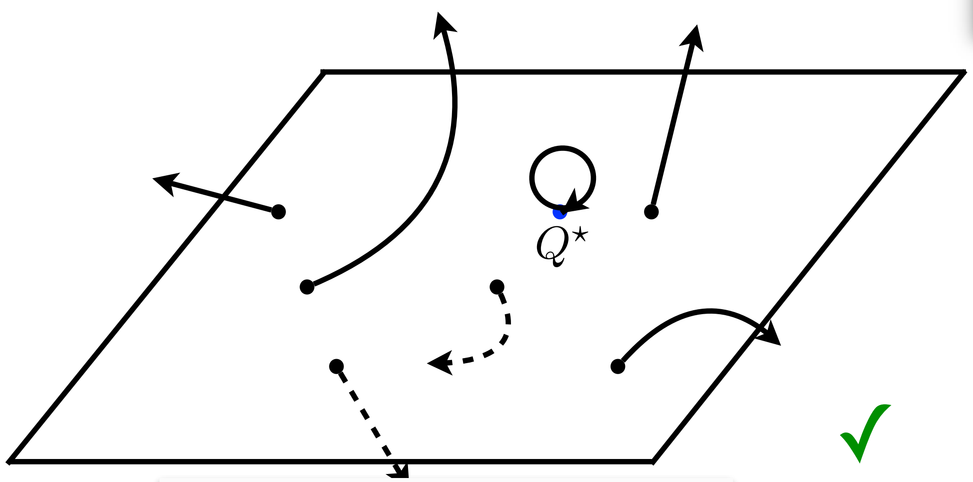

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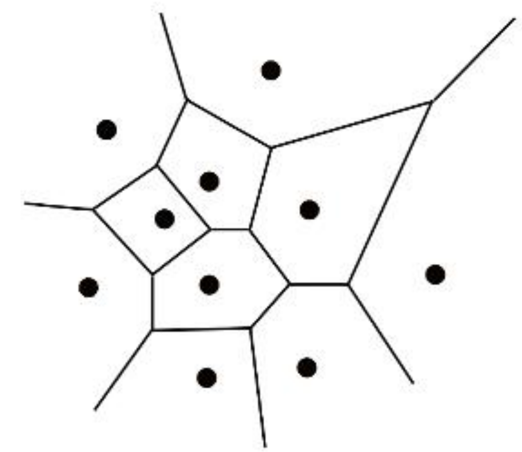

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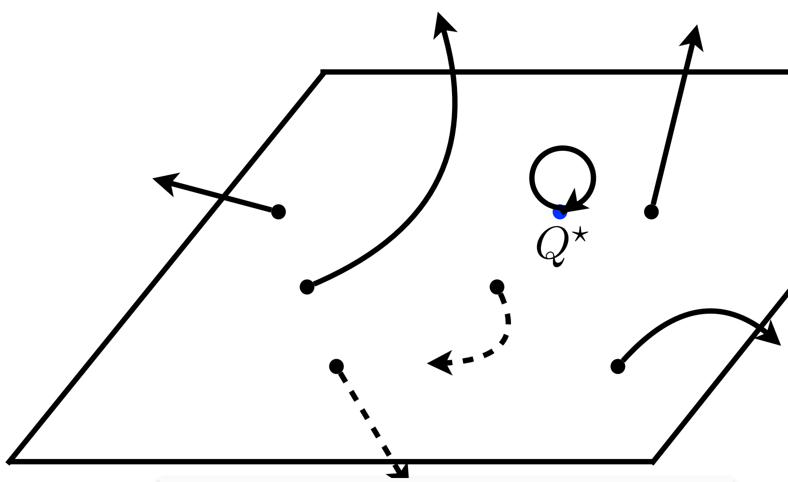

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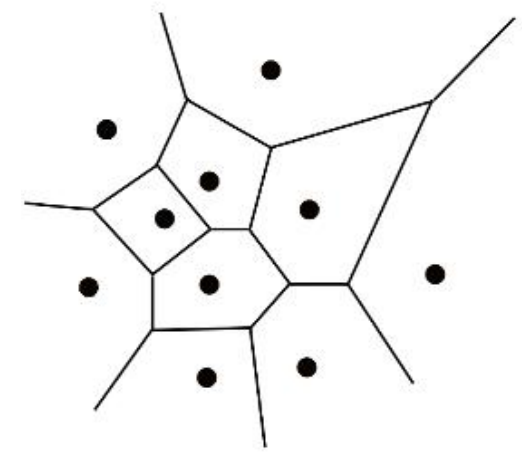

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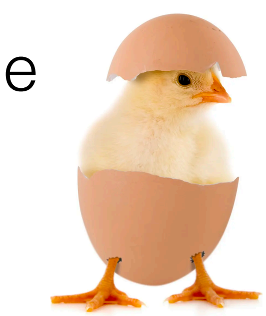

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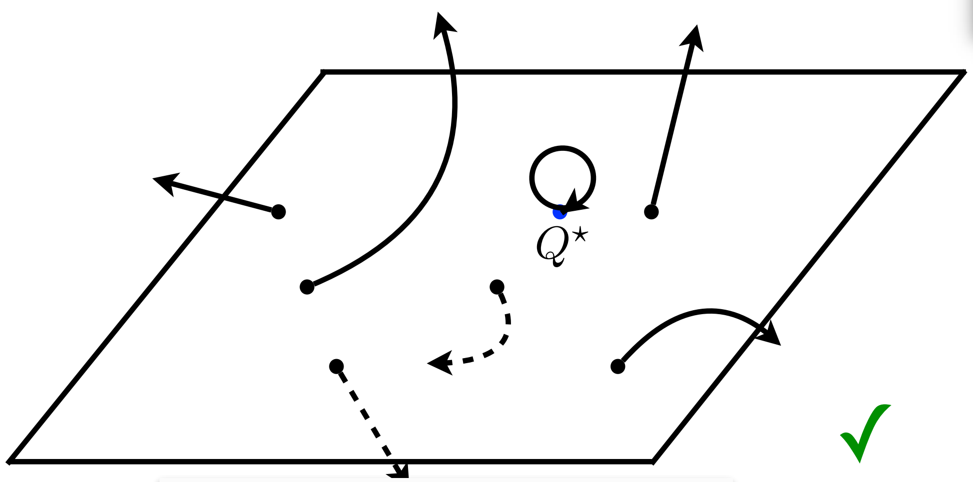

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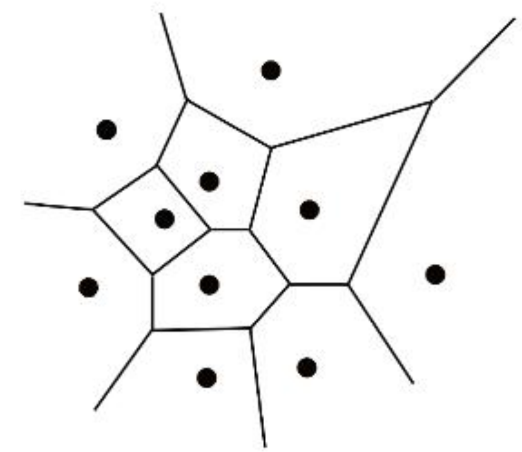

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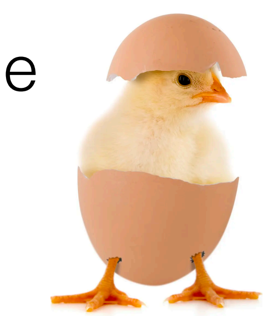

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Our method: create such a magical G "out of nothing"!

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Does a magical G always exist?

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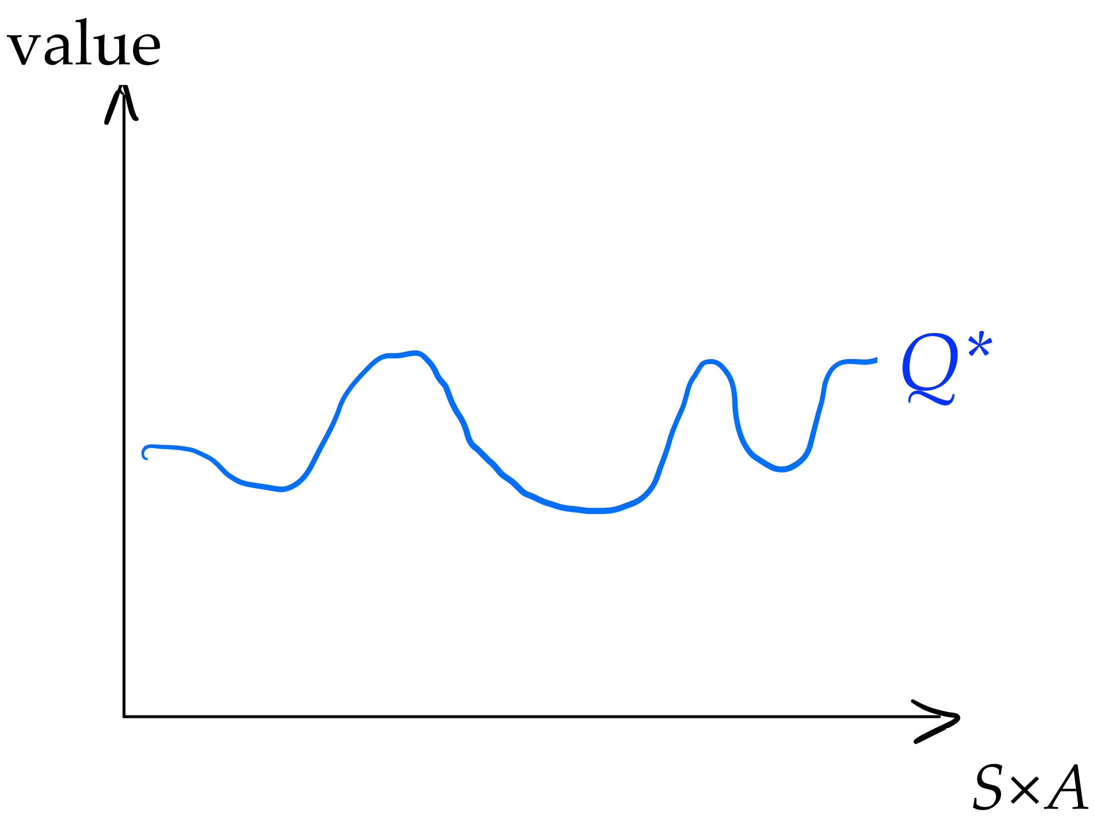
Does a magical G always exist?

- YES! Just partition SxA according to **output** of Q^*



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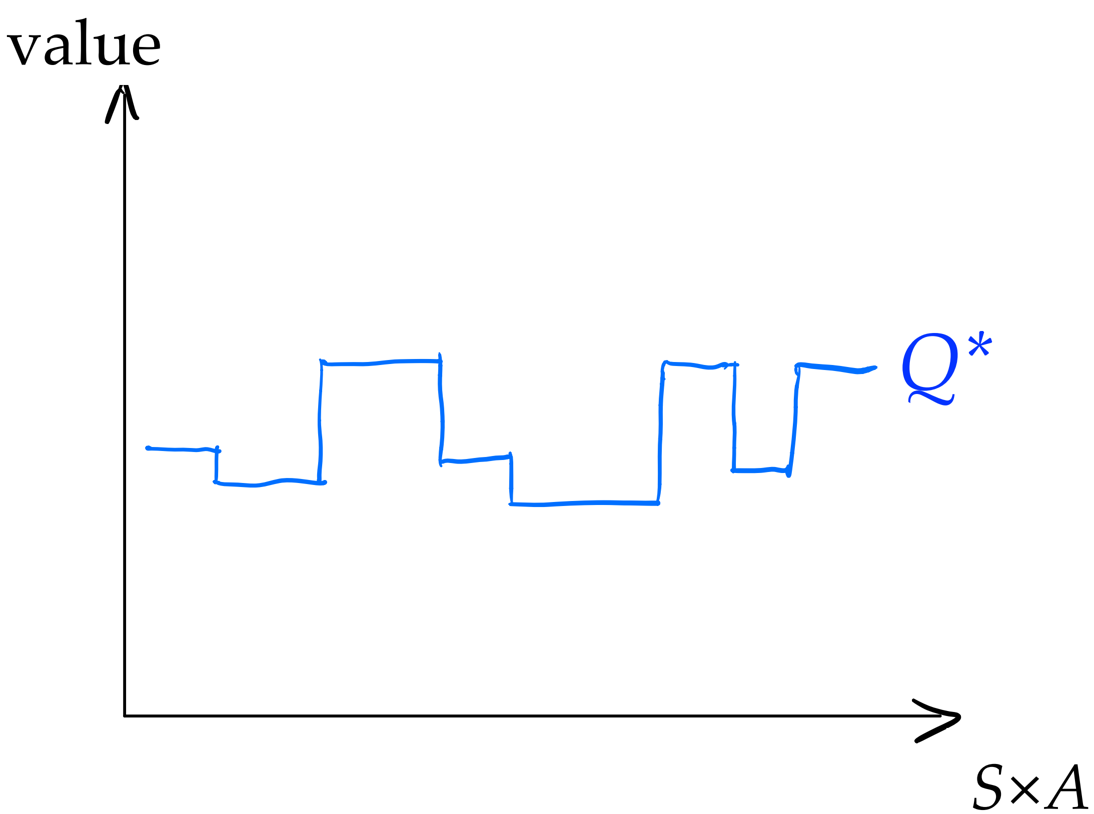

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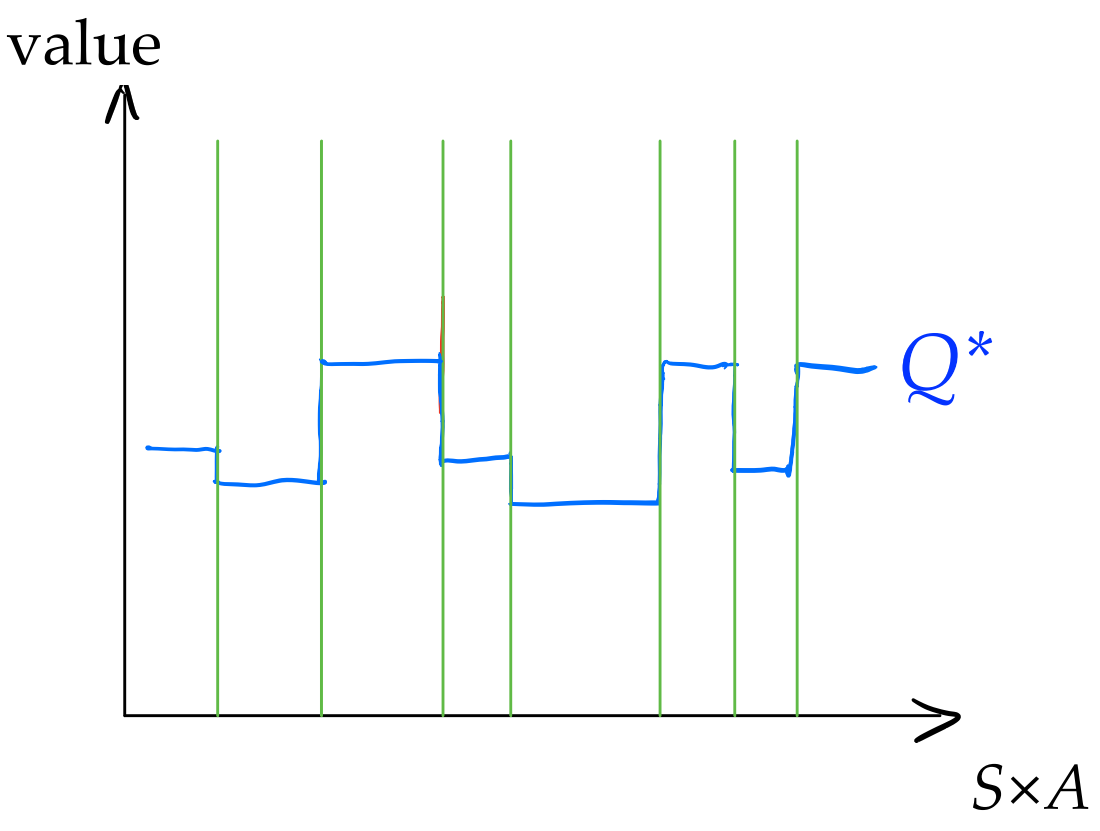

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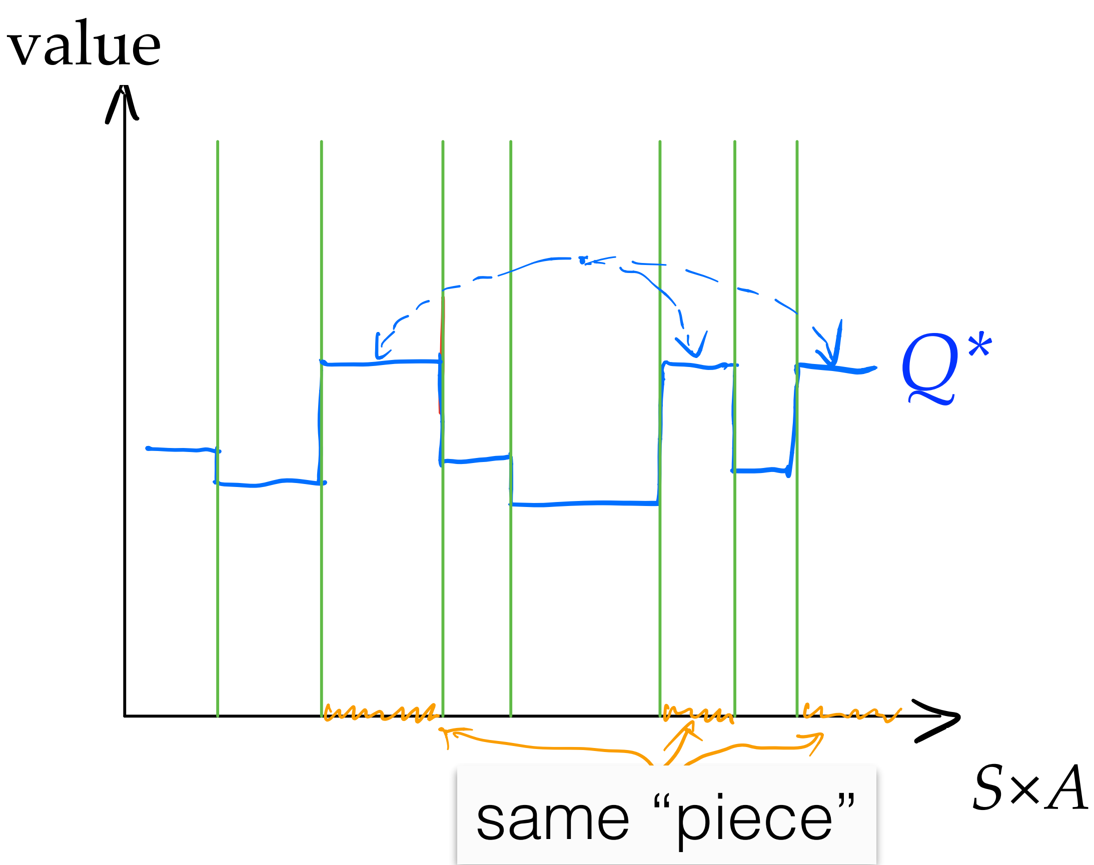

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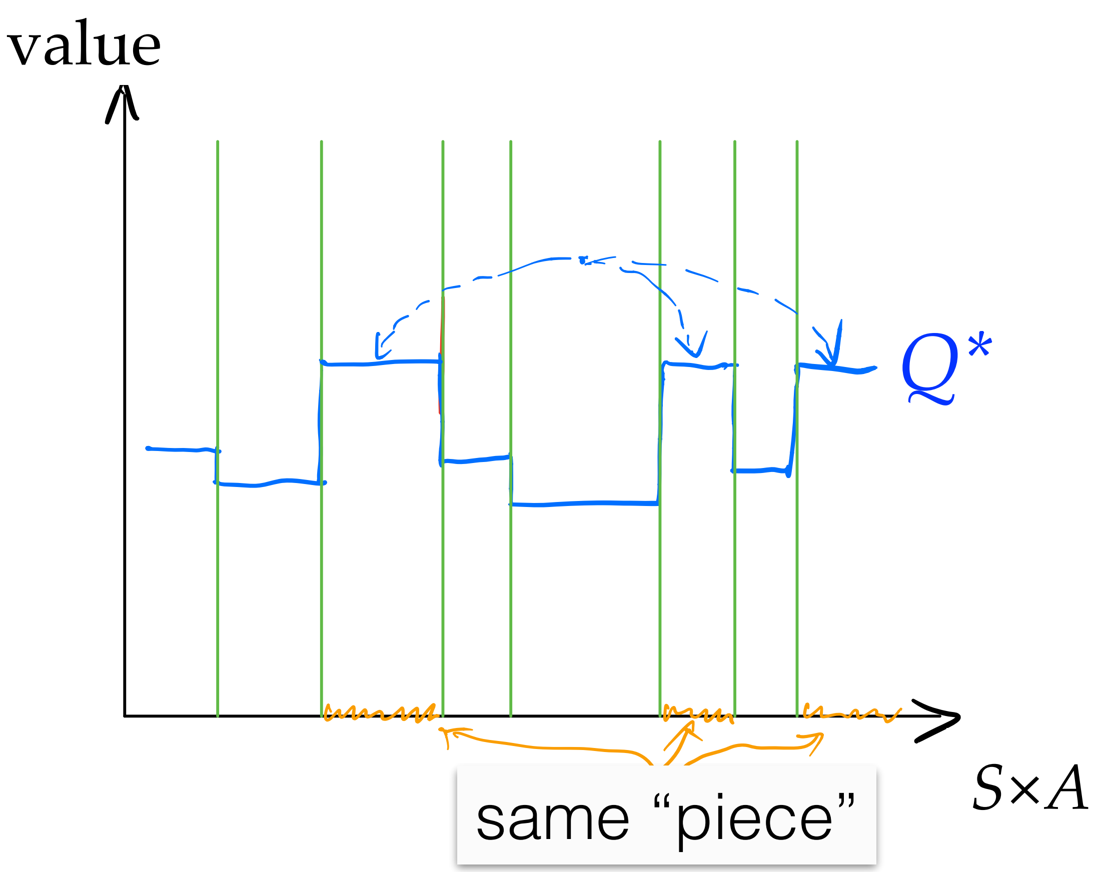

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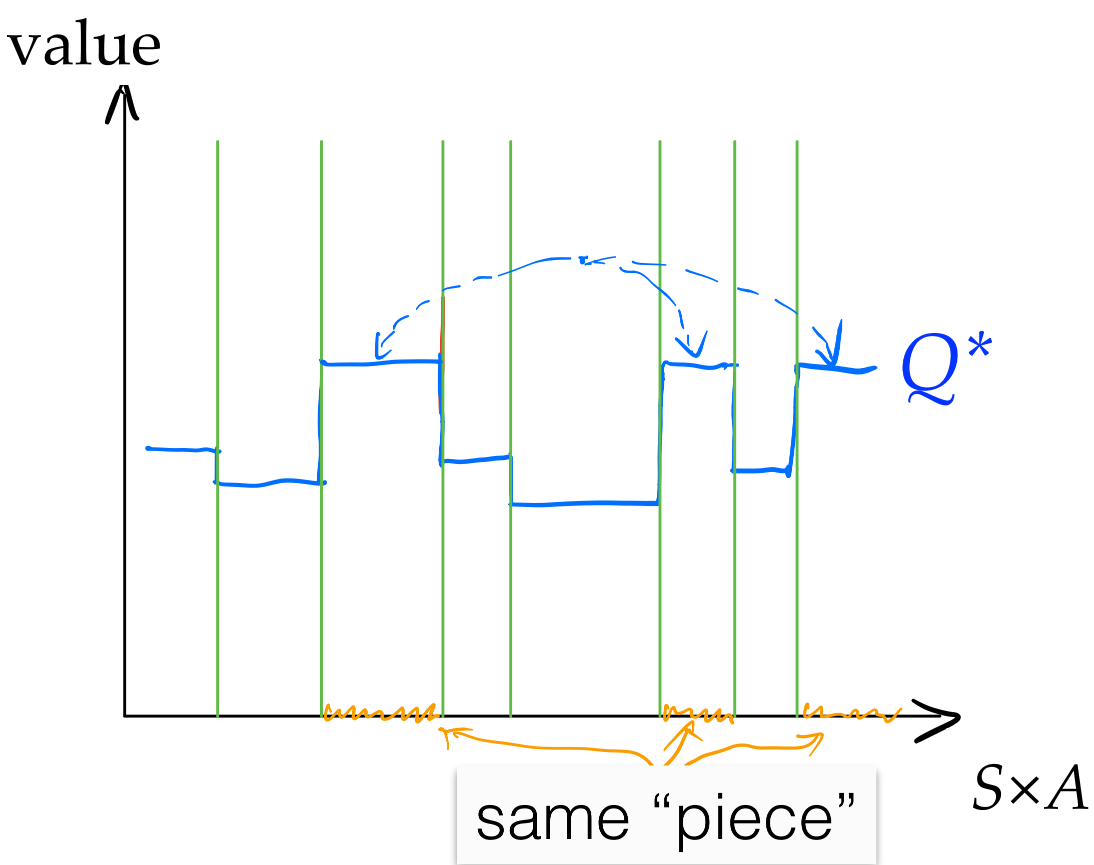

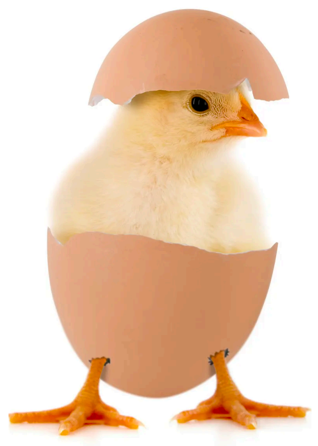

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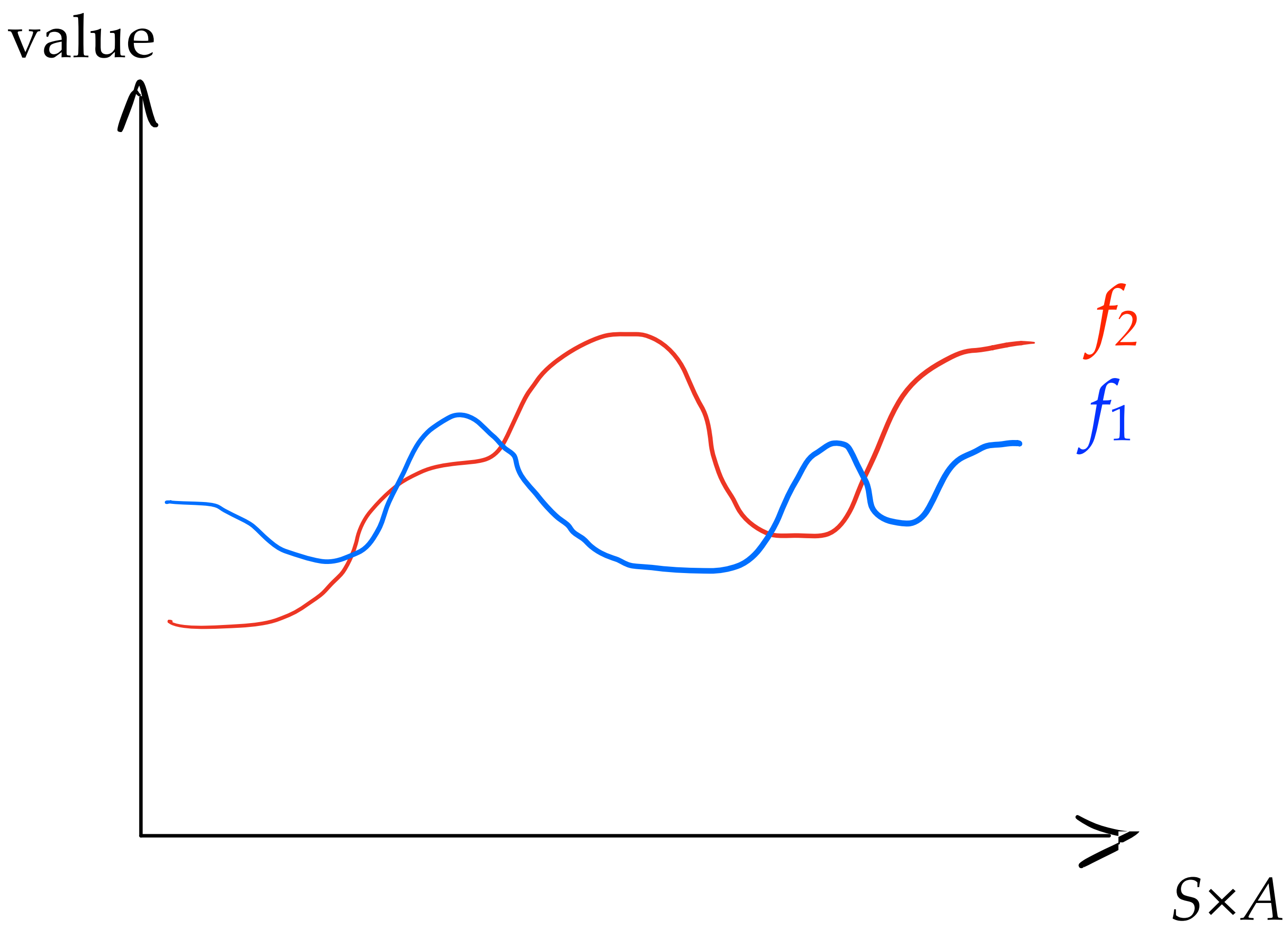
Batch Value-Function Tournament [XJ, ICML-21]



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Xie & **Jiang**. ICML-21. Batch Value-function Approximation with Only Realizability.

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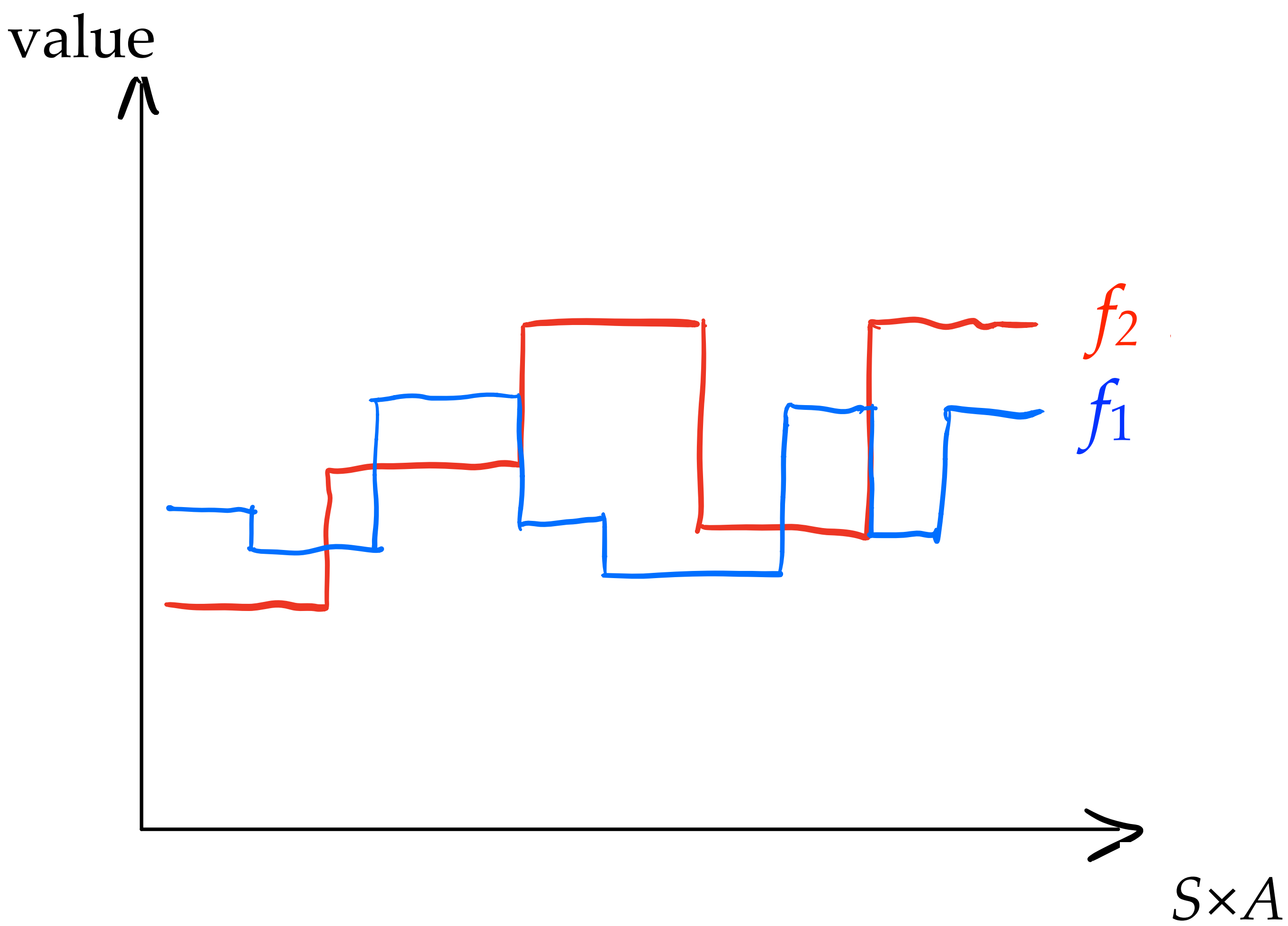

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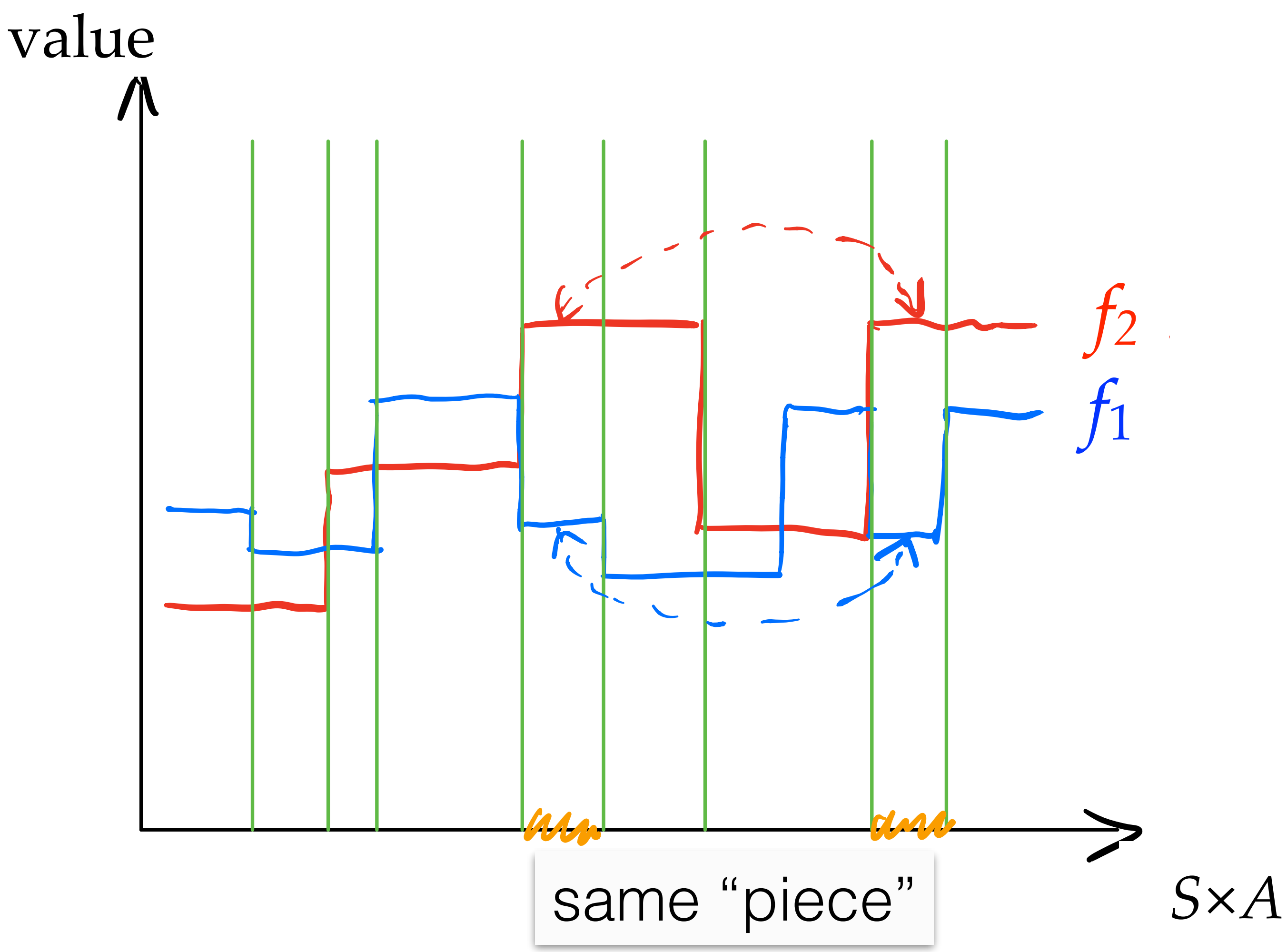

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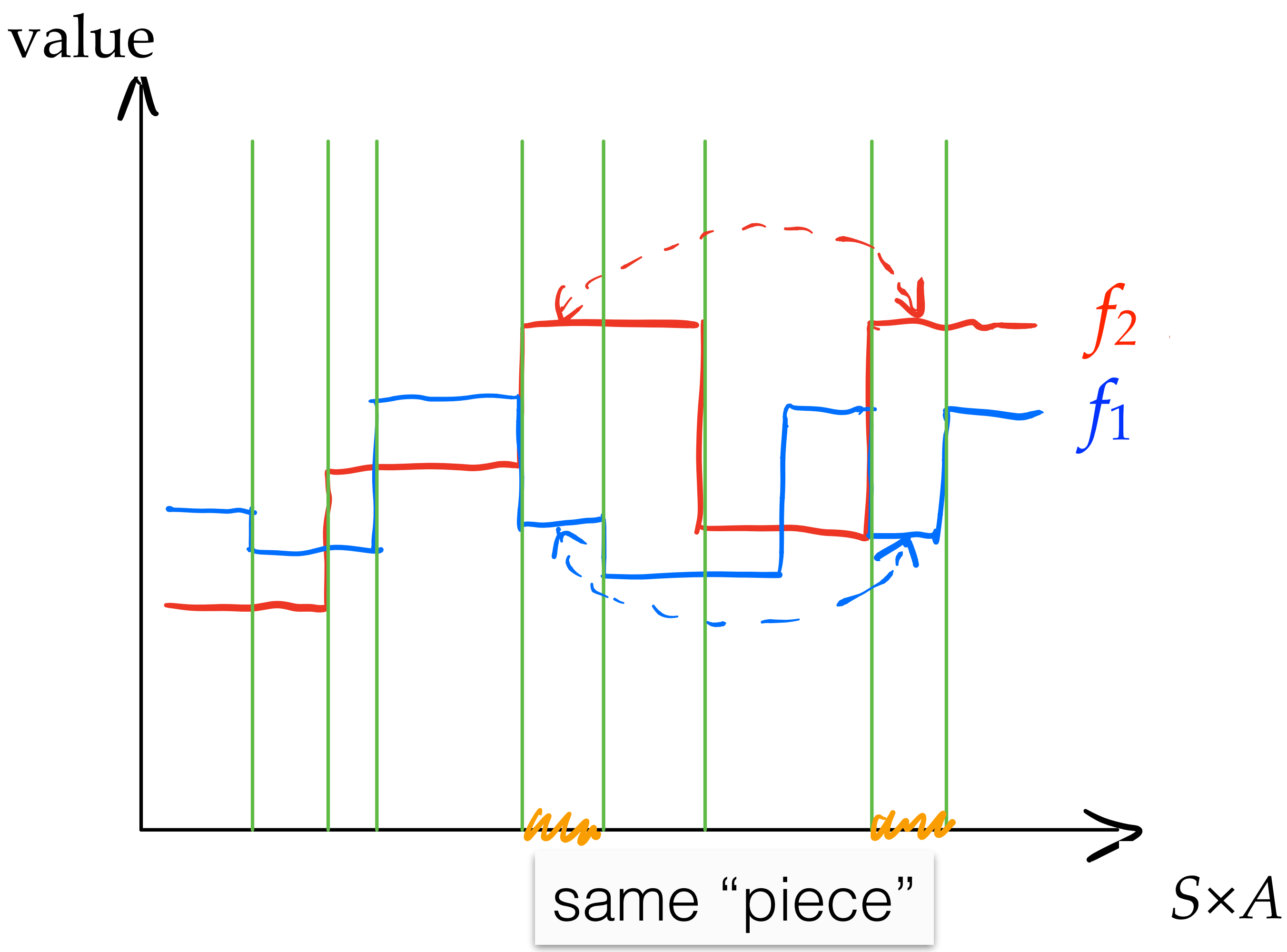

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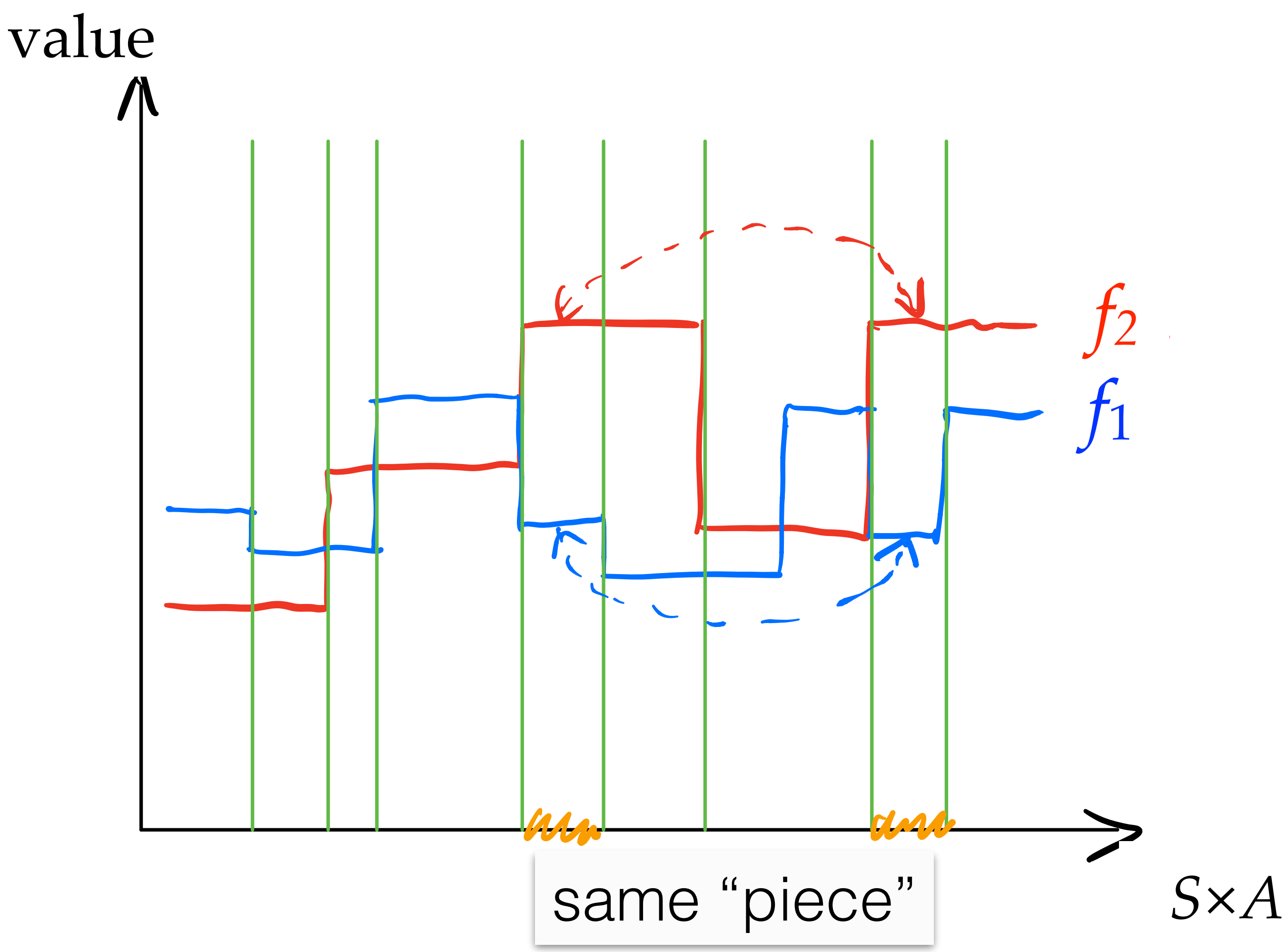

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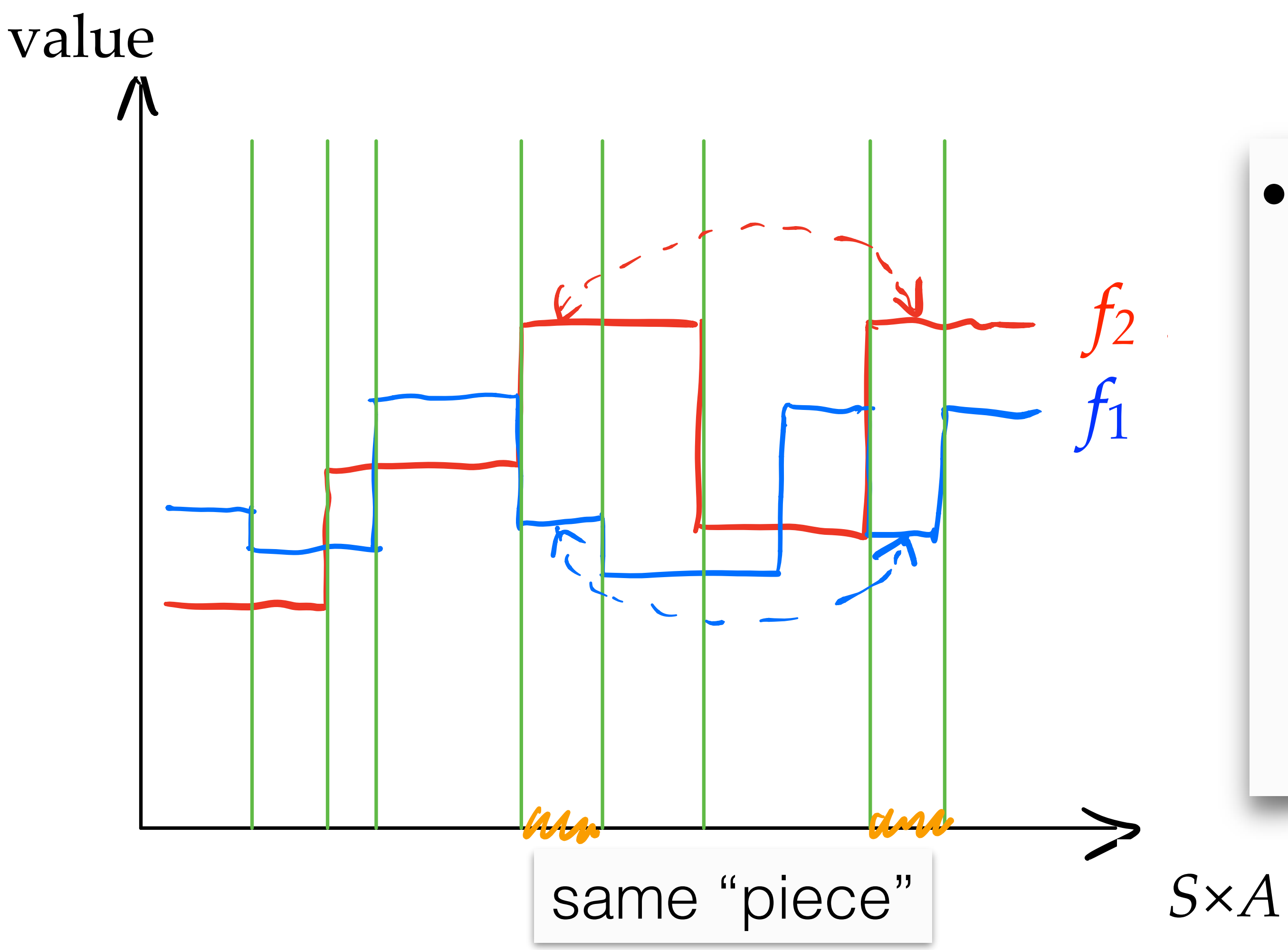

- Algorithm: **BVFT**

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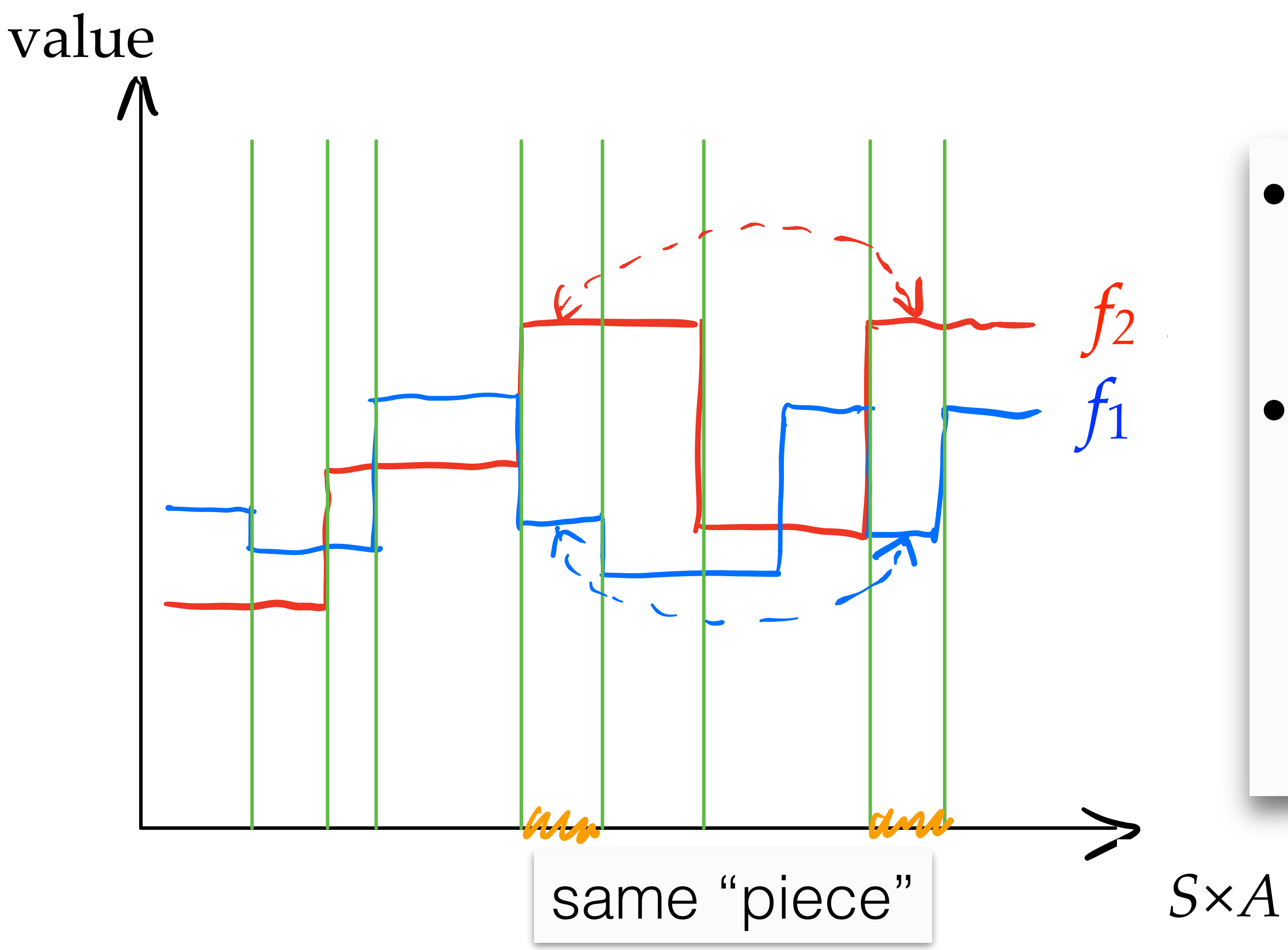

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Formal guarantee in backup slide

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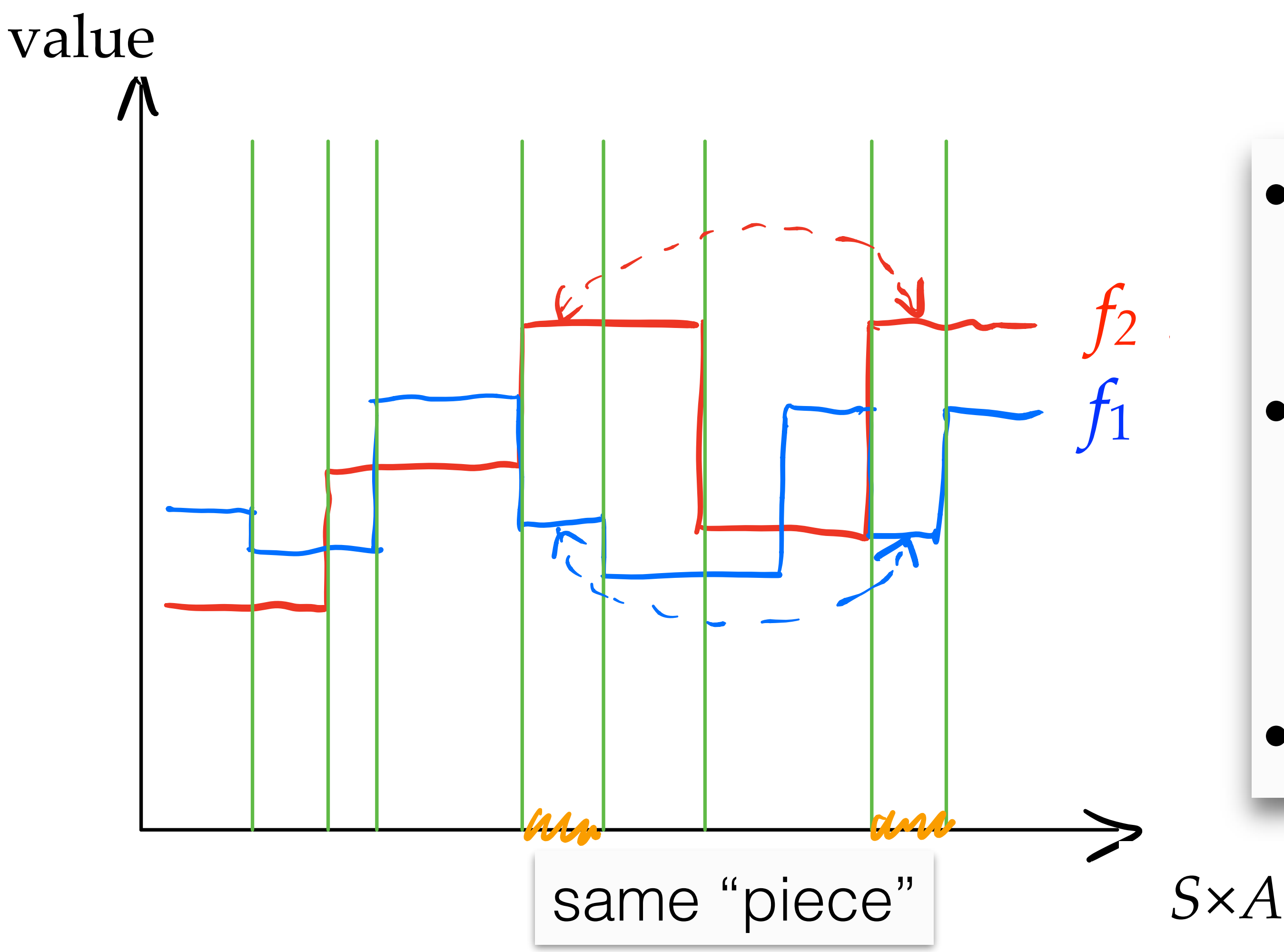

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- Computation: #data points * $|F|^2$

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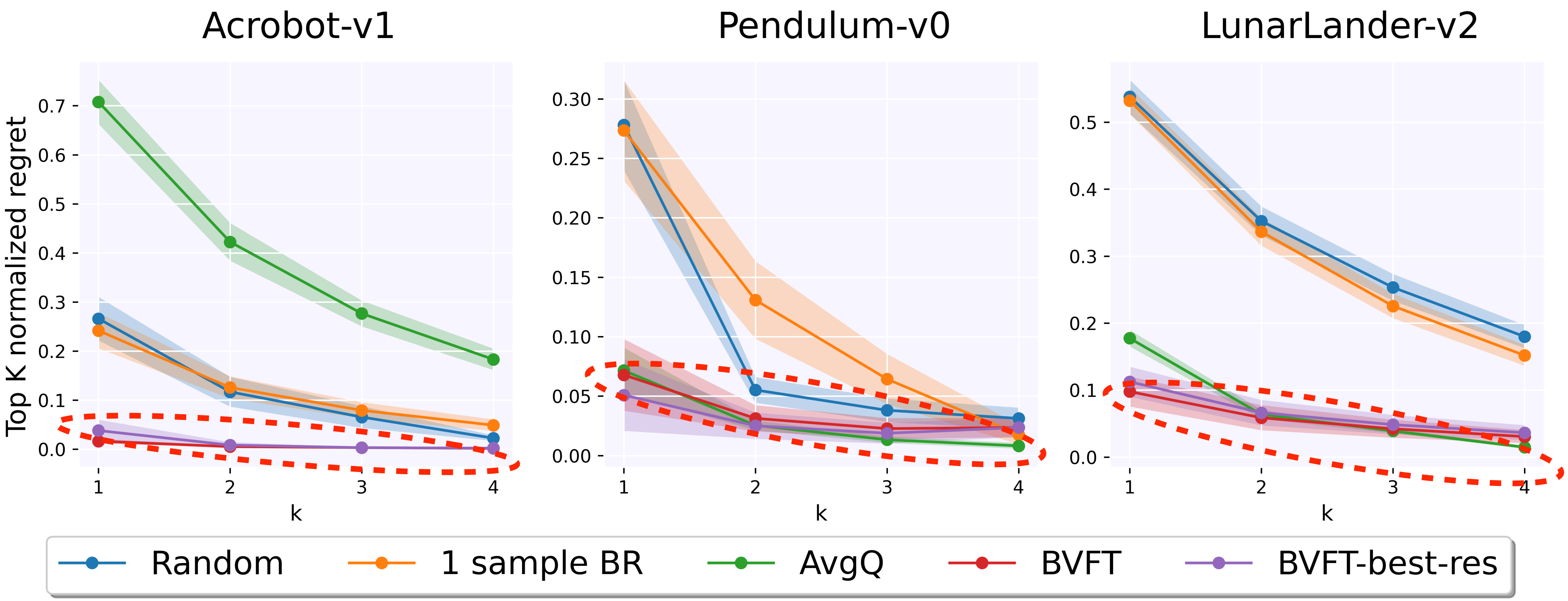
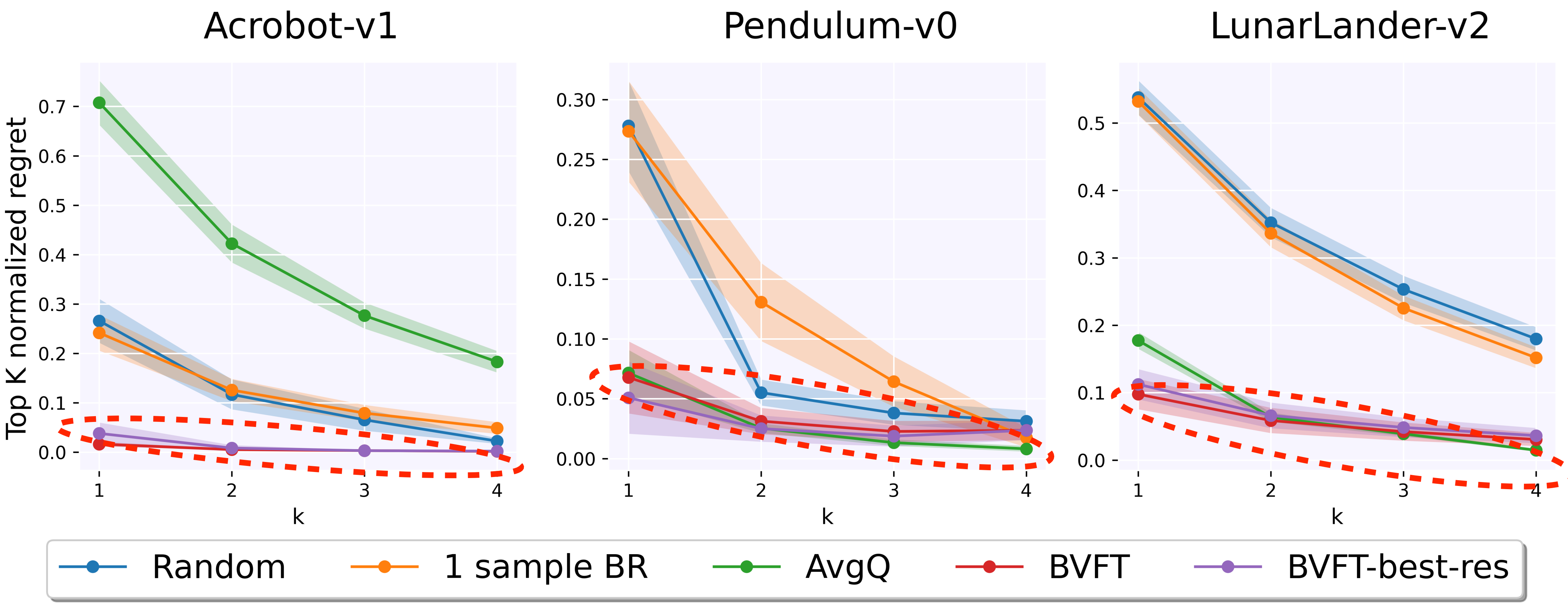
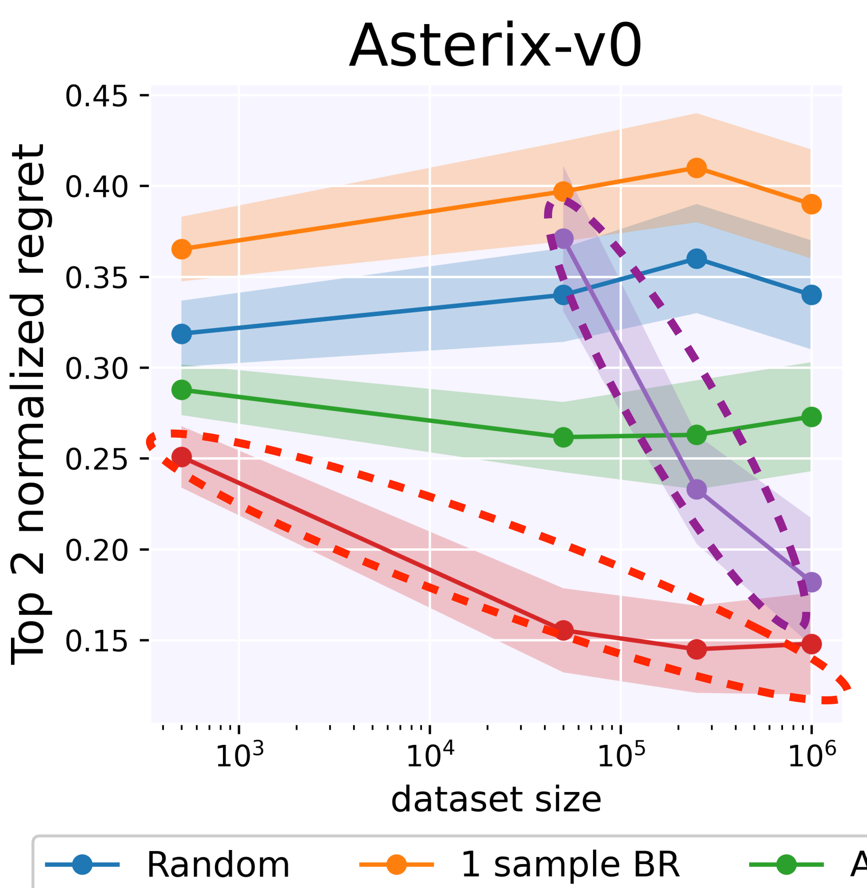


Zhang & **Jiang**. NeurIPS-21. Towards Hyperparameter-free Policy Selection for Offline Reinforcement Learning.





$$f_k \leftarrow \arg\min_{f_\theta} \mathbb{E}_D \left[\left(f_\theta(s,a) - r - \gamma f_{k-1}(s', \pi) \right)^2 \right]$$

Neural architecture designed by "cheating"

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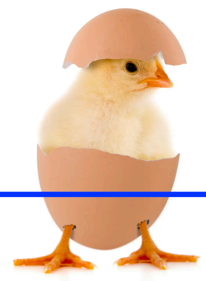
$\Downarrow$ π = greedy w.r.t. $\hat{f}$



Validation:

(FQE: learn Q^π)

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- BVFT: H-P free solution for value-function selection

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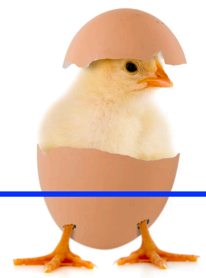

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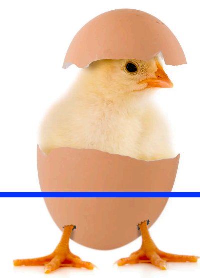

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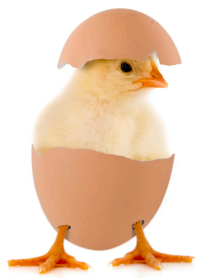

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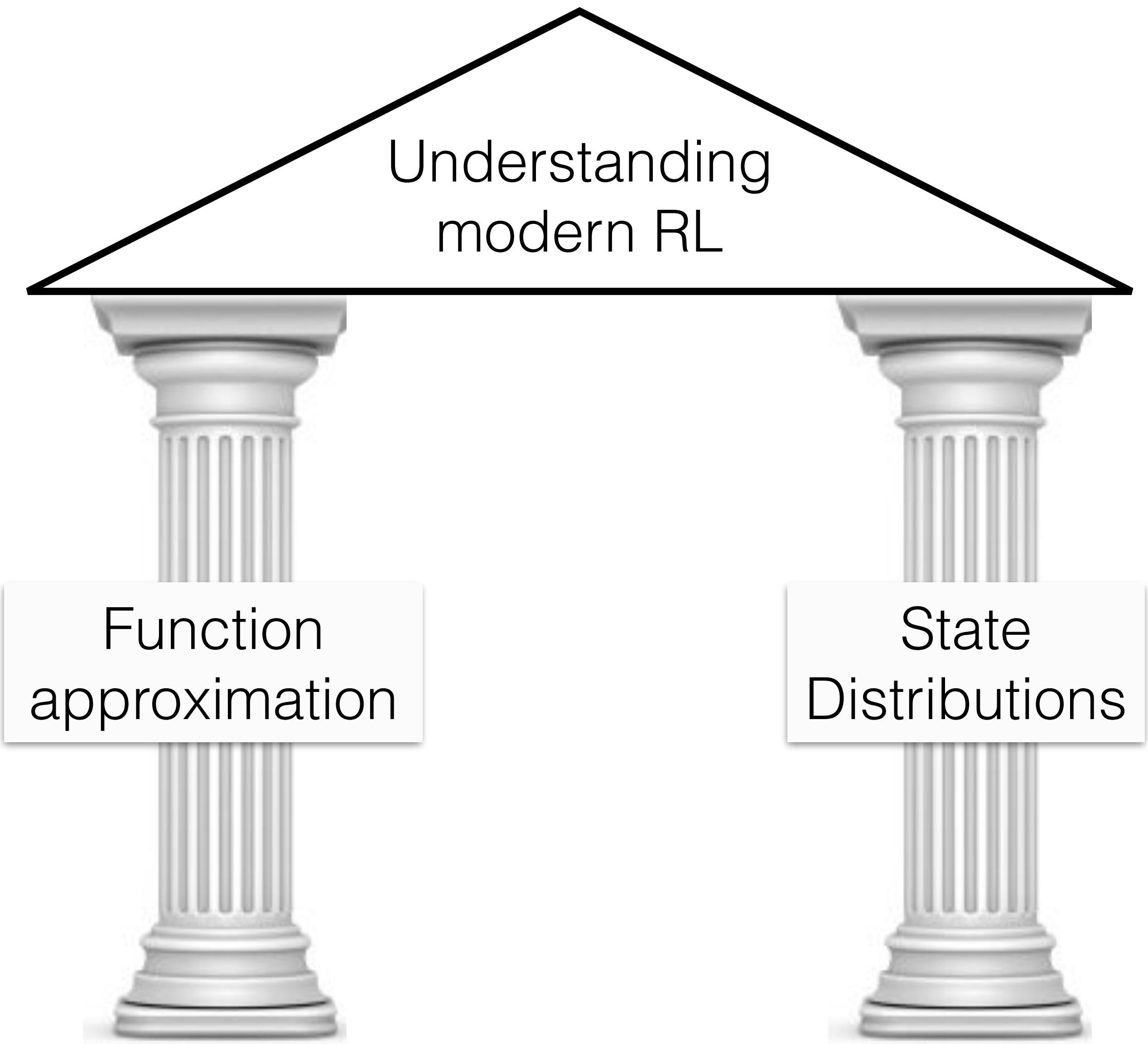

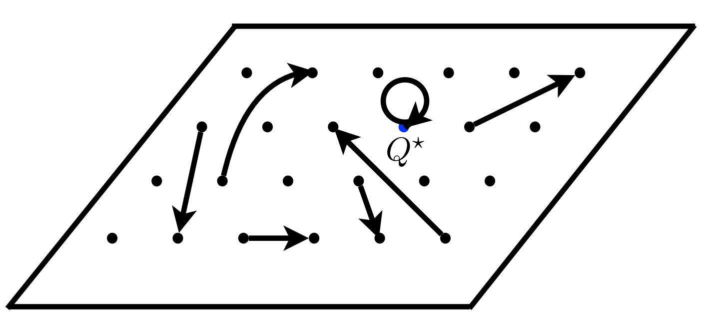

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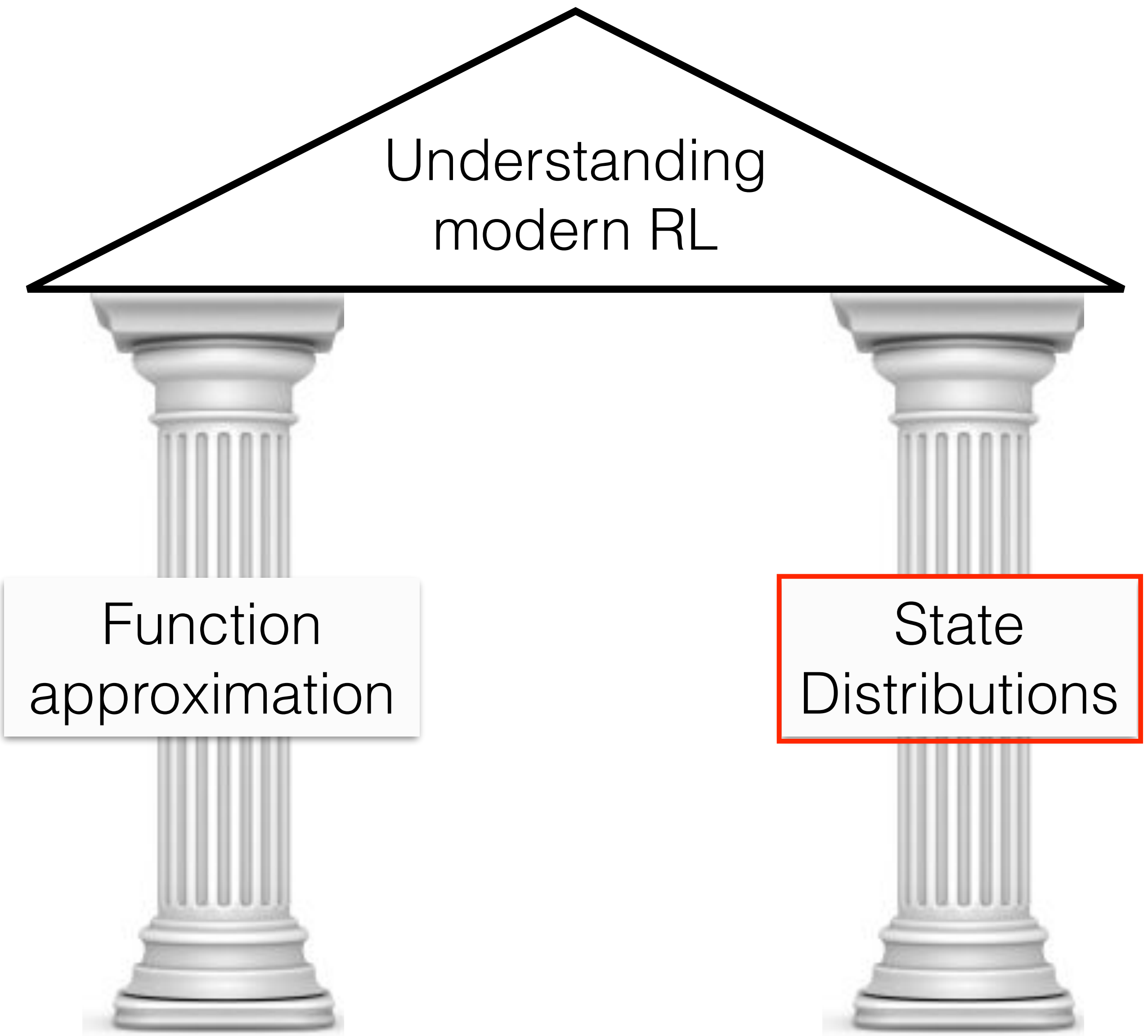

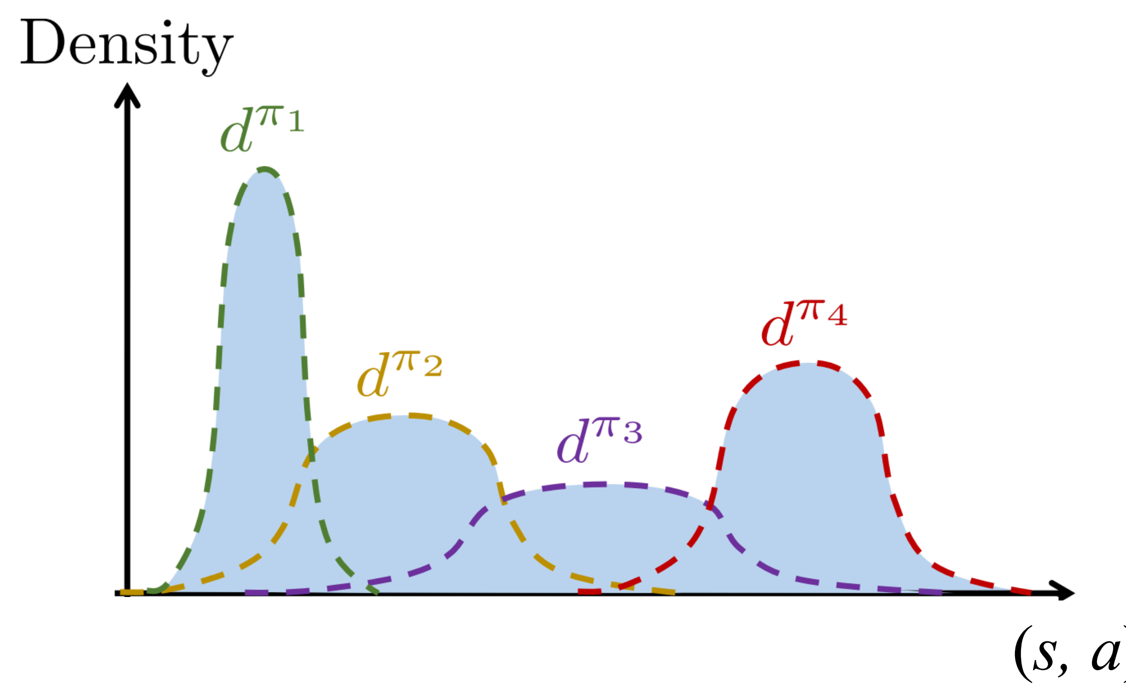

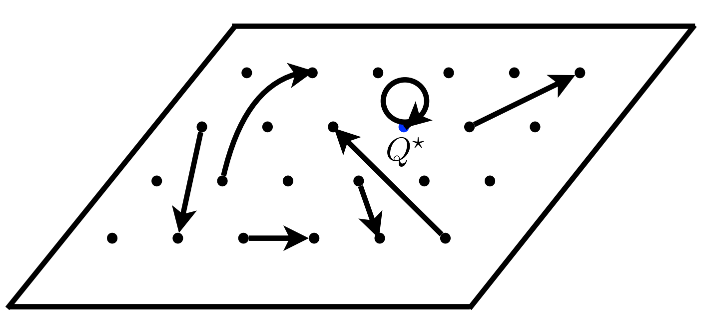

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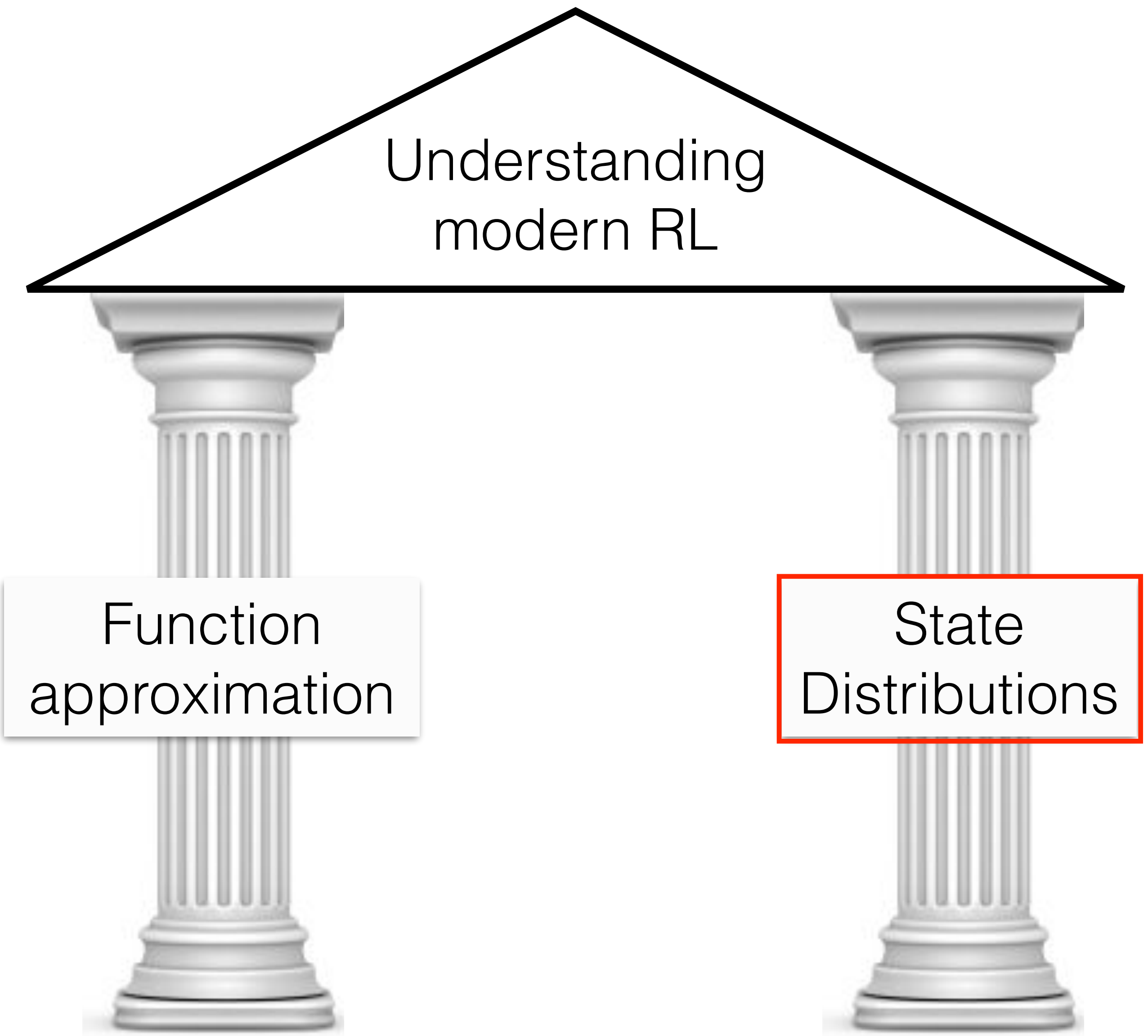

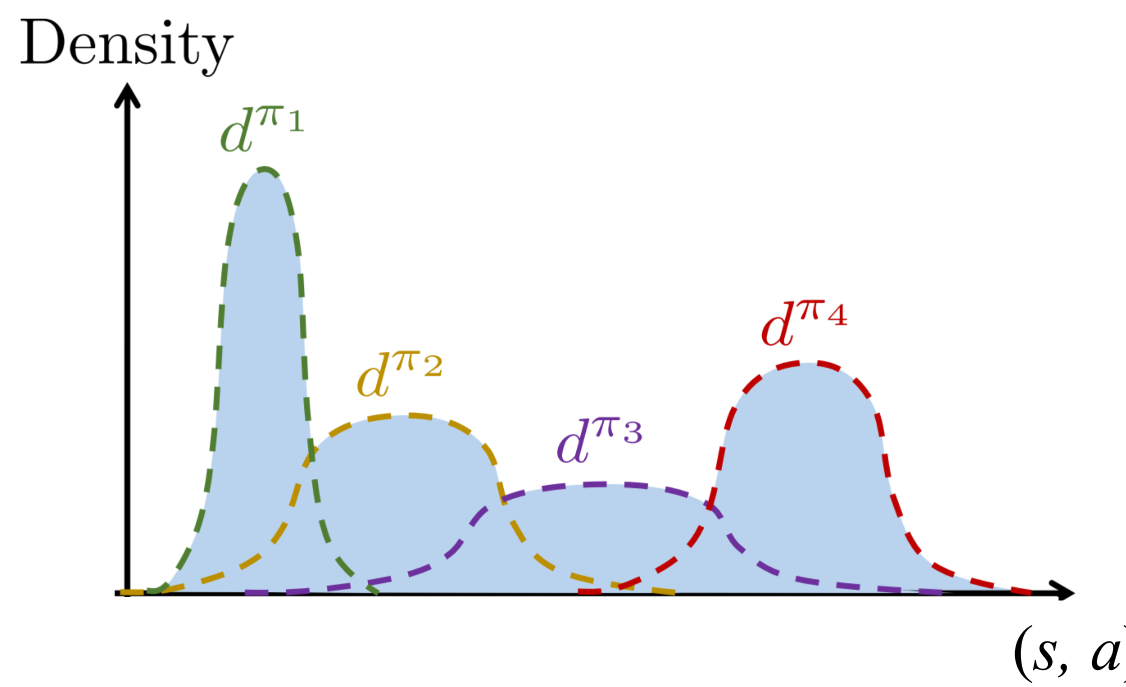

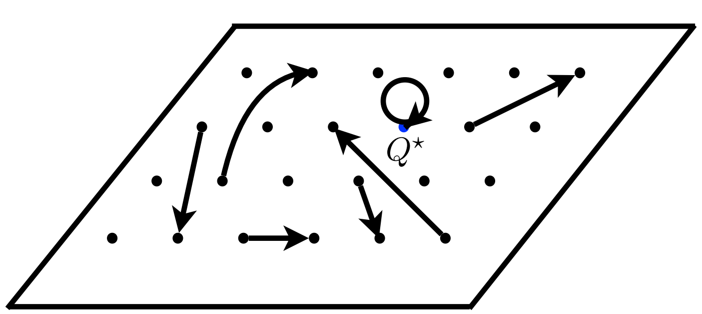

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Outstanding Paper Runner Up

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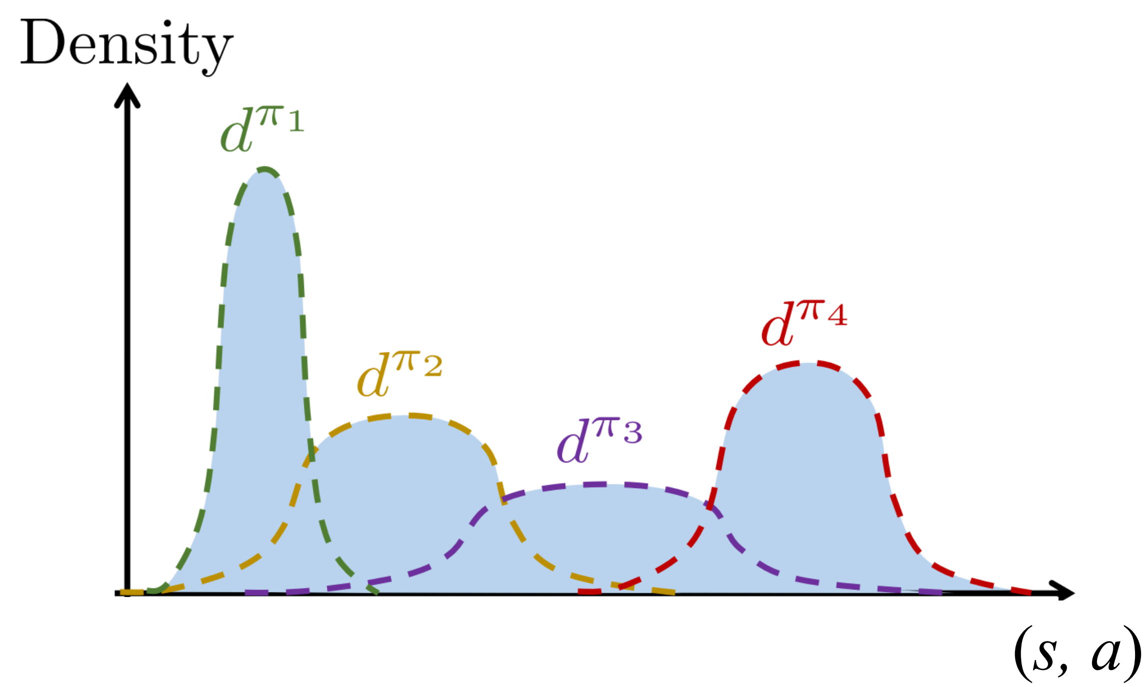

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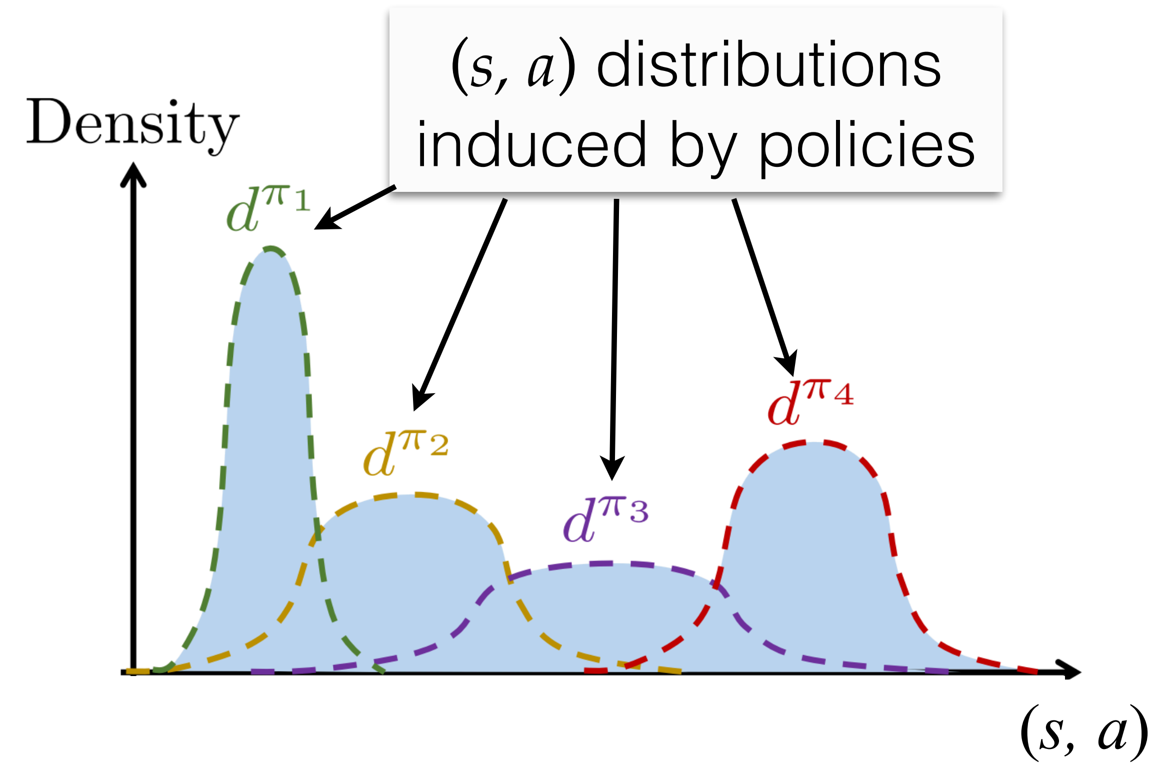

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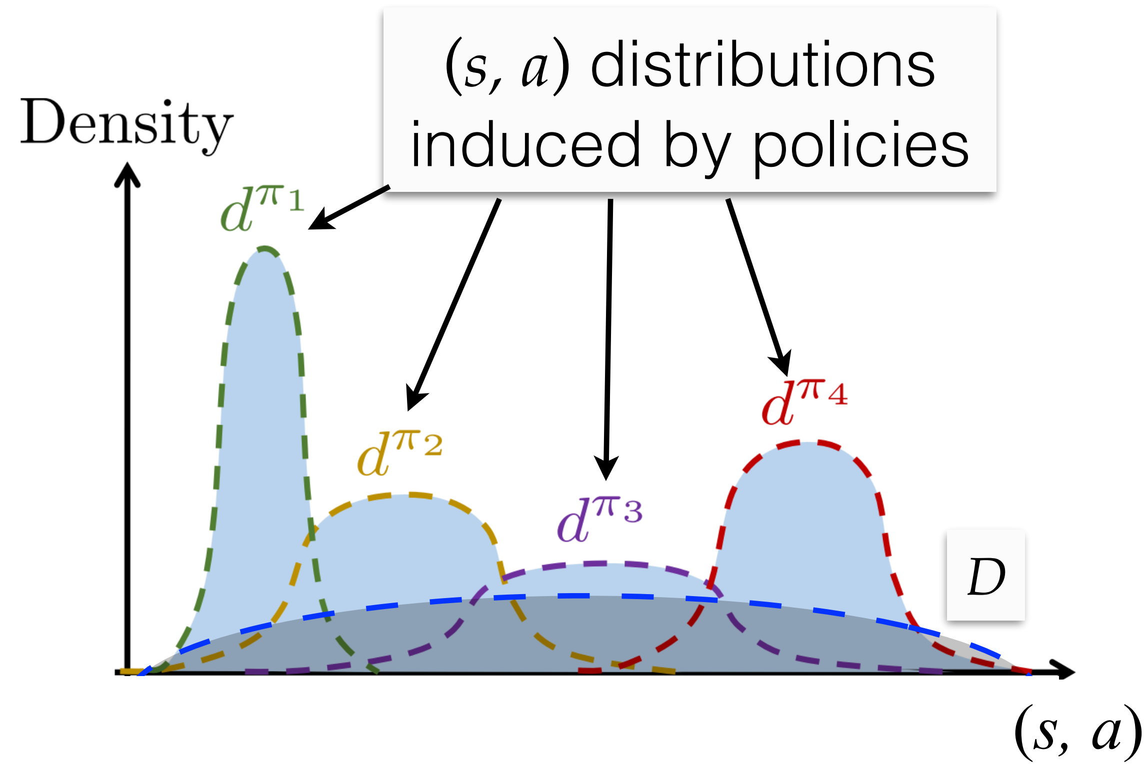

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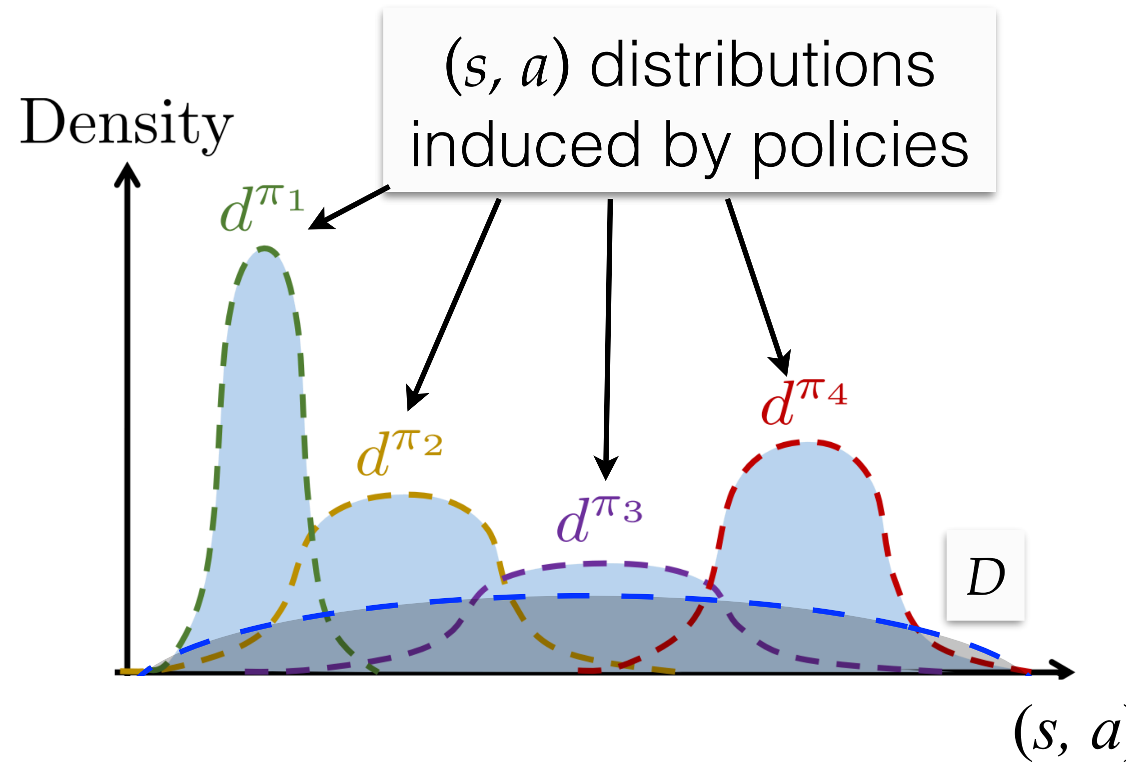

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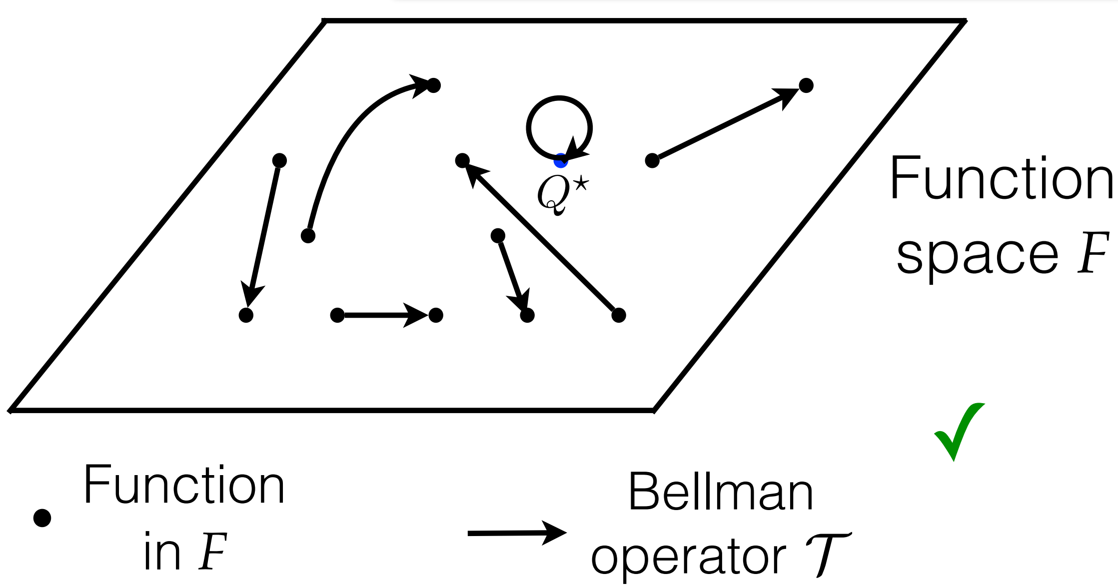

✓

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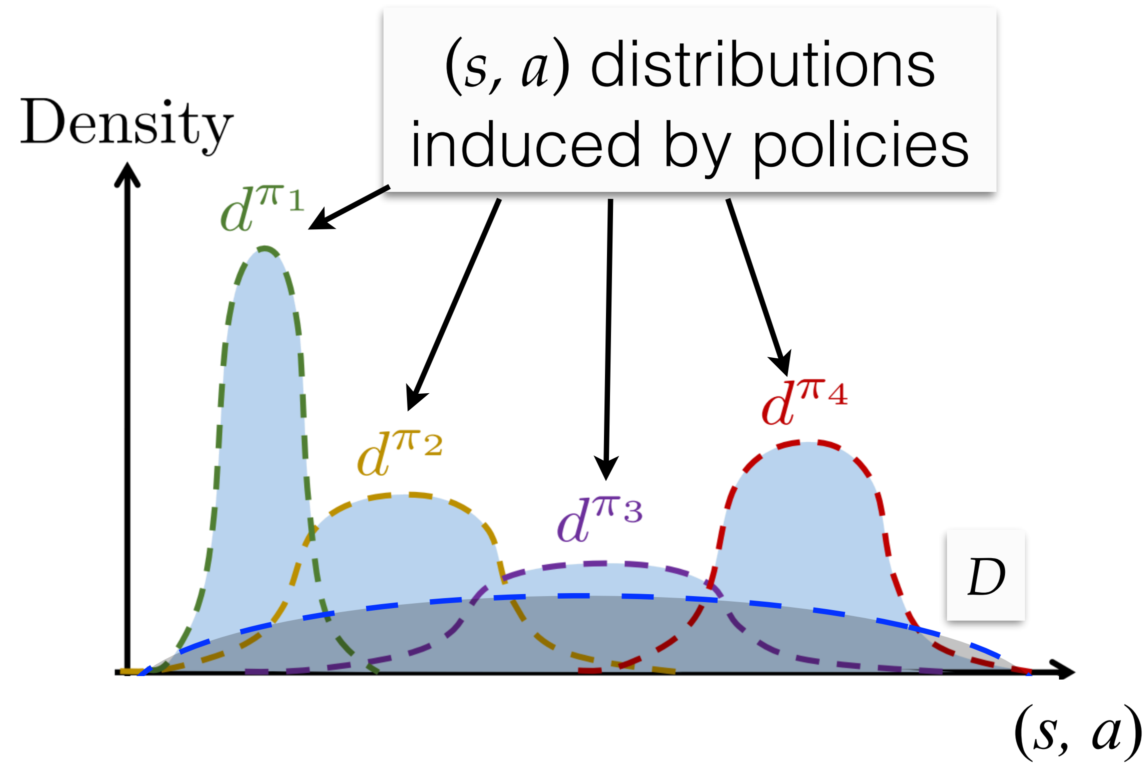

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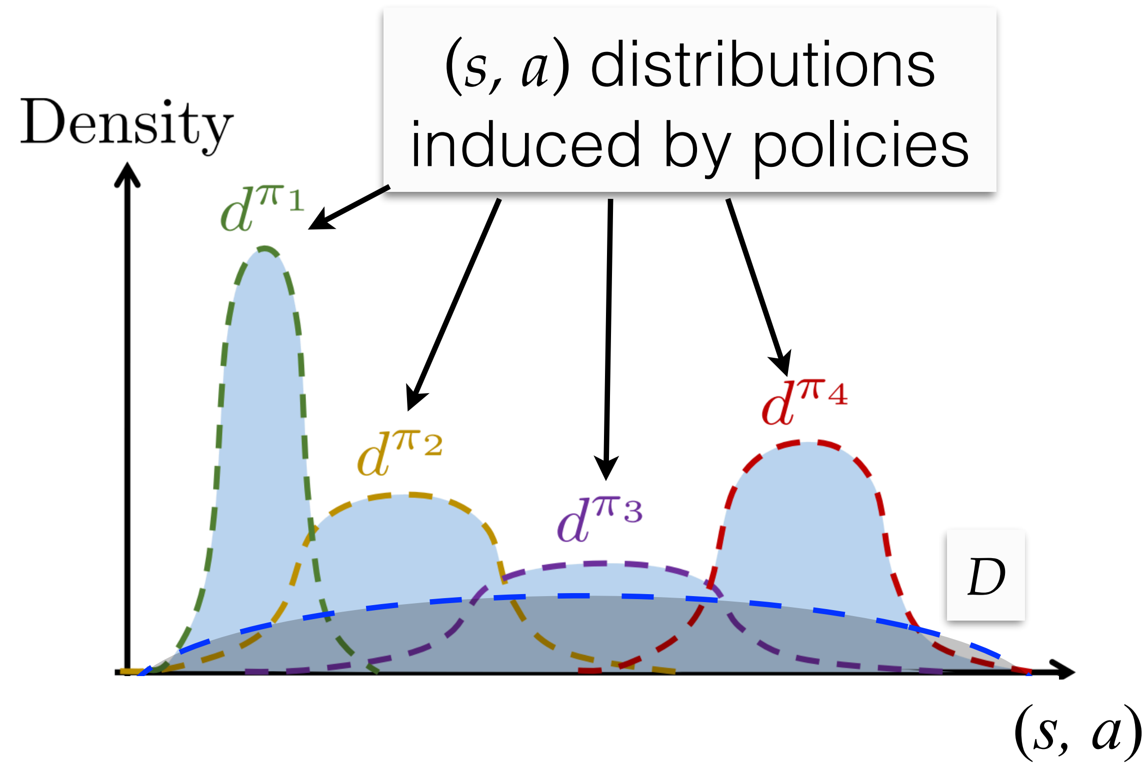

Challenge: real-world data lacks exploration!

- Data may not contain all bad behaviors
- Alg may **over-estimate** their performance

Desirable assumption

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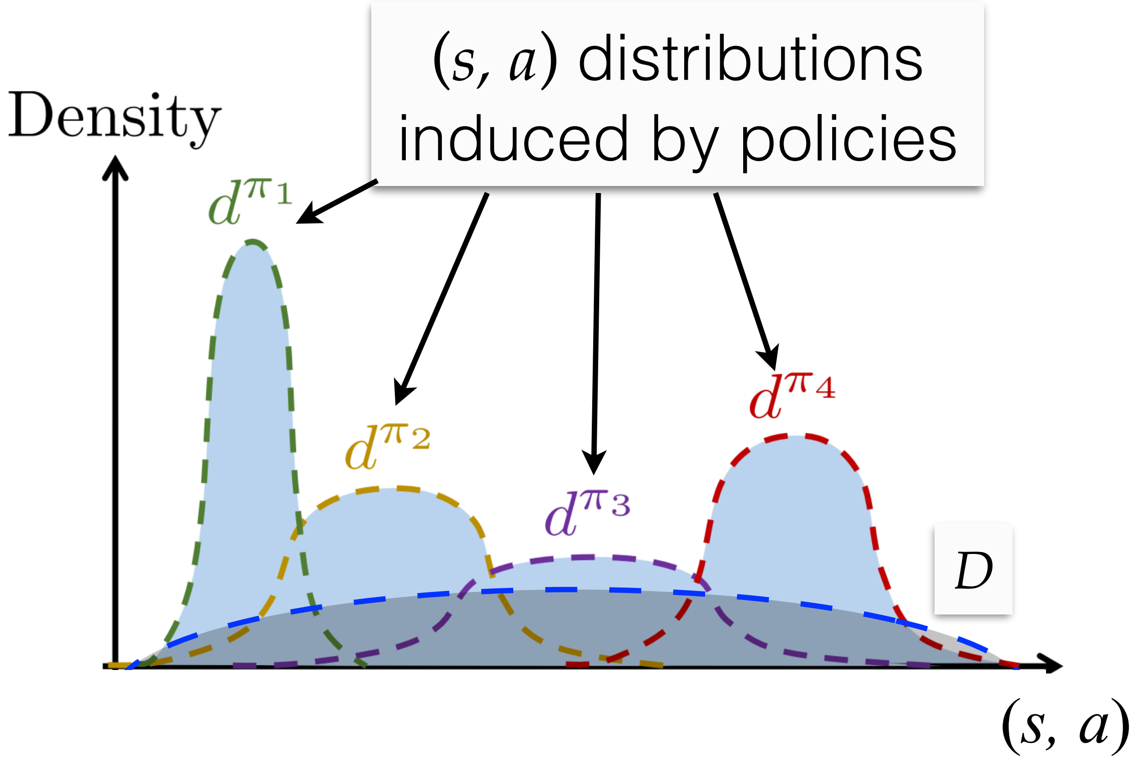

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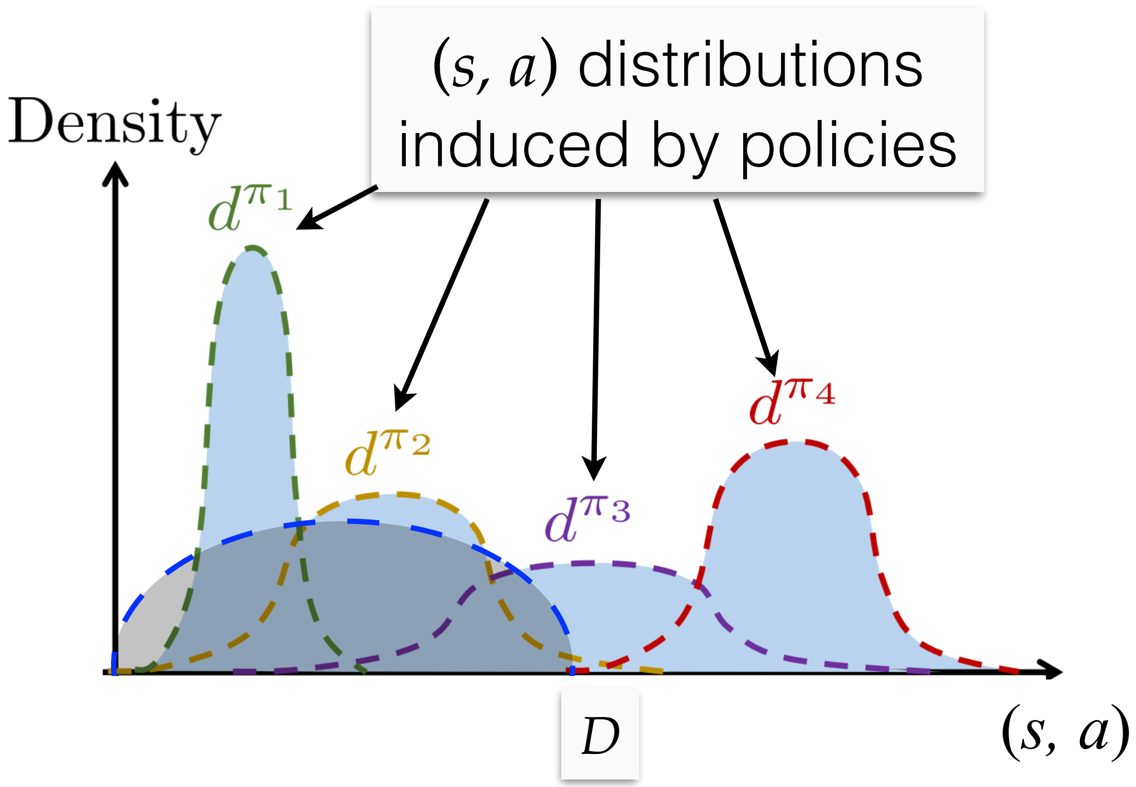

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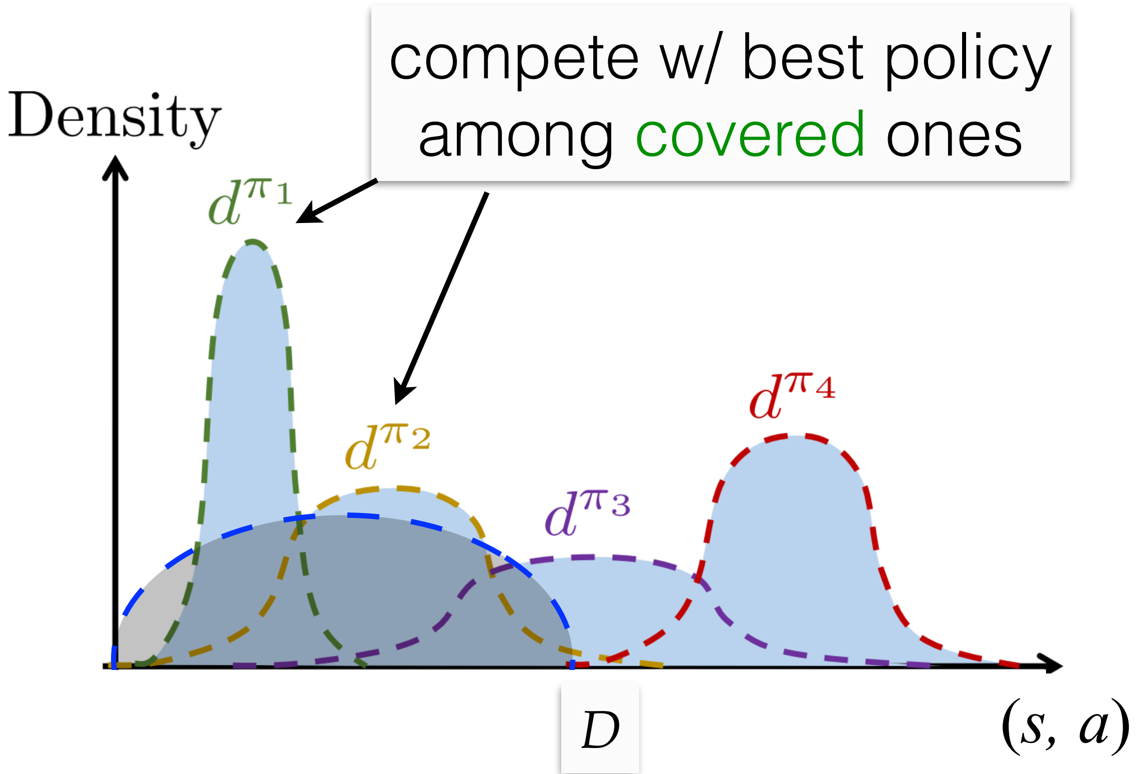

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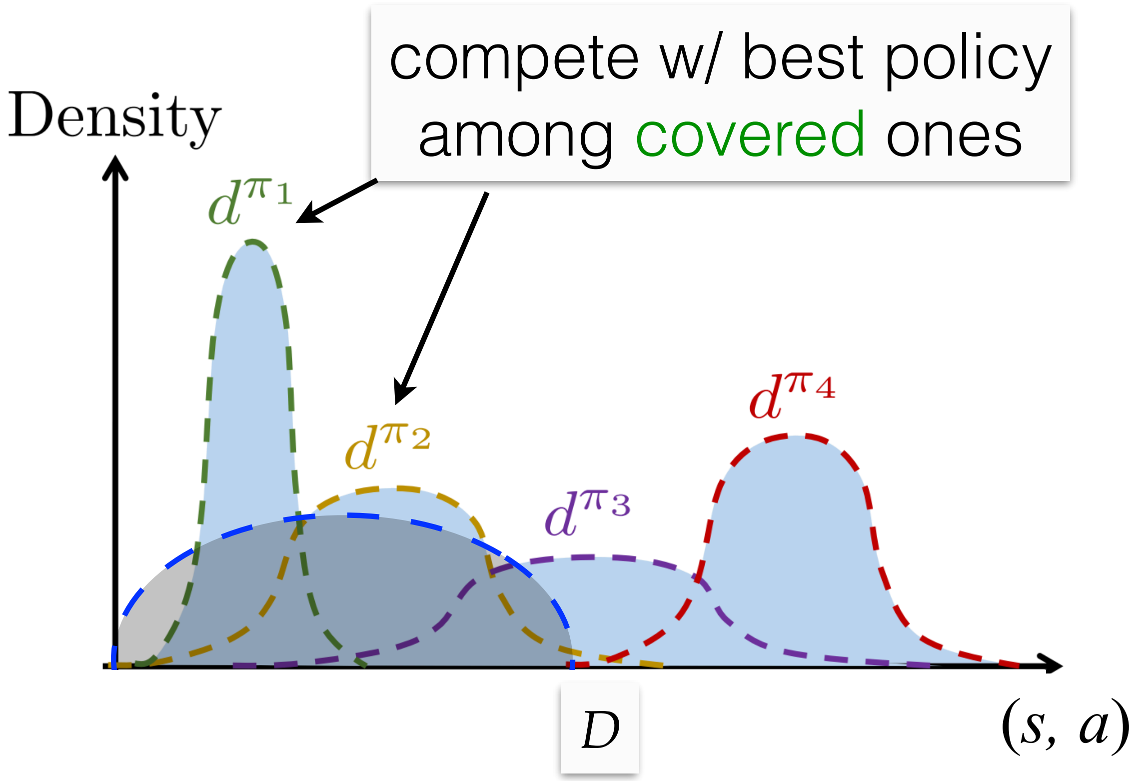

Offline RL (exploitation)

Goal: stay within data distribution

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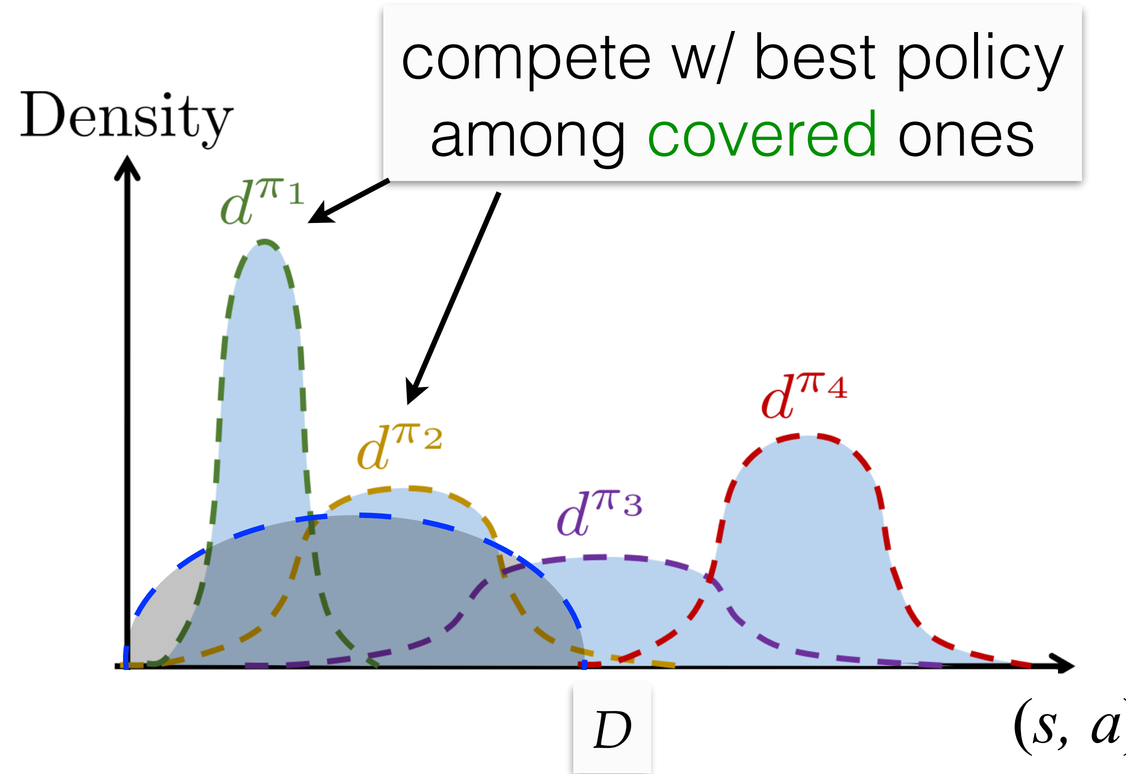

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Principle: *optimism*

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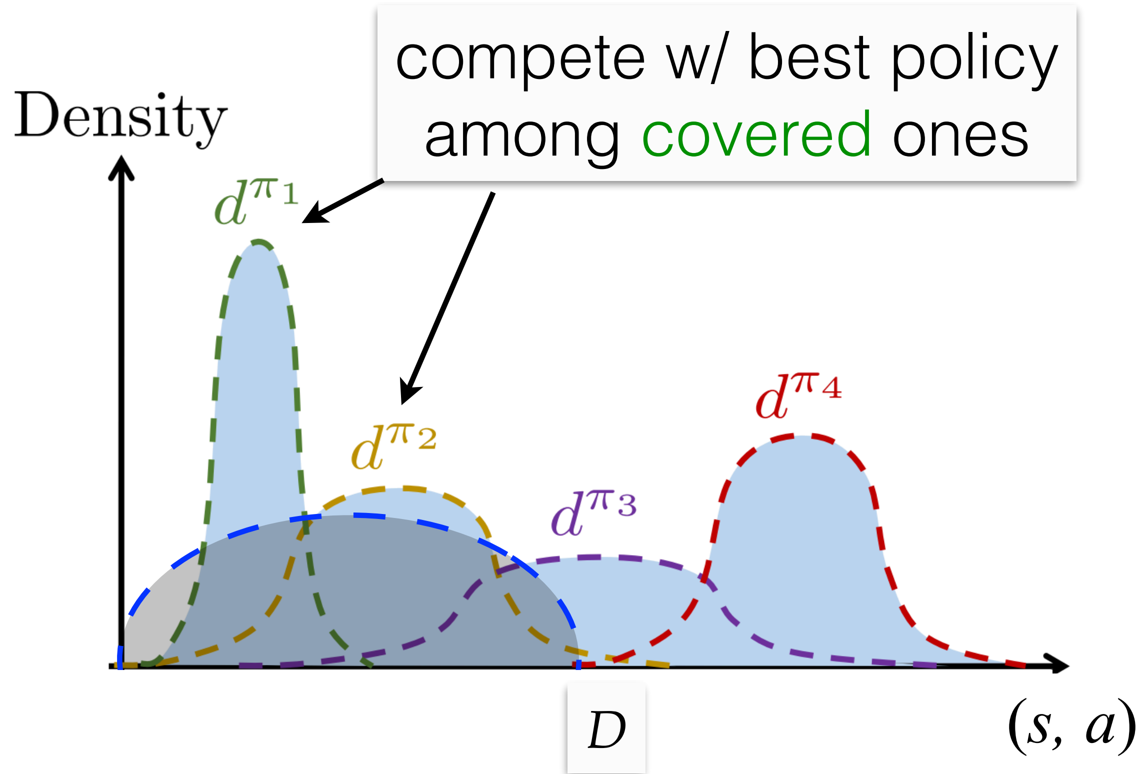

Offline RL (exploitation)

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Principle: *pessimism*

Online RL (exploration)

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Pessimism in face of uncertainty

- Policy optimization: $\arg\max_{\pi \in \Pi} J(\pi) := Q^{\pi}(s_0, \pi)$
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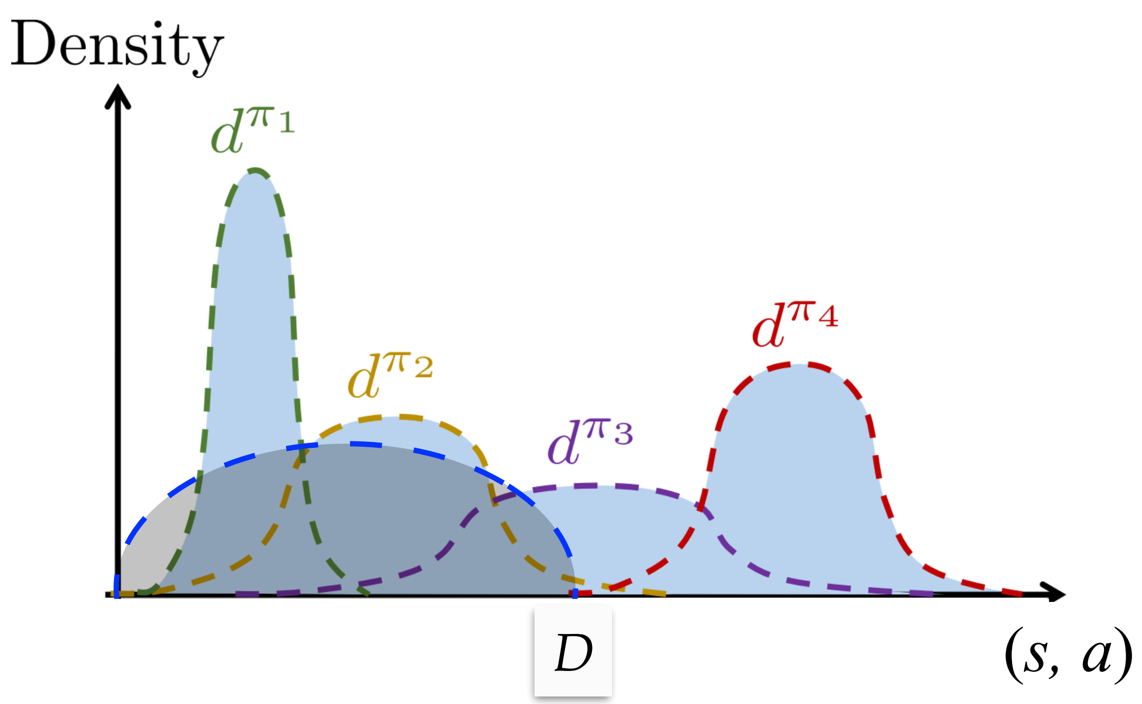

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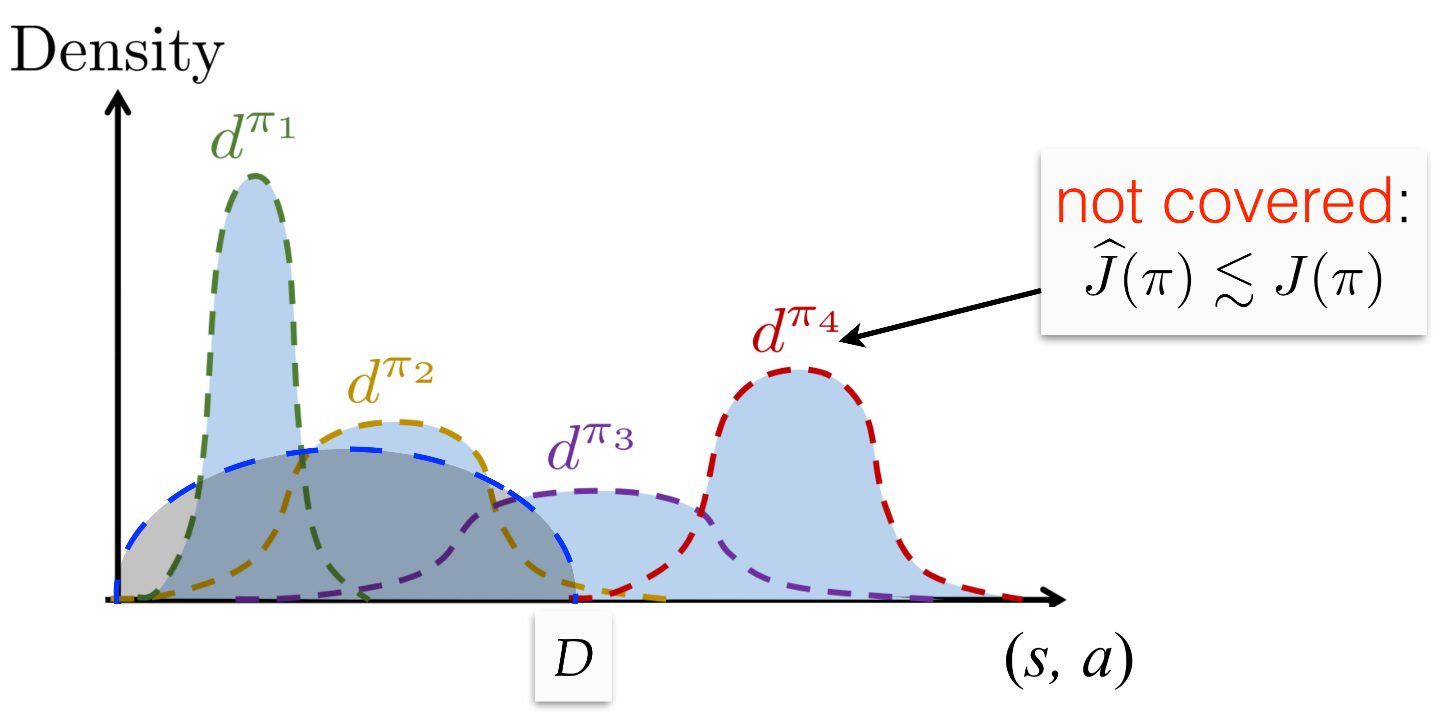

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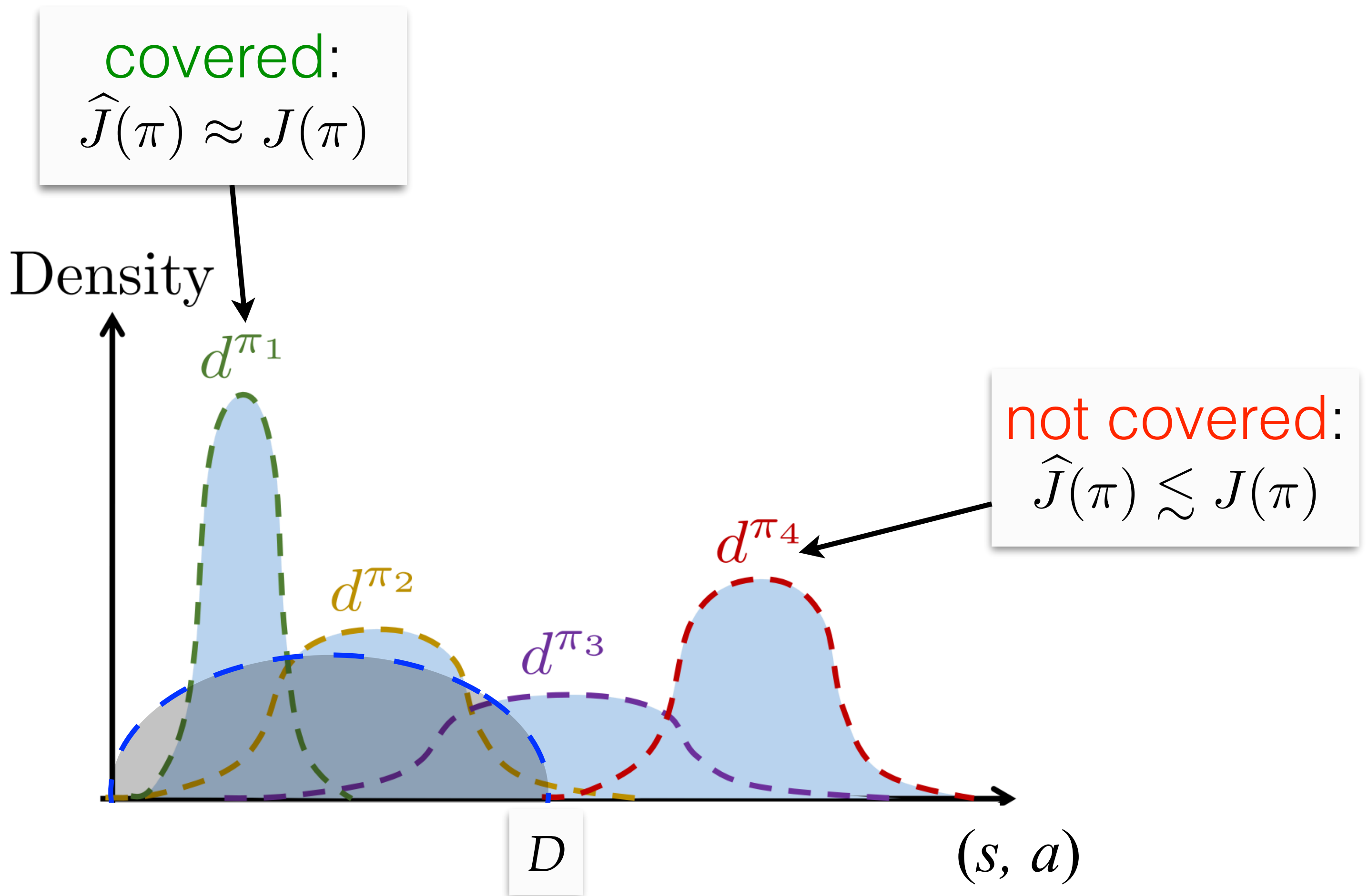

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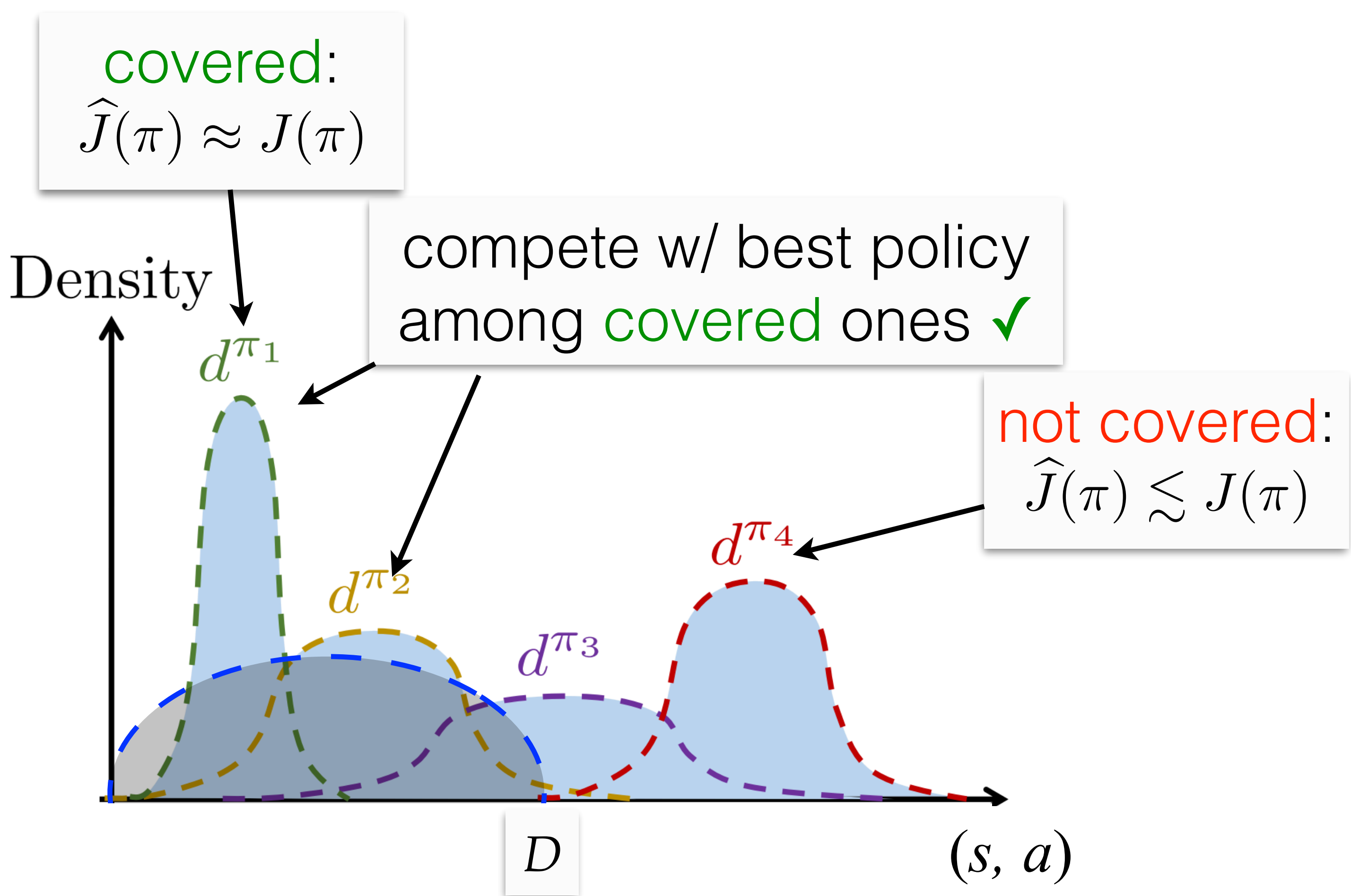

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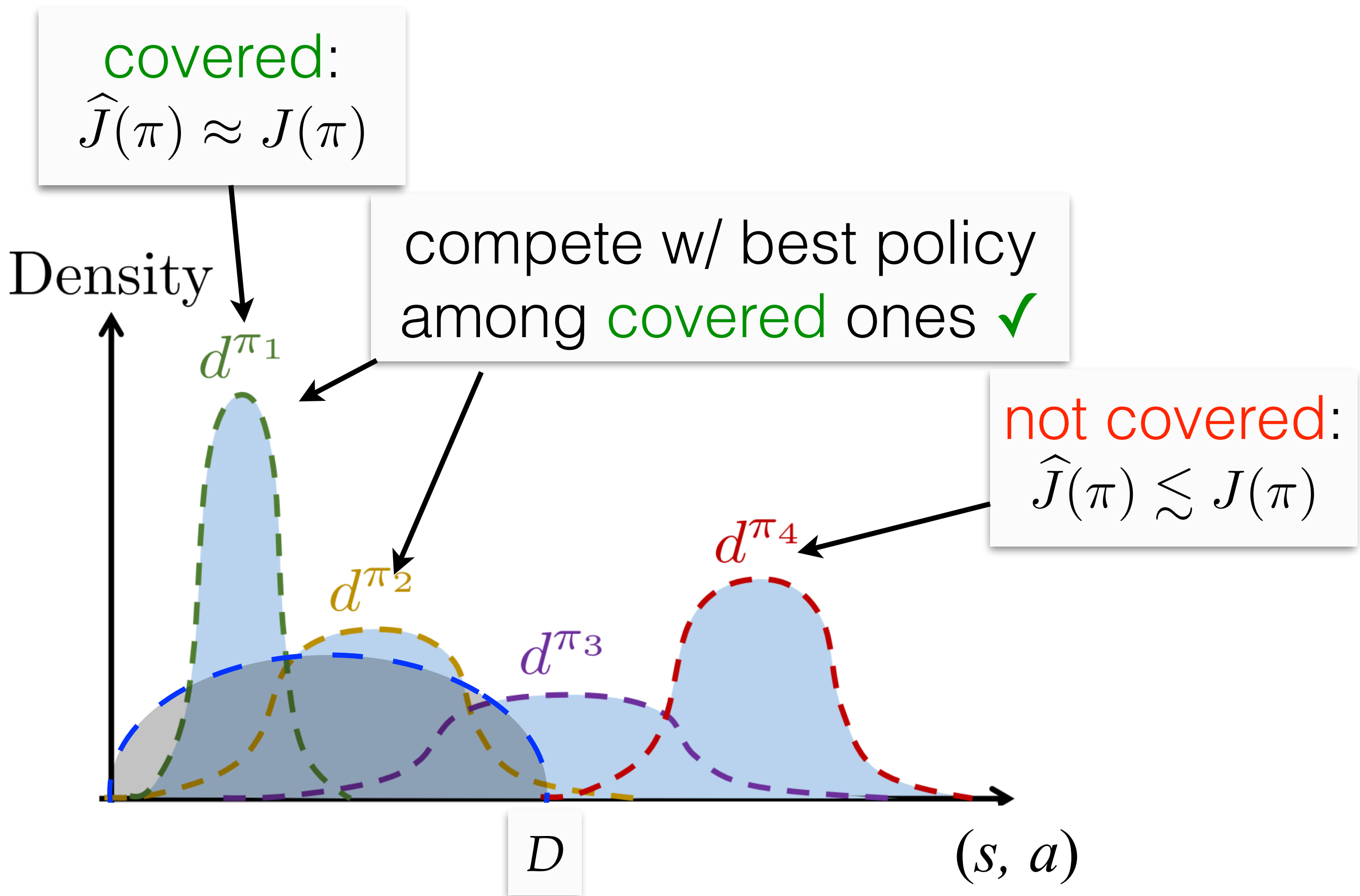

 Nilim and Ghaoui. OR-05. Robust control of Markov decision processes with uncertain transition matrices.

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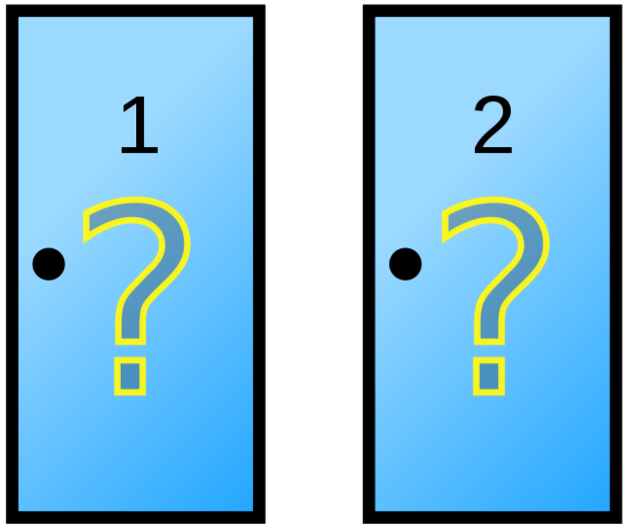

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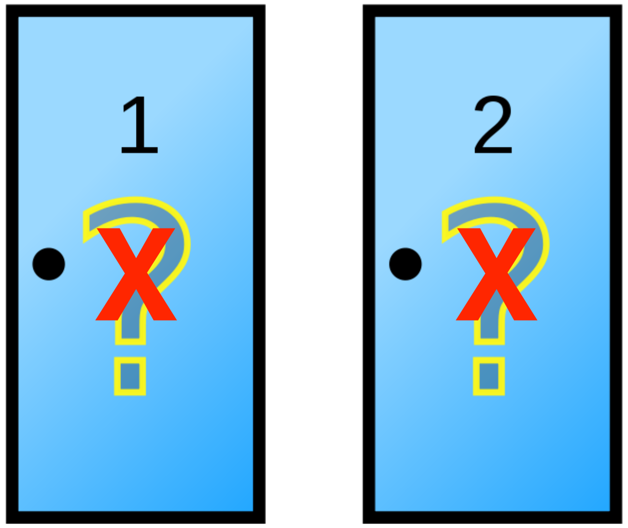

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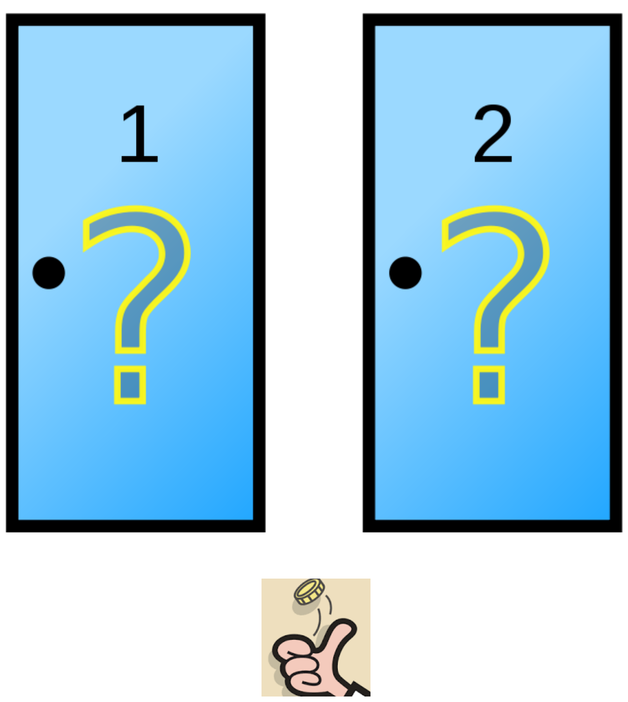

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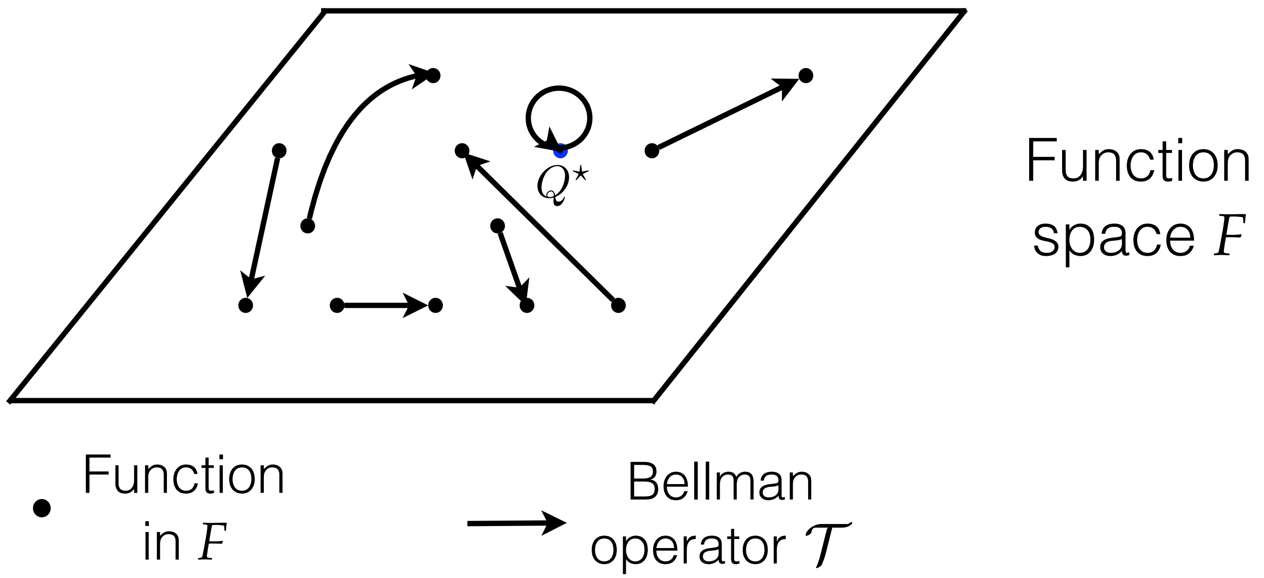

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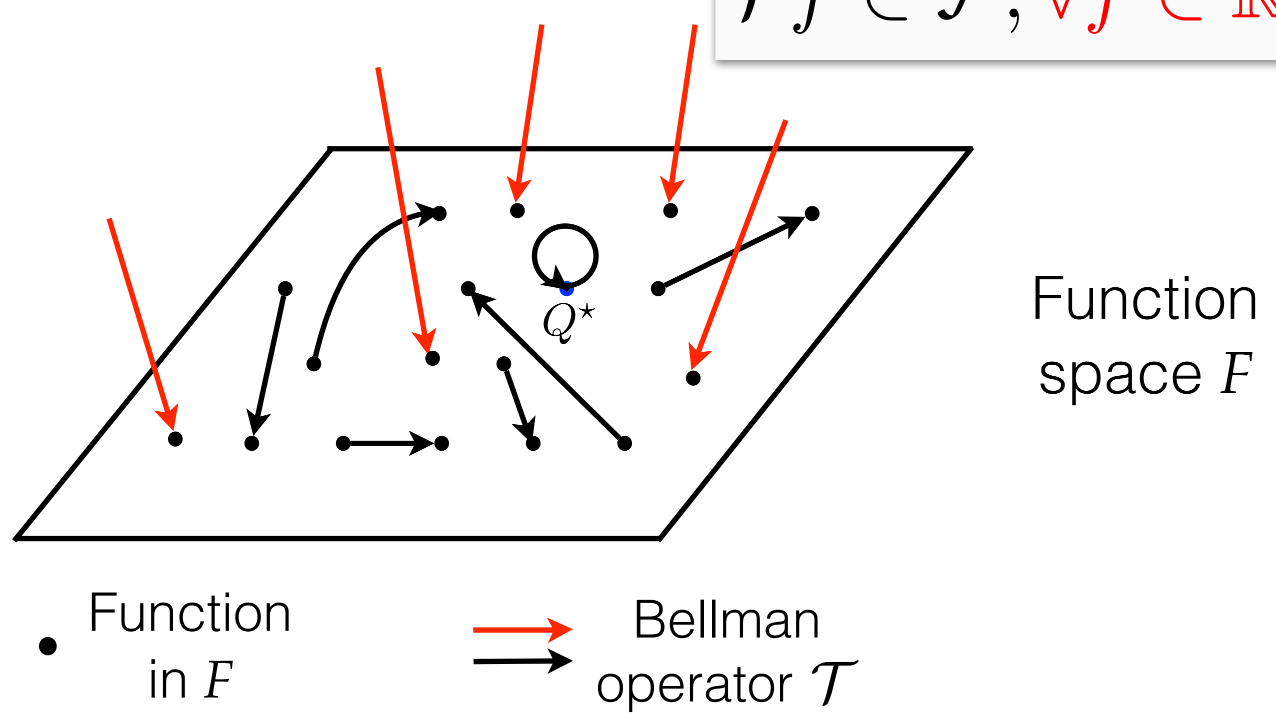

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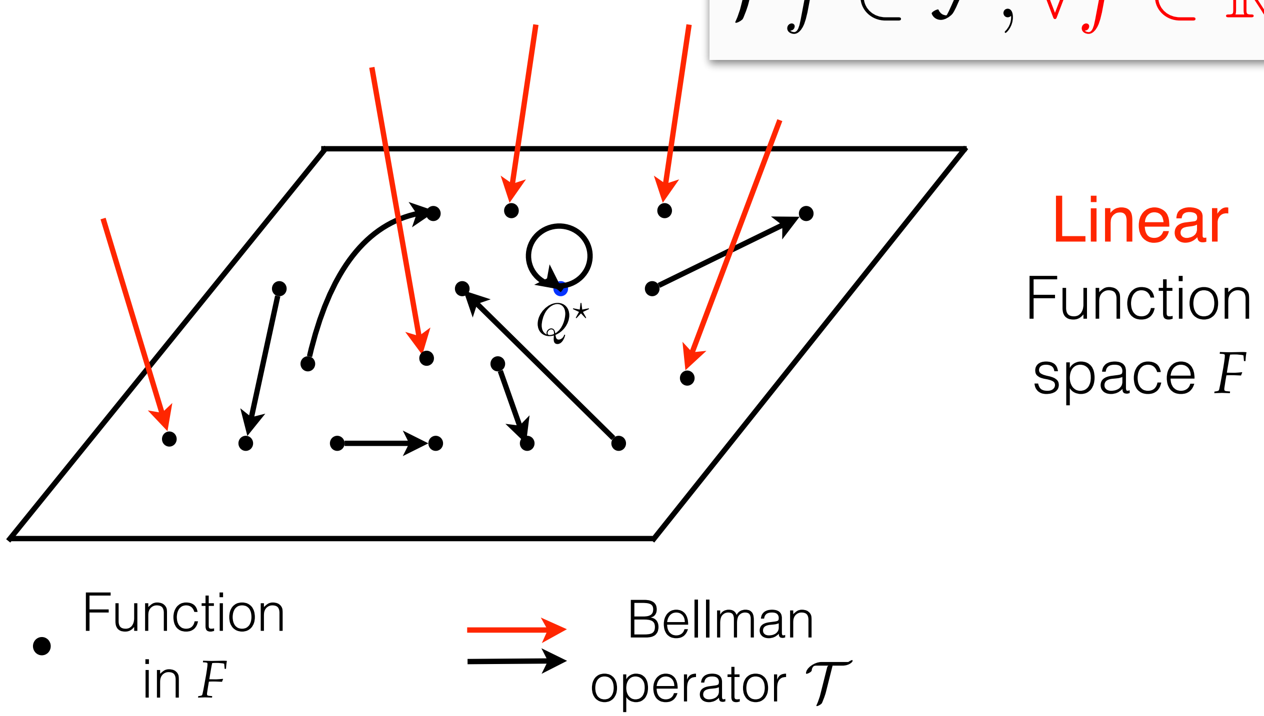

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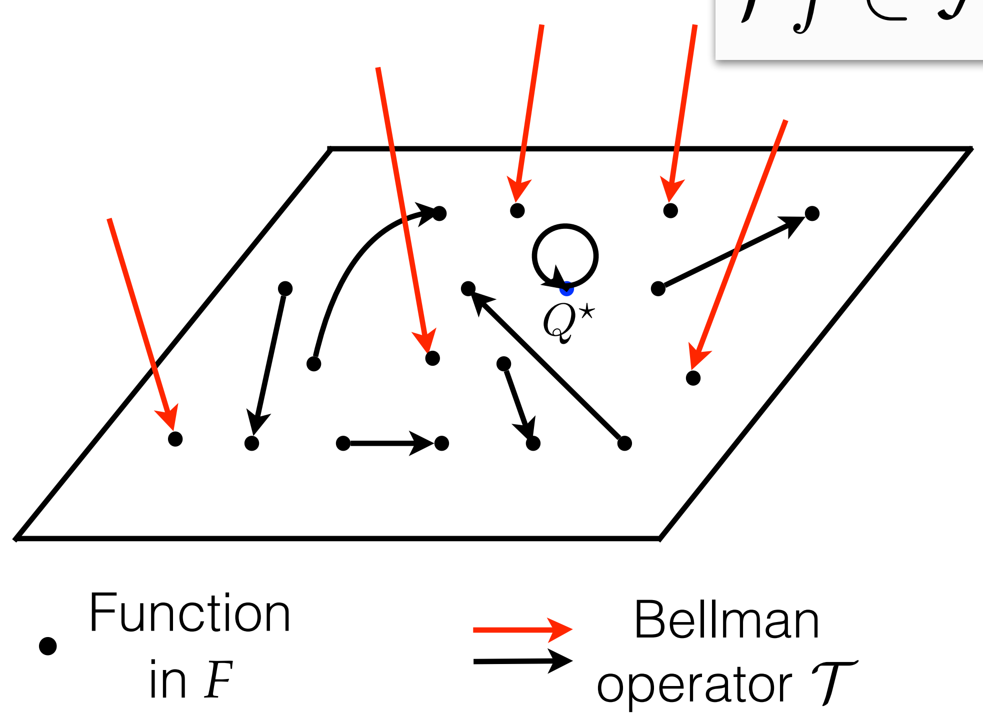

Linear Function space F

$\Big\} \Longleftrightarrow$ Linear MDP [JYWJ'20]

Restrict MDP dynamics

P(s'|s,a) = ✓ x ?

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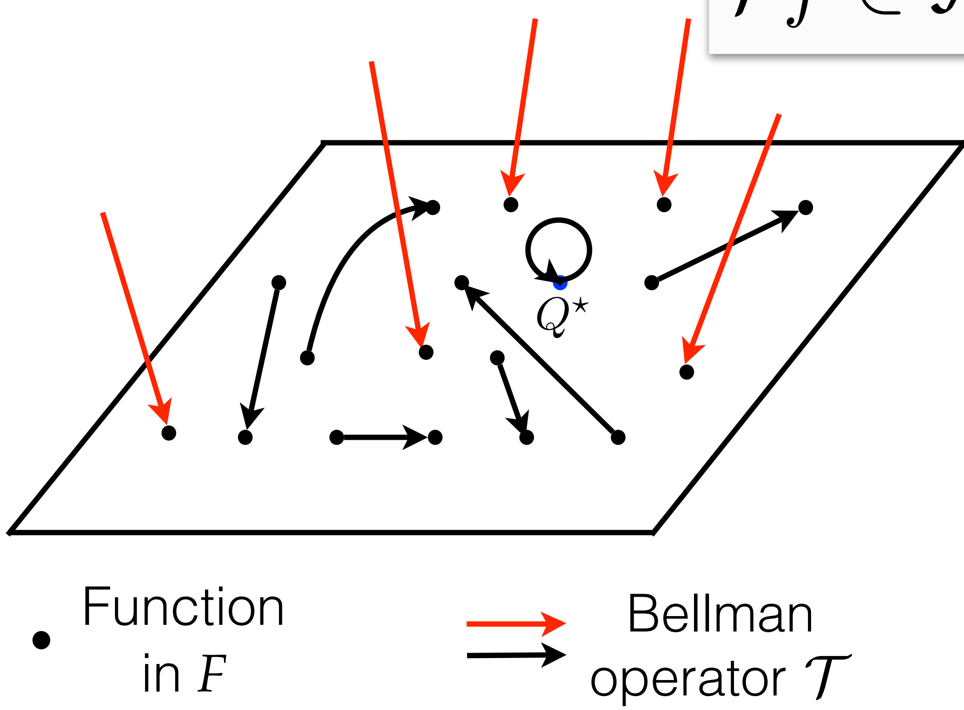

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$\Leftrightarrow$ Linear MDP [JYWJ'20]

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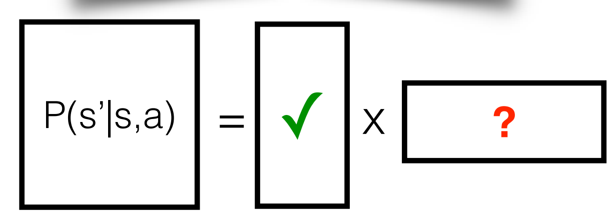

Beyond linear MDPs?
- No restriction on MDP
- Allow non-linear F

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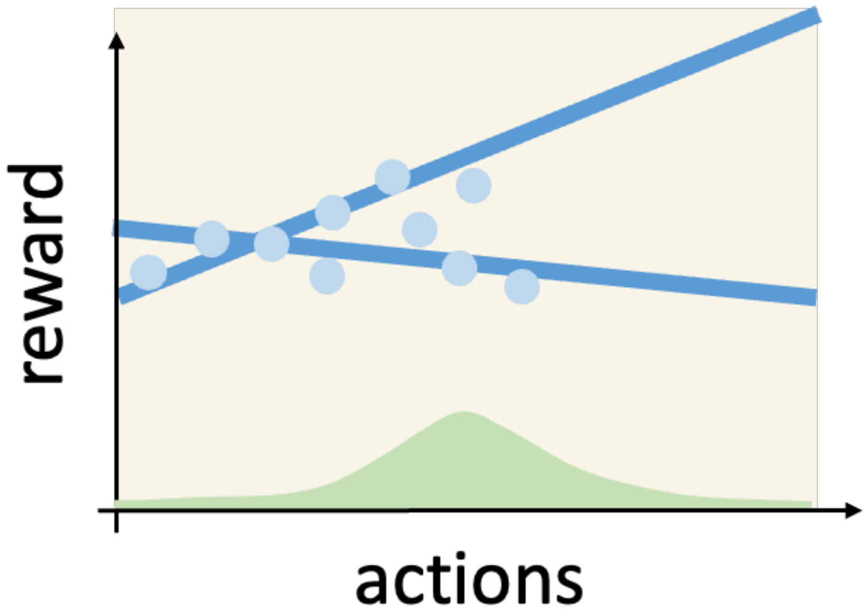
Bellman-consistent pessimism [XCJMA’21]



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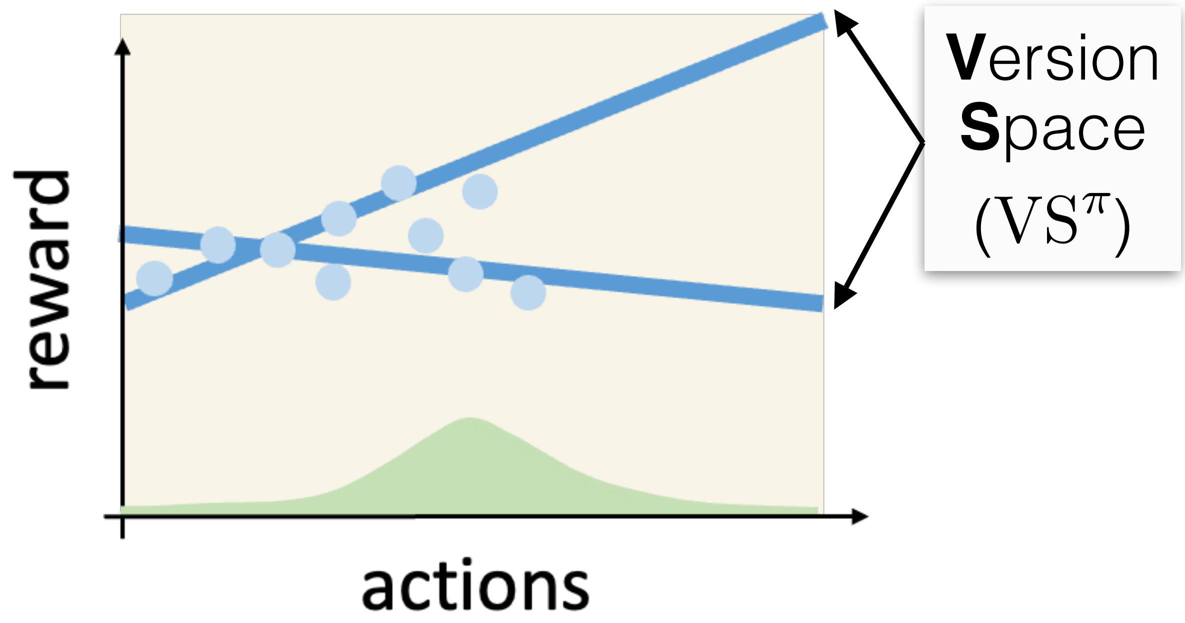

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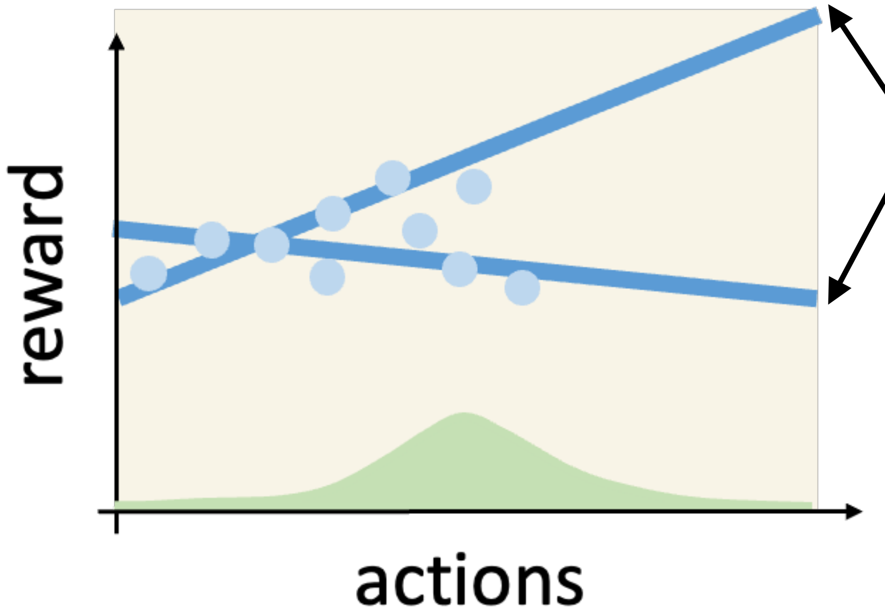

Version **S**pace (VS^π)

$$\mathrm{VS}^\pi := \{f : \mathbb{E}_D[(f - \mathcal{T}^\pi f)^2] \approx 0\}$$

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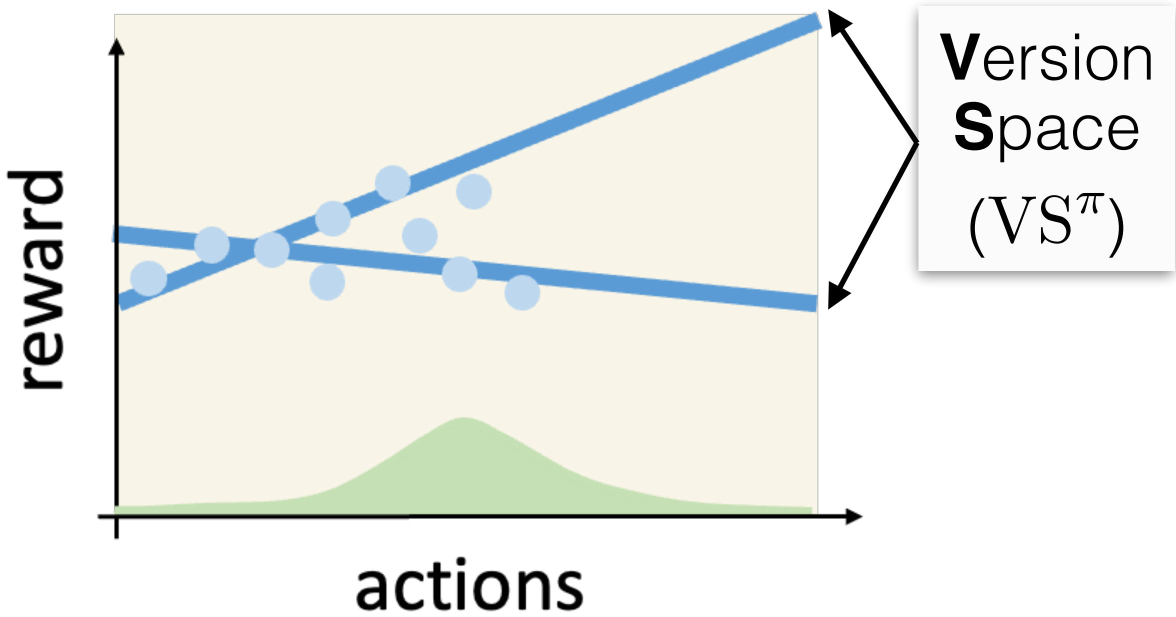

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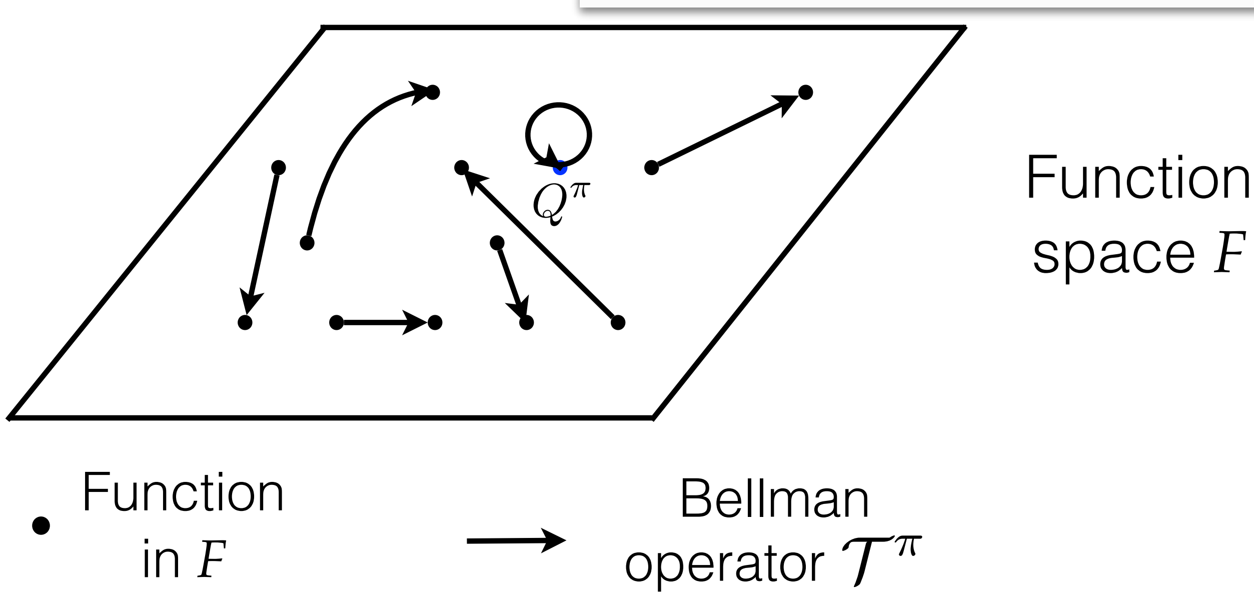
"Bellman-completeness" [AMS'08]

$\mathcal{T}^\pi f \in \mathcal{F}, \forall f \in \mathcal{F}, \pi \in \Pi$



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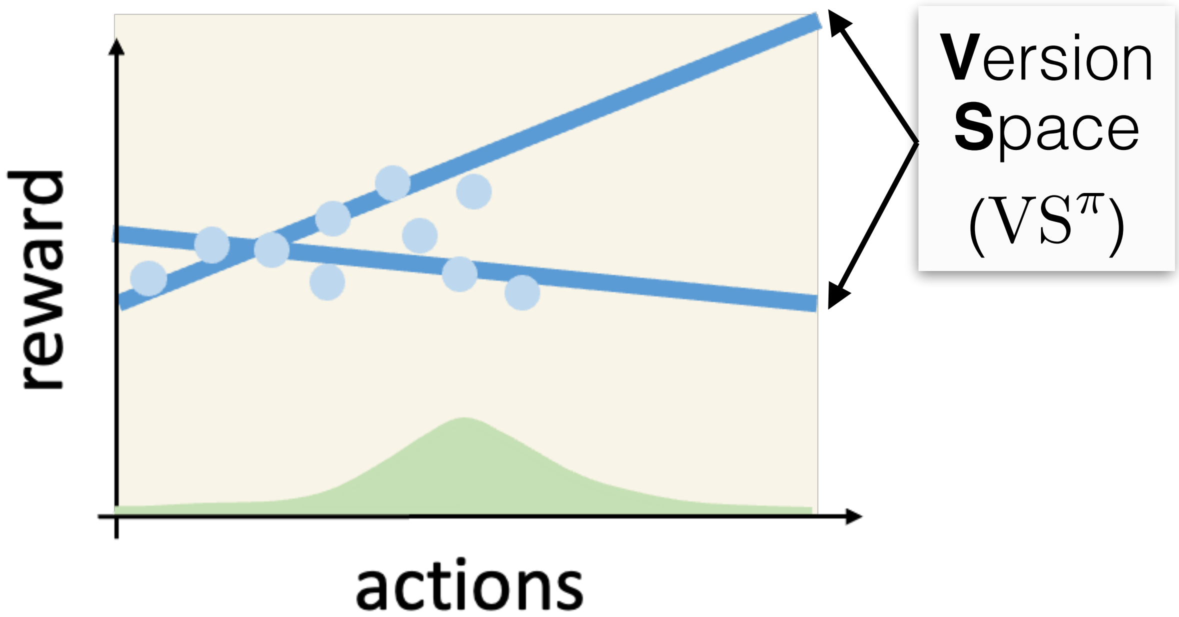

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- Overly-conservative by imagining impossible scenarios
  - *Ex.* In linear MDPs, true Q^π linear, but $\widehat{Q}^\pi$ quadratic

- Strong assumptions for point-wise uncertainty
  - Restrict MDP dynamics

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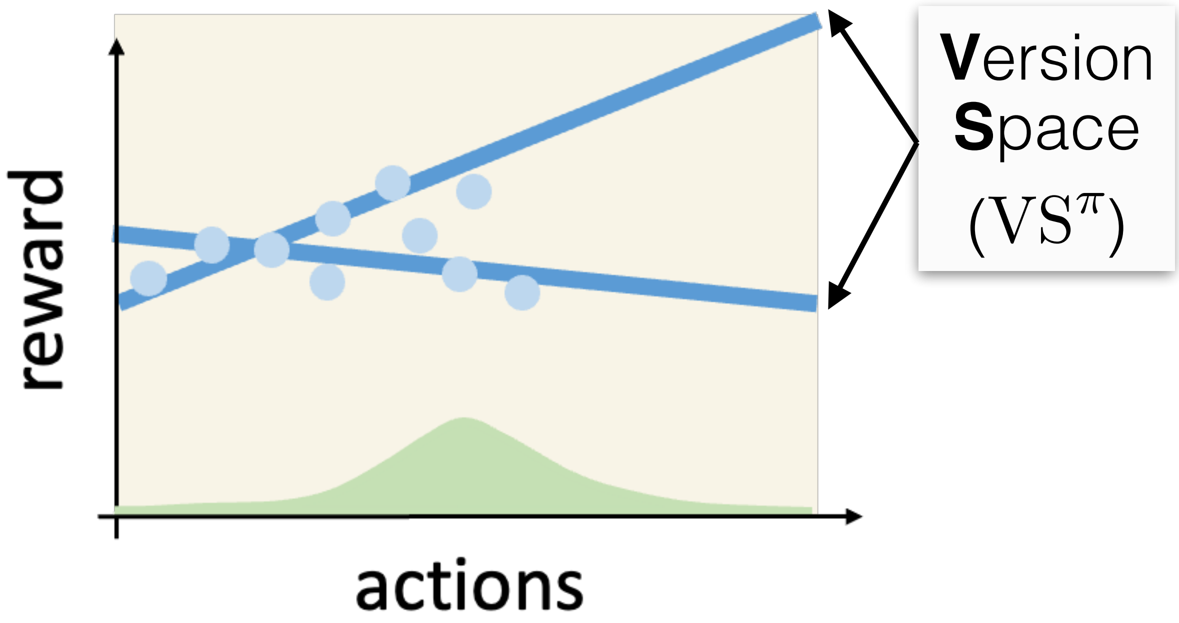

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- Less conservative: only plausible scenarios
  - *Ex.* In linear MDPs, argmin f is linear

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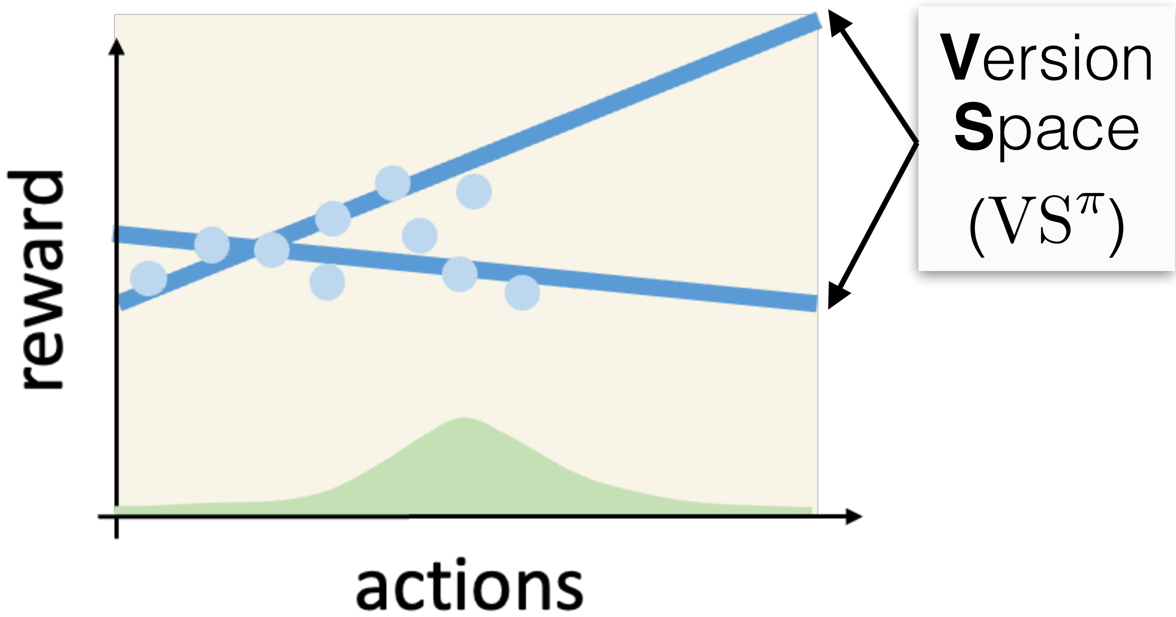

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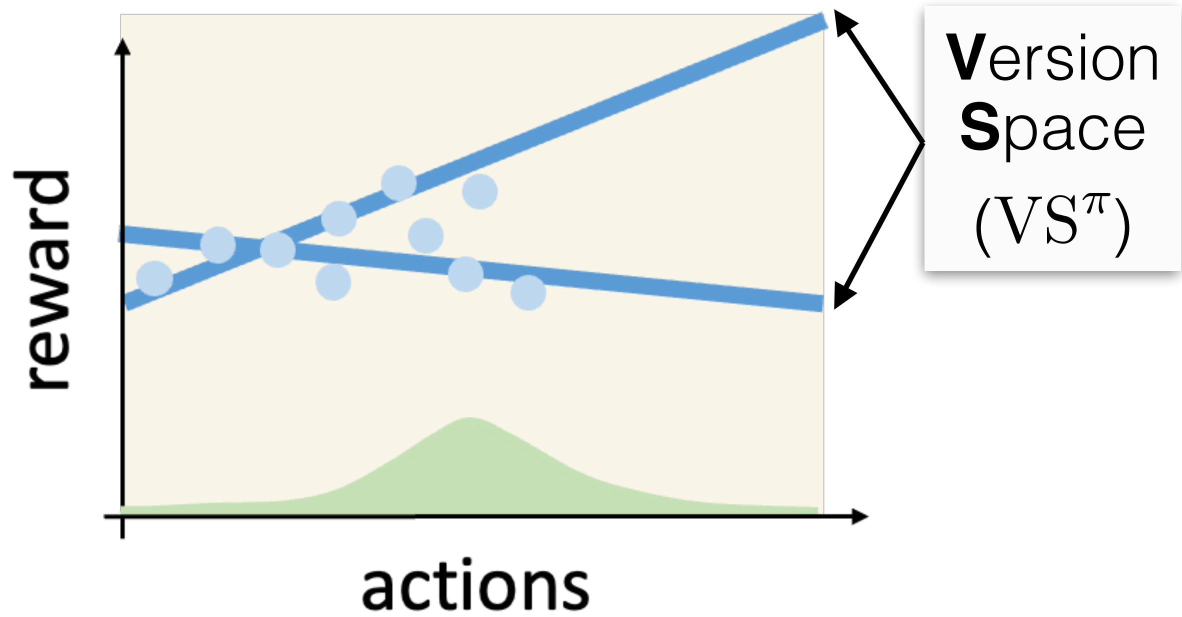

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Formal guarantee in backup slide

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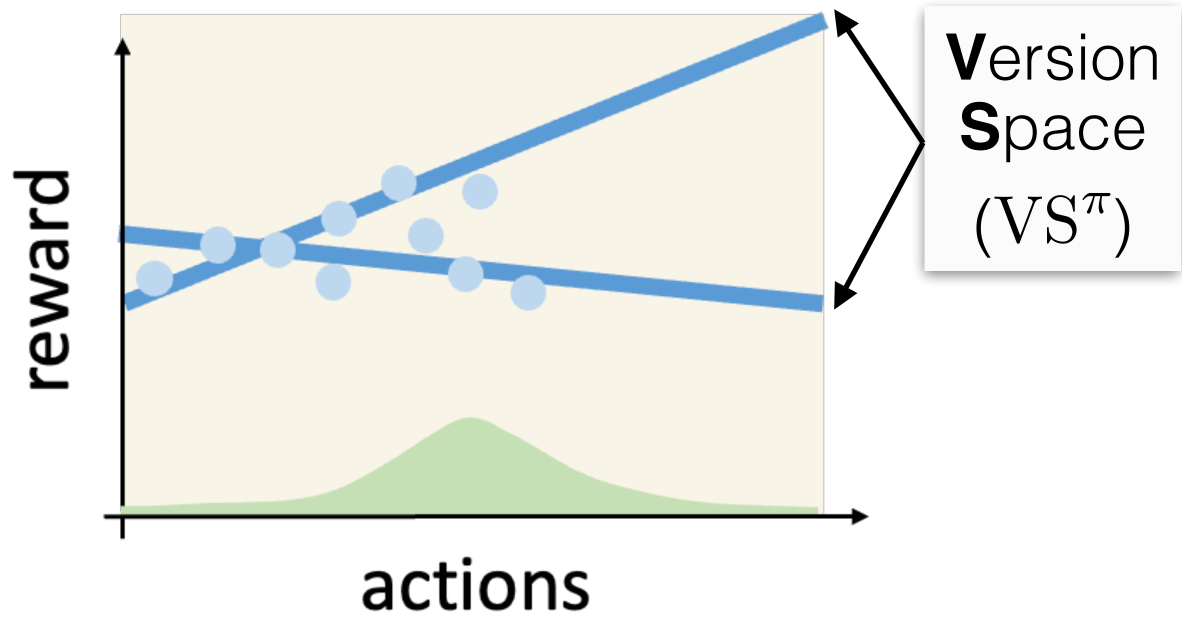

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Computational efficiency?

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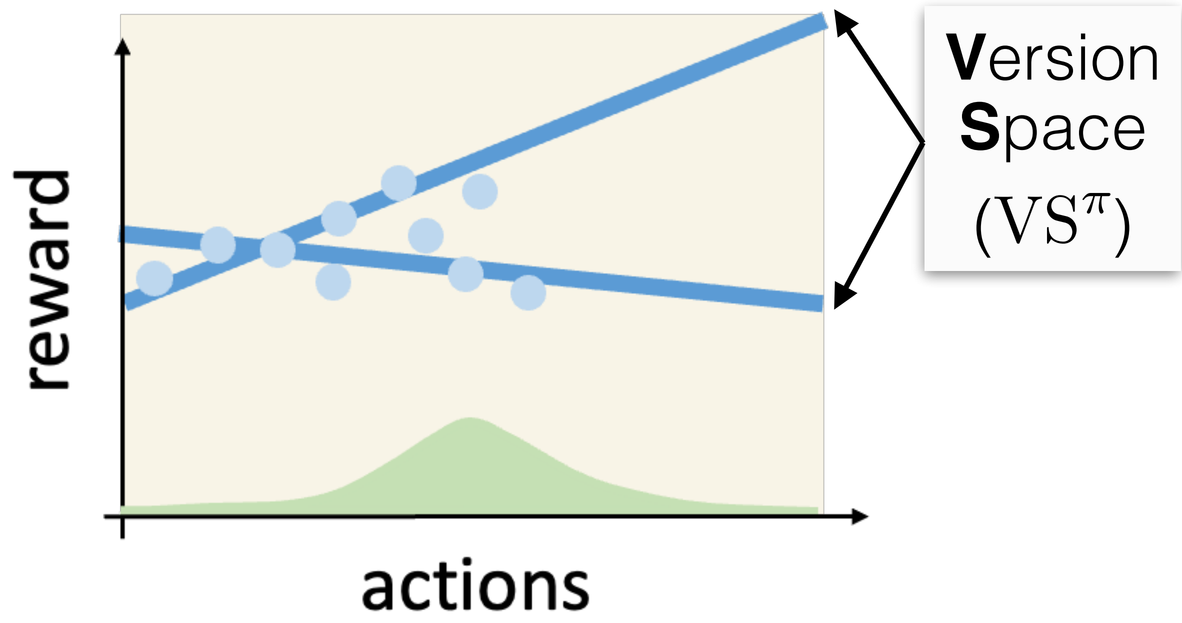

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$$\arg\max_{\pi\in\Pi} \min_{f\in\mathcal{F}} \max_{f'\in\mathcal{F}} f(s_0,\pi) + \lambda \mathbb{E}_D[(f(s,a) - r - \gamma f(s',\pi))^2] - \mathbb{E}_D[(f'(s,a) - r - \gamma f(s',\pi))^2]$$

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Online RL

Offline RL

$$\arg\max_{\pi \in \Pi} \max_{f \in \mathrm{VS}^\pi} f(s_0, \pi)$$

$$\arg\max_{\pi \in \Pi} \min_{f \in \mathrm{VS}^\pi} f(s_0, \pi)$$

Bellman-consistent
(version space)

Point-wise

	Online RL	Offline RL
Bellman-consistent (version space)	$\arg\max_{\pi\in\Pi} \max_{f\in \mathrm{VS}^\pi} f(s_0, \pi)$	$\arg\max_{\pi\in\Pi} \min_{f\in \mathrm{VS}^\pi} f(s_0, \pi)$
Point-wise		

	Online RL	Offline RL
Bellman-consistent (version space)	$\arg\max_{\pi\in\Pi} \max_{f\in\mathrm{VS}^\pi} f(s_0,\pi)$ • Statistical guarantee in very general settings [JKALS'17] ✓	$\arg\max_{\pi\in\Pi} \min_{f\in\mathrm{VS}^\pi} f(s_0,\pi)$
Point-wise		

Online RL | Offline RL

Bellman-consistent (version space)

$$\arg\max_{\pi \in \Pi} \max_{f \in \mathrm{VS}^\pi} f(s_0, \pi)$$

- Statistical guarantee in very general settings [JKALS'17] ✓
- NP-hardness under strong oracles [DJKALS'18] X

$$\arg\max_{\pi \in \Pi} \min_{f \in \mathrm{VS}^\pi} f(s_0, \pi)$$

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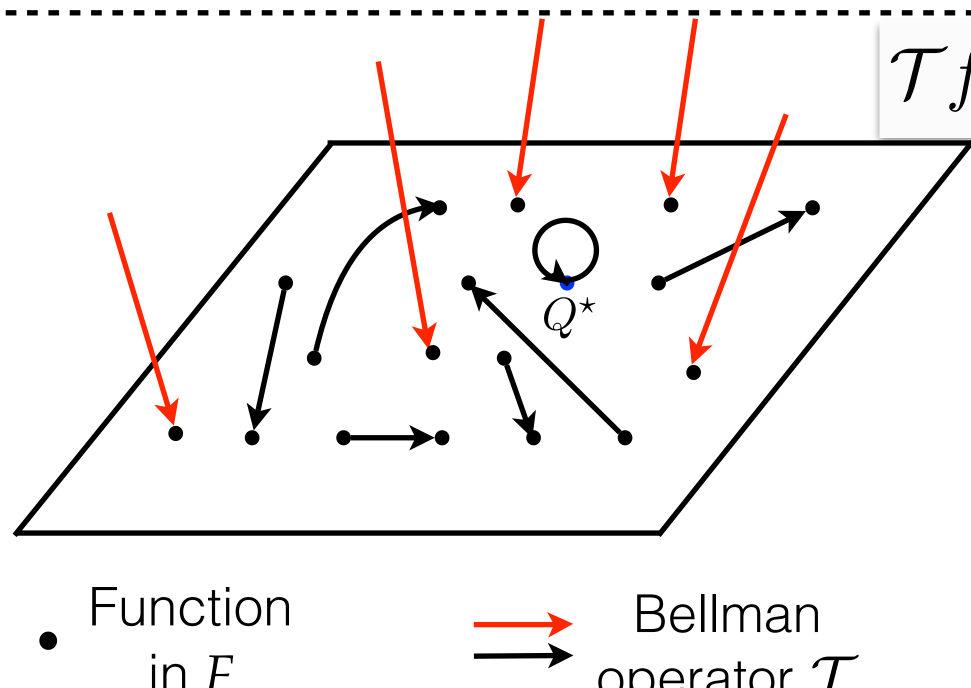

$\mathcal{T}f \in \mathcal{F}, \forall f \in \mathbb{R}^{\mathcal{S}\times\mathcal{A}}$

Linear Function space F

$\Leftrightarrow$ Linear MDP [JYWJ'20]

Online RL	Offline RL
$\arg\max_{\pi \in \Pi} \max_{f \in \mathrm{VS}^\pi} f(s_0, \pi)$	$\arg\max_{\pi \in \Pi} \min_{f \in \mathrm{VS}^\pi} f(s_0, \pi)$

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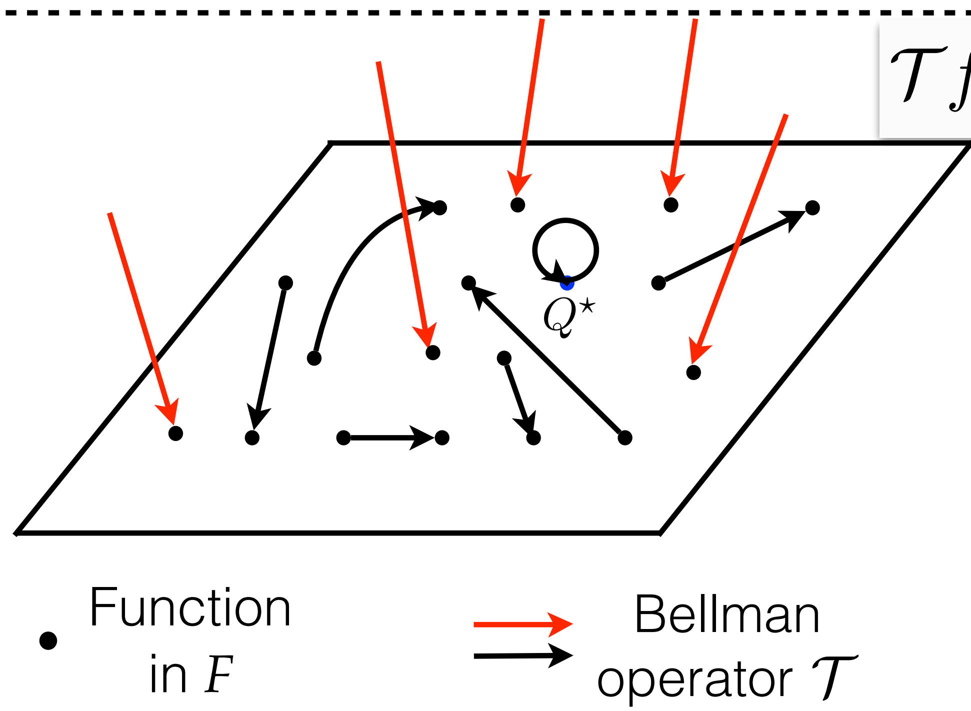

Linear Function space F

$\mathcal{T} f \in \mathcal{F}, \forall f \in \mathbb{R}^{\mathcal{S}\times\mathcal{A}}$ } $\Leftrightarrow$ Linear MDP [JYWJ'20]

Computationally efficient ✓

Online RL

Offline RL

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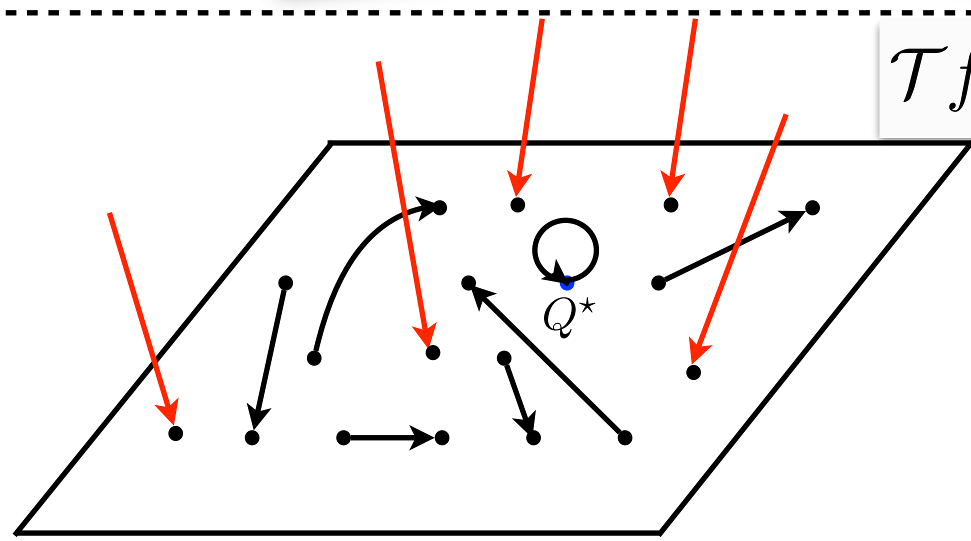
Statistical generality
vs
Computational tractability?

Point-wise



$\mathcal{T} f \in \mathcal{F}, \forall f \in \mathbb{R}^{\mathcal{S}\times\mathcal{A}}$ } ⇔ Linear MDP [JYWJ'20]

Linear Function space F

Computationally efficient ✓

• Function in F

Bellman operator $\mathcal{T}$

Online RL

Offline RL

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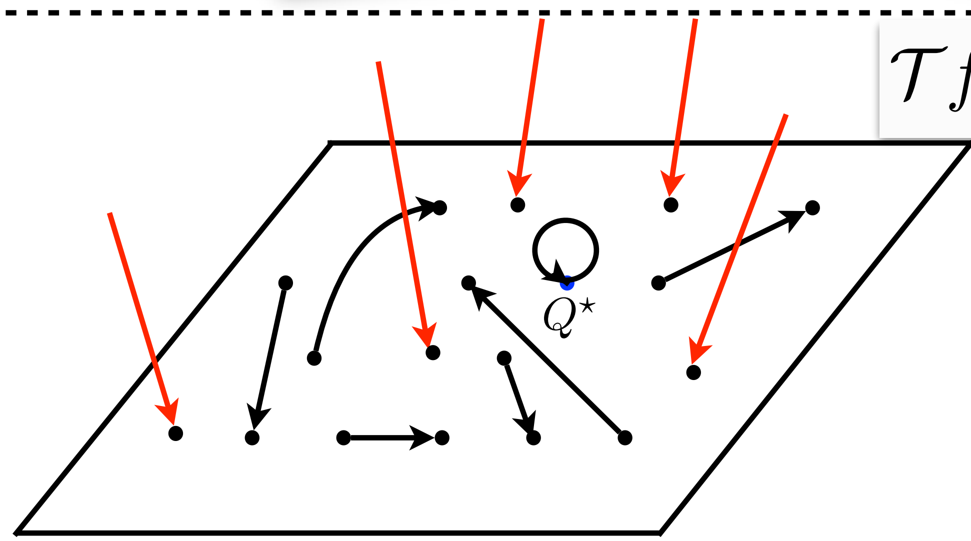

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• Function in F

→ Bellman operator $\mathcal{T}$

Freund and Schapire.1999. Adaptive game playing using multiplicative weights.

Bellman-consistent (version space)

Online RL

$$\arg\max_{\pi \in \Pi} \max_{f \in \mathrm{VS}^{\pi}} f(s_0, \pi)$$

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Statistical generality
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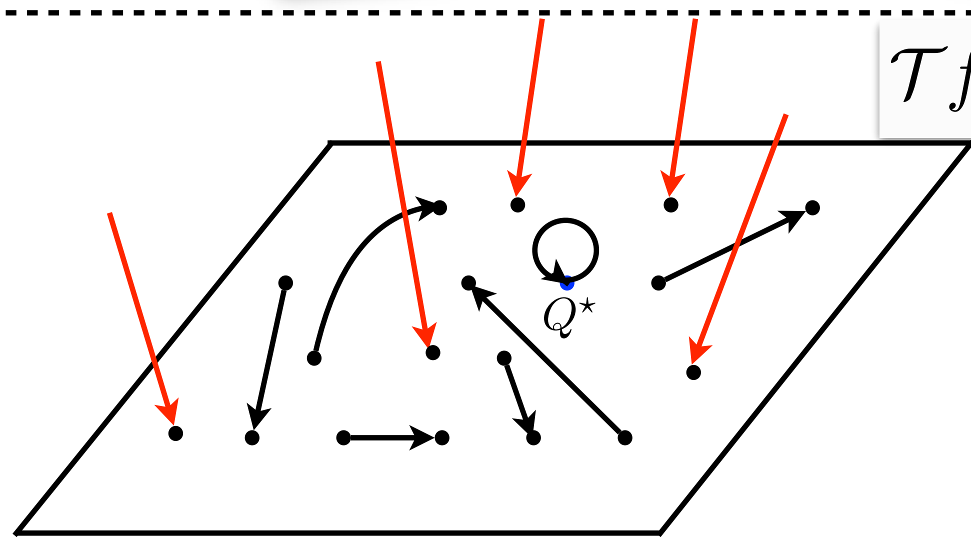
- Oracle-efficient ! ✓
- Oracle itself is efficient in the linear setting (pessimistic LSTD)

Point-wise



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Robustness of offline RL

Example: network control

- Status quo: time-tested heuristics

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Robustness of offline RL

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Imitation Learning

WHEN SHOULD WE PREFER OFFLINE REINFORCEMENT LEARNING OVER BEHAVIORAL CLONING?

Aviral Kumar[*,1,2], Joey Hong[*,1], Anikait Singh[1], Sergey Levine[1,2]

Robustness of offline RL

Example: network control

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- RL: training instability
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Imitation Learning

- Reliably learn the data policy

Offline RL

- Can be worse than data policy

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- Performance ceiling

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Best of both worlds?

ATAC: *Relative* Pessimism [CXJA'22]

$$\arg\max_{\pi \in \Pi} \text{tight lower bound of } J(\pi)$$

Cheng et al. ICML-22. Adversarially Trained Actor Critic for Offline Reinforcement Learning.
Swamy et al. ICML-21. Of Moments and Matching: A Game-Theoretic Framework for Closing the Imitation Gap.

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data policy

$$\arg\max_{\pi \in \Pi} \text{tight lower bound of } J(\pi) - J(\pi_D)$$

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"Performance-diff Lemma" [Langford & Kakade'02]

$$J(\pi) - J(\pi_D) \propto \mathbb{E}_{(s,a)\sim D}[Q^\pi(s,\pi) - Q^\pi(s,a)]$$

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- λ small (≈ 0): (adversarial) **Imitation Learning**!
  - strong discriminator (π must imitate π_D)
  - IL requires weaker assumptions ($\pi_D \in \Pi + Q^\pi \in \mathcal{F}, \forall \pi \in \Pi$)

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Bellman-error regularization

- λ small (≈ 0): (adversarial) **Imitation Learning**!
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  - IL requires weaker assumptions ($\pi_D \in \Pi + Q^\pi \in \mathcal{F}, \forall \pi \in \Pi$)
- Well-specified λ: offline RL
  - weakens discriminator, allowing π to further improve

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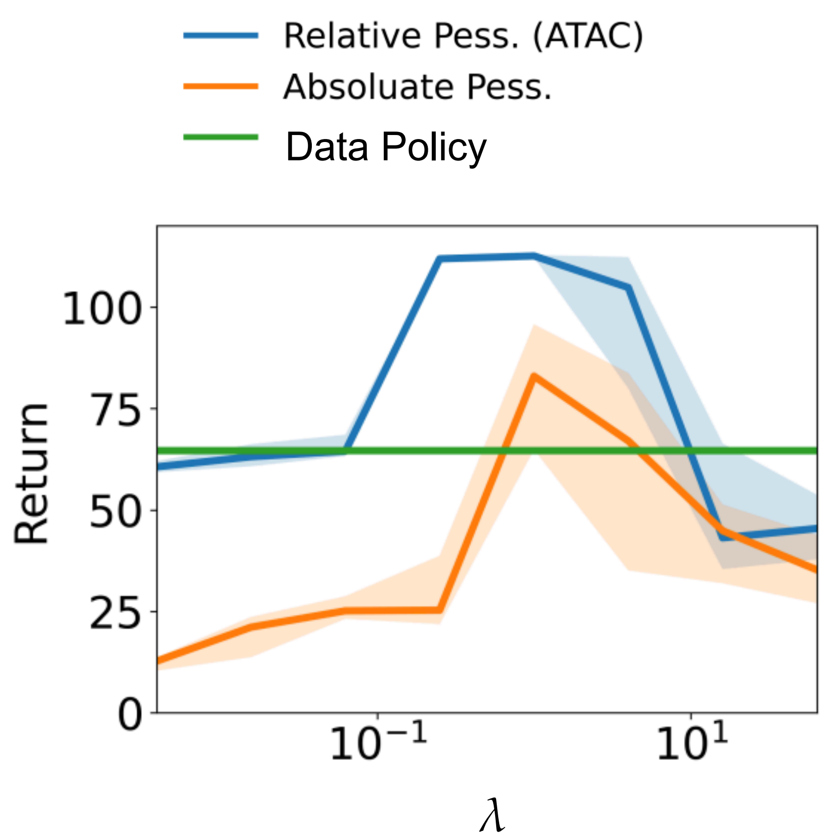
Empirical evaluation



(d) hopper-medium-expert

	Behavior	ATAC*	CQL	COMBO	TD3BC	IQL	BC
halfcheetah-rand	-0.1	4.8	35.4	**38.8**	10.2	-	2.1
walker2d-rand	0.0	**8.0**	7.0	7.0	1.4	-	1.6
hopper-rand	1.2	**31.8**	10.8	17.9	11.0	-	9.8
halfcheetah-med	40.6	**54.3**	44.4	**54.2**	42.8	47.4	36.1
walker2d-med	62.0	**91.0**	74.5	75.5	79.7	78.3	6.6
hopper-med	44.2	**102.8**	86.6	94.9	**99.5**	66.3	29.0
halfcheetah-med-replay	27.1	49.5	46.2	**55.1**	43.3	44.2	38.4
walker2d-med-replay	14.8	**94.1**	32.6	56.0	25.2	73.9	11.3
hopper-med-replay	14.9	**102.8**	48.6	73.1	31.4	94.7	11.8
halfcheetah-med-exp	64.3	**95.5**	62.4	90.0	**97.9**	86.7	35.8
walker2d-med-exp	82.6	**116.3**	98.7	96.1	101.1	**109.6**	6.4
hopper-med-exp	64.7	**112.6**	**111.0**	**111.1**	**112.2**	91.5	**111.9**
pen-human	207.8	79.3	37.5	-	-	71.5	34.4
hammer-human	25.4	**6.7**	**4.4**	-	-	1.4	1.5
door-human	28.6	8.7	**9.9**	-	-	4.3	0.5
relocate-human	86.1	**0.3**	**0.2**	-	-	**0.1**	**0.0**
pen-cloned	107.7	73.9	39.2	-	-	37.3	56.9
hammer-cloned	8.1	2.3	2.1	-	-	2.1	0.8
door-cloned	12.1	**8.2**	0.4	-	-	1.6	-0.1
relocate-cloned	28.7	**0.8**	**-0.1**	-	-	**-0.2**	**-0.1**
pen-exp	105.7	**159.5**	107.0	-	-	-	85.1
hammer-exp	96.3	**128.4**	86.7	-	-	-	**125.6**
door-exp	100.5	**105.5**	**101.5**	-	-	-	34.9
relocate-exp	101.6	**106.5**	95.0	-	-	-	**101.3**

New perspective that bridges IL and offline RL

- IL ($\lambda \approx 0$): strong discriminator (π must imitate π_D)
- RL weakens discriminator, allowing π to further improve

Cheng et al. ICML-22. Adversarially Trained Actor Critic for Offline Reinforcement Learning.

Online RL

Offline RL

Bellman-consistent (version space)

Online RL:

$$\arg\max_{\pi \in \Pi} \max_{f \in \mathrm{VS}^{\pi}} f(s_0, \pi)$$

- Statistical guarantee in very general settings [JKALS'17] ✓
- NP-hardness under strong oracles [DJKALS'18] ✗

Offline RL:

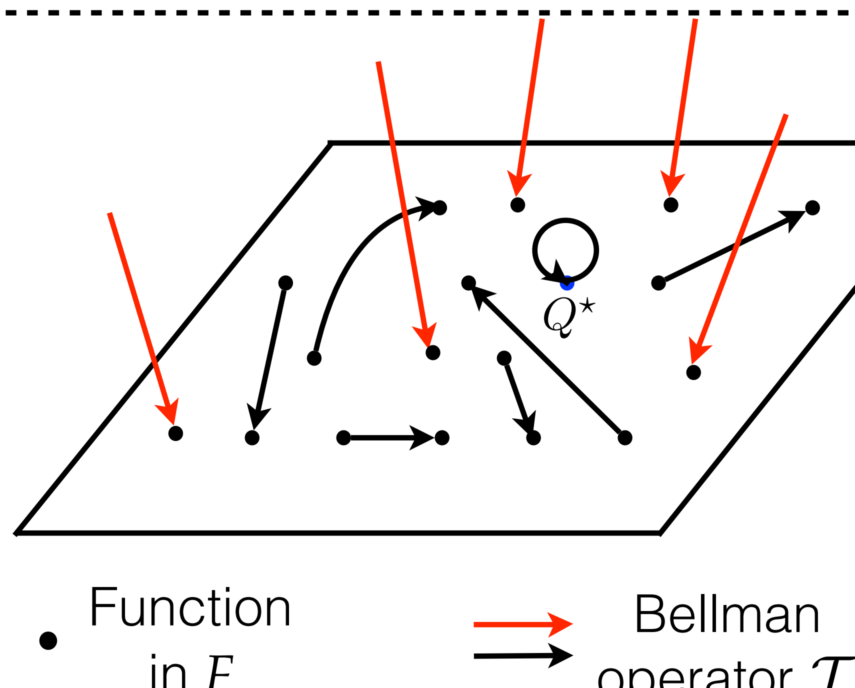
$$\arg\max_{\pi \in \Pi} \min_{f \in \mathrm{VS}^{\pi}} f(s_0, \pi)$$

$$f_k \leftarrow \arg\min_{f \in \mathrm{VS}^{\pi_k}} f(s_0, \pi_k)$$

$$\pi_{k+1} \leftarrow \mathrm{NoRegret}(\pi_k, f_k)$$

- Oracle-efficient ! ✓
- Oracle itself is efficient in the linear setting (pessimistic LSTD)

Point-wise



Linear Function space F

$$\mathcal{T} f \in \mathcal{F}, \forall f \in \mathbb{R}^{\mathcal{S} \times \mathcal{A}}$$

$\Leftrightarrow$ Linear MDP [JYWJ'20]

Computationally efficient ✓

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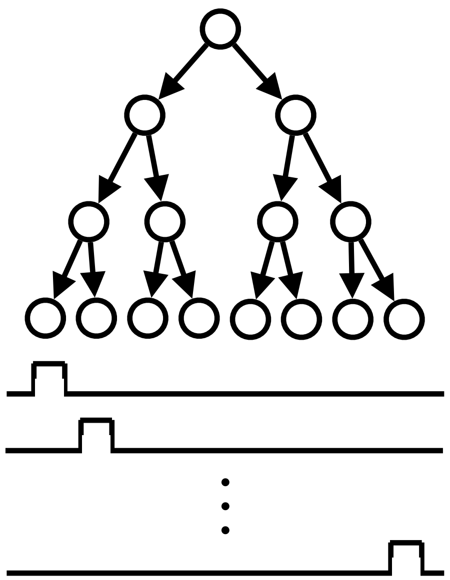

Bounded statistical complexities (e.g., VC-type dim) are insufficient! [KAL'16, JKALS'17]

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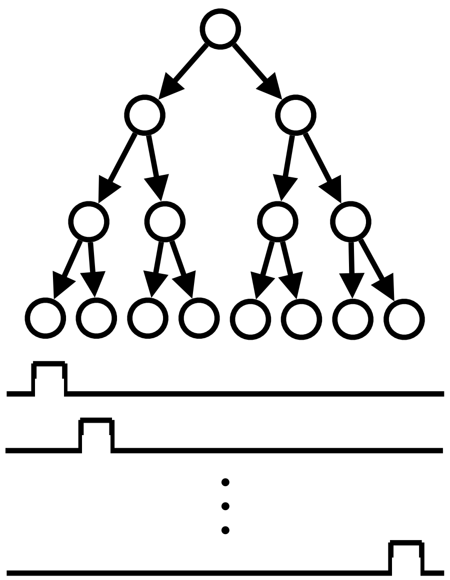
Structural assumptions



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Structural assumptions

Low Bellman rank [JKALS'17]

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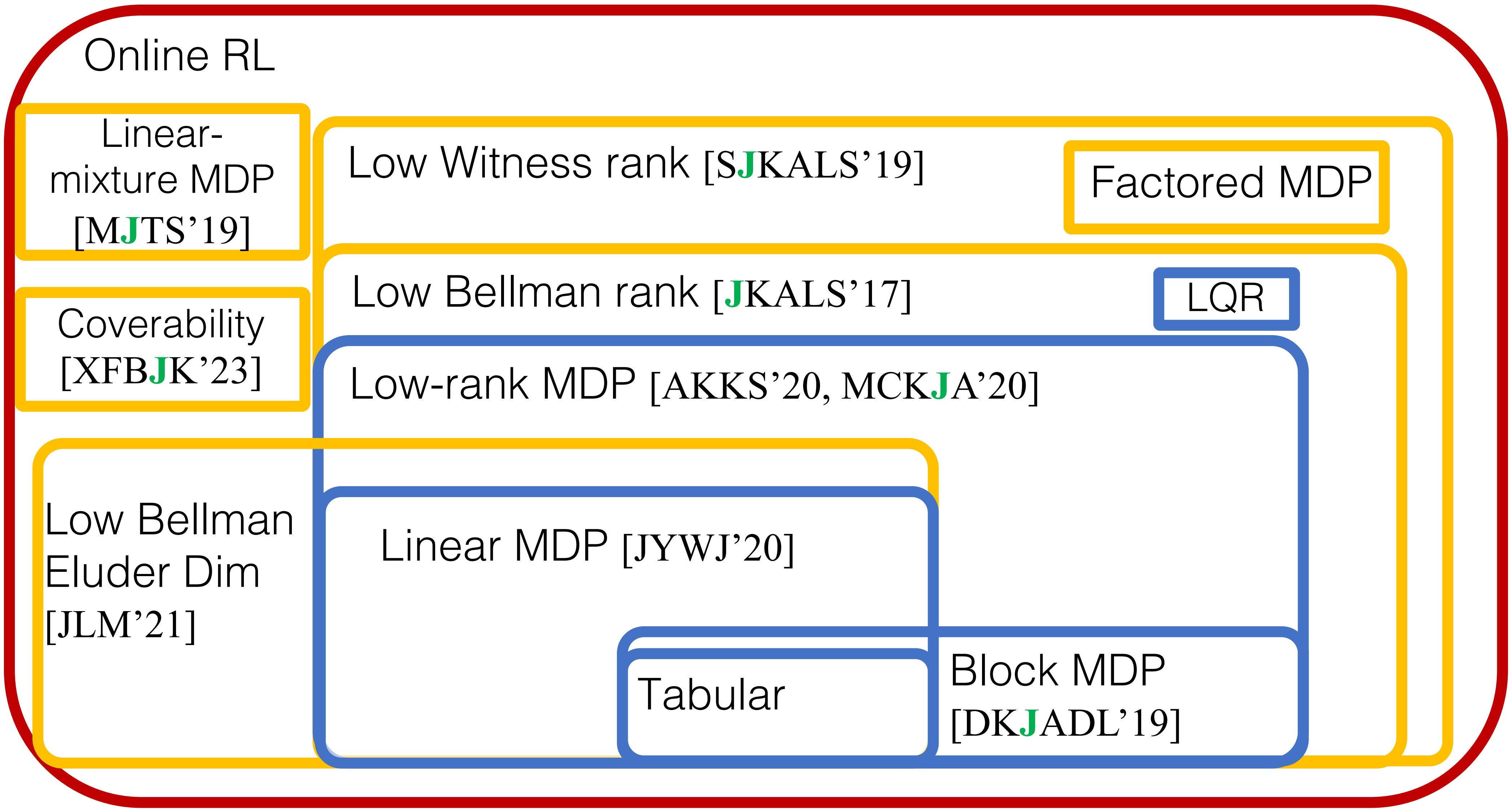

- Adapted from FOCS'20 Tutorial by Agarwal, Krishnamurthy, and Langford
- Also related: bilinear classes [DKLLMSW'21], DEC [FKQR'21]
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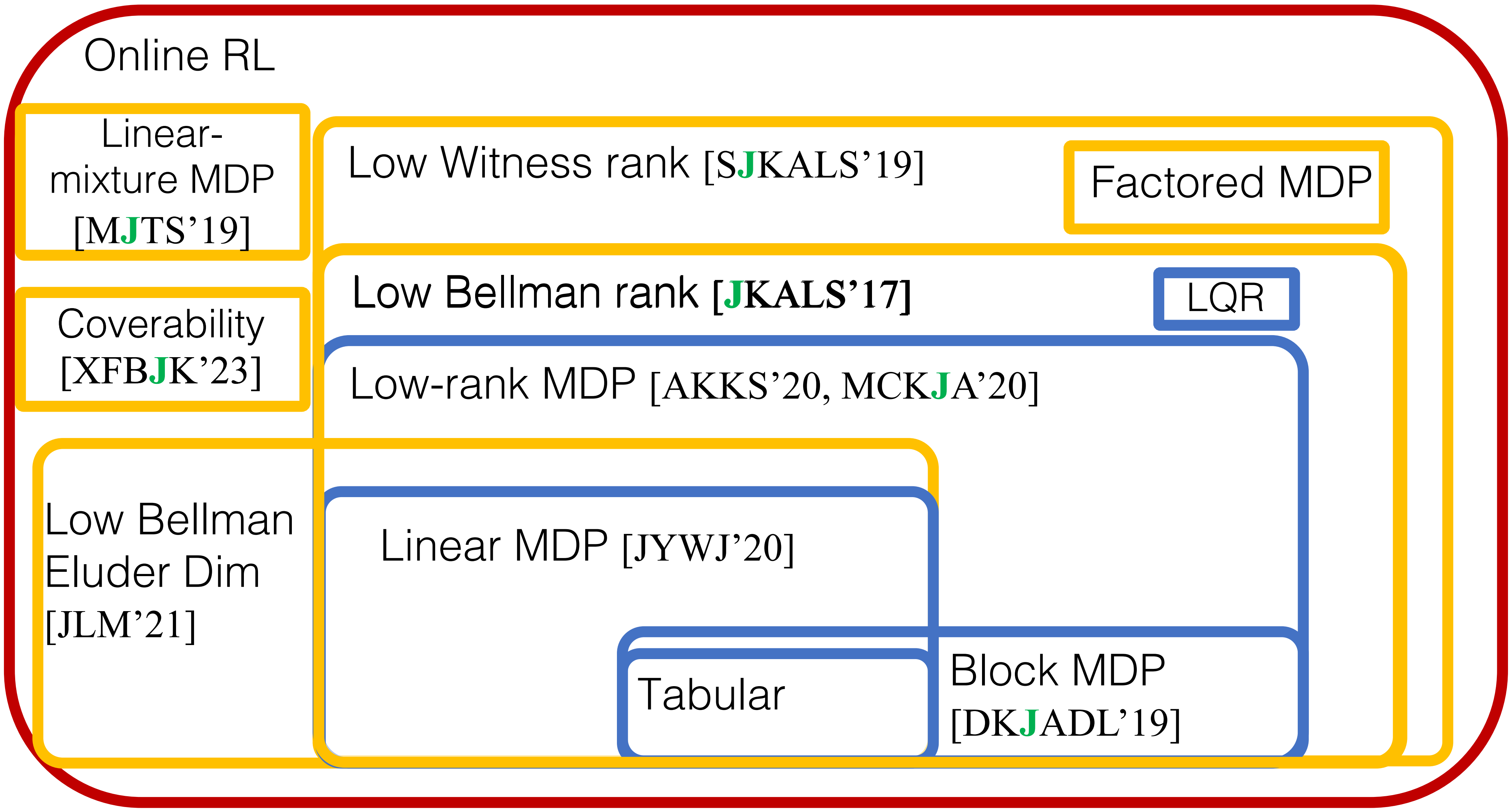
- All consider modeling value functions



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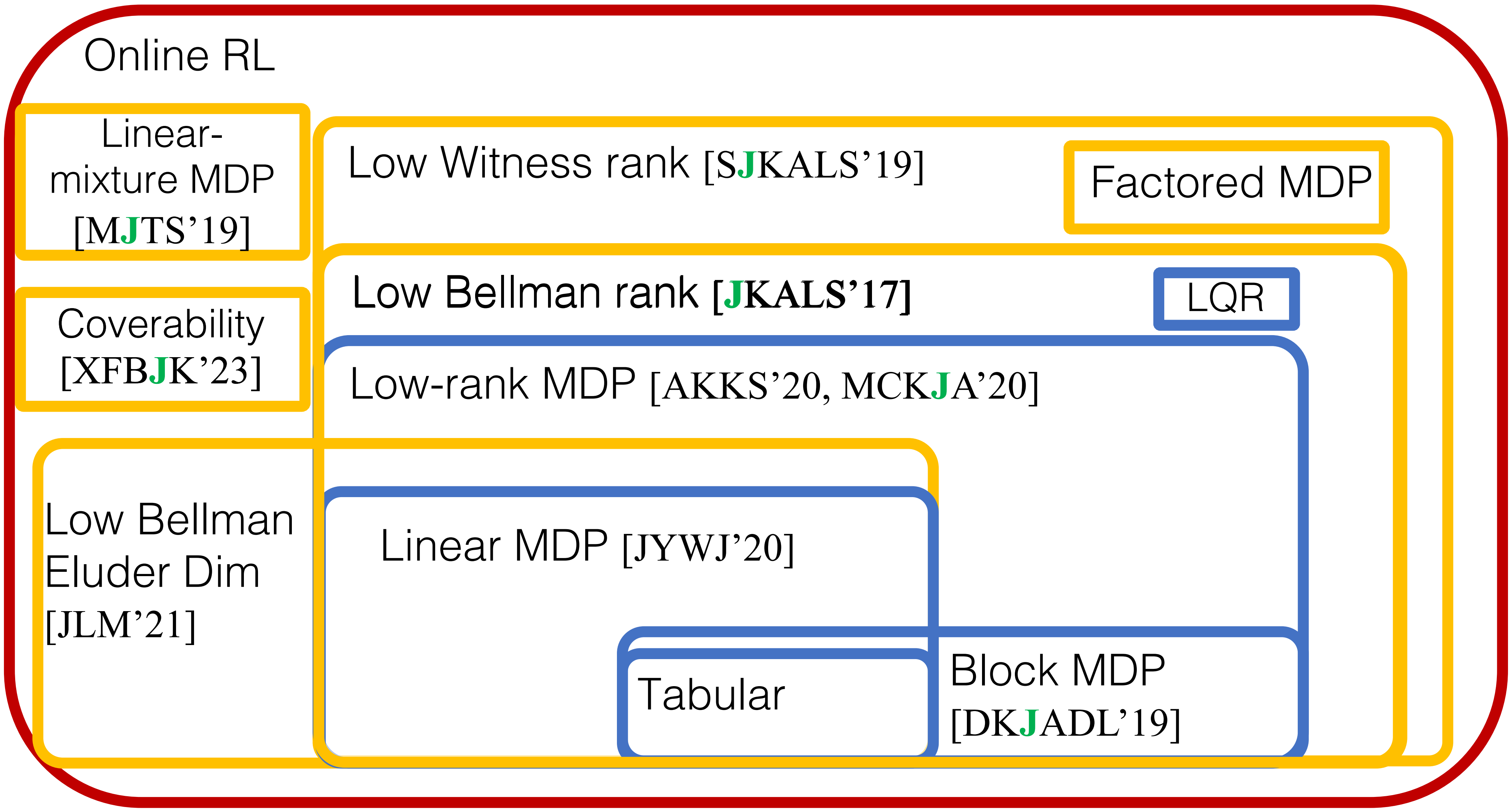
Discriminative learning

Generative learning?



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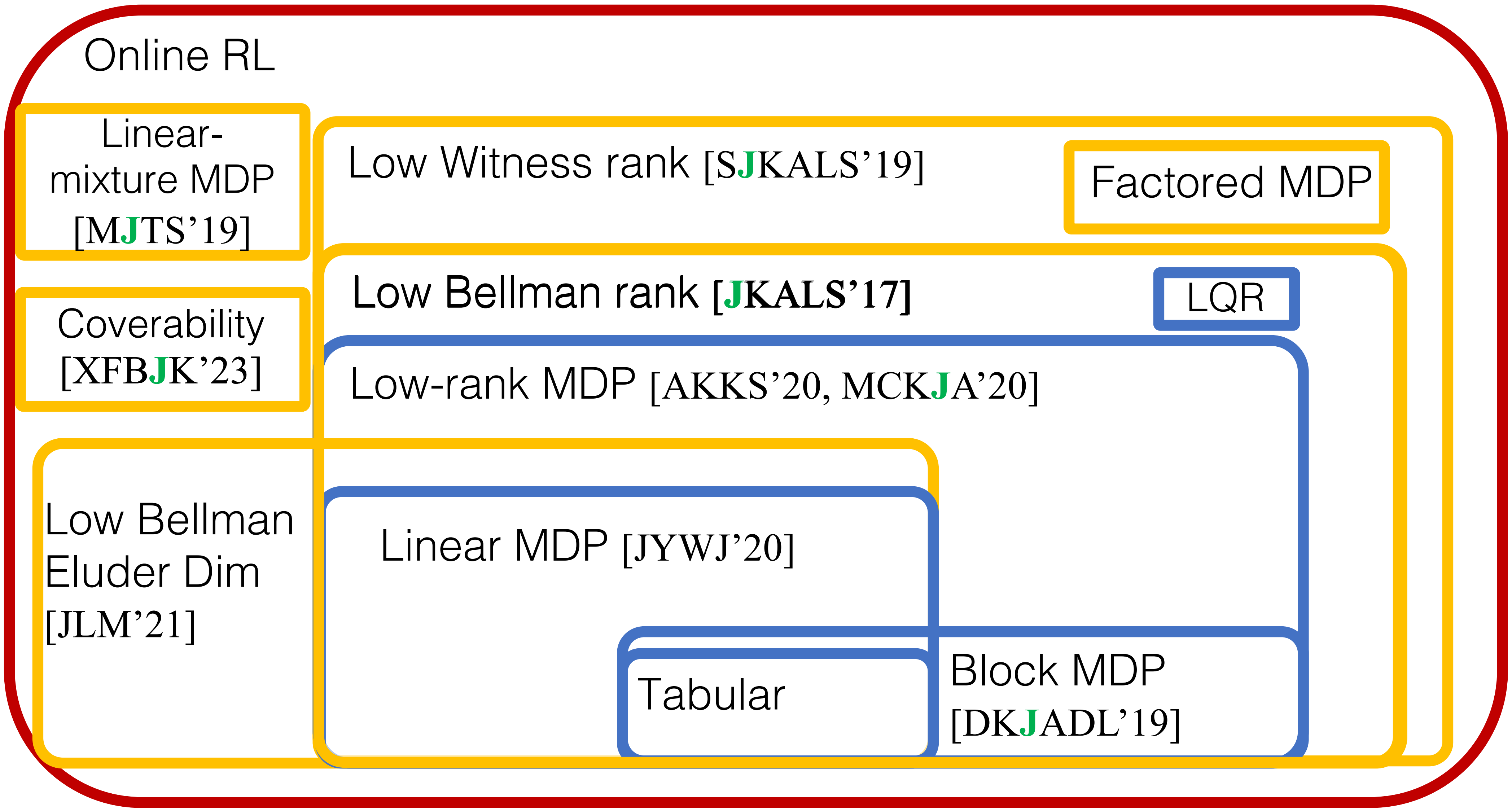

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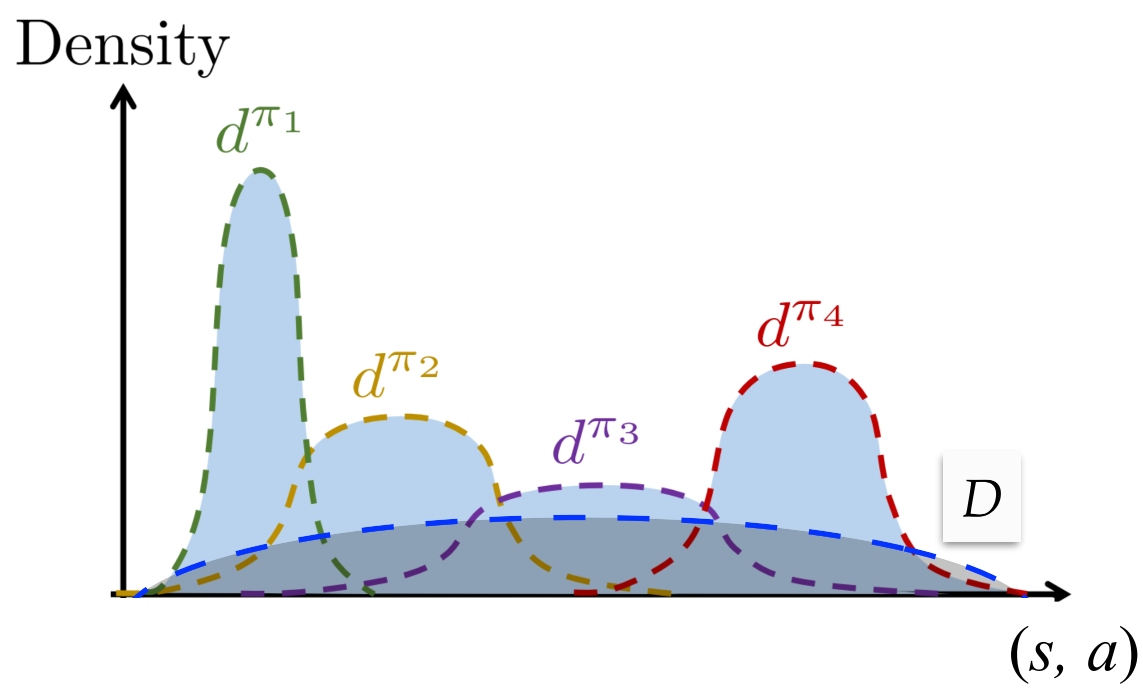

Coverability [XFBJK'23]

- Online vs. Offline: Unification

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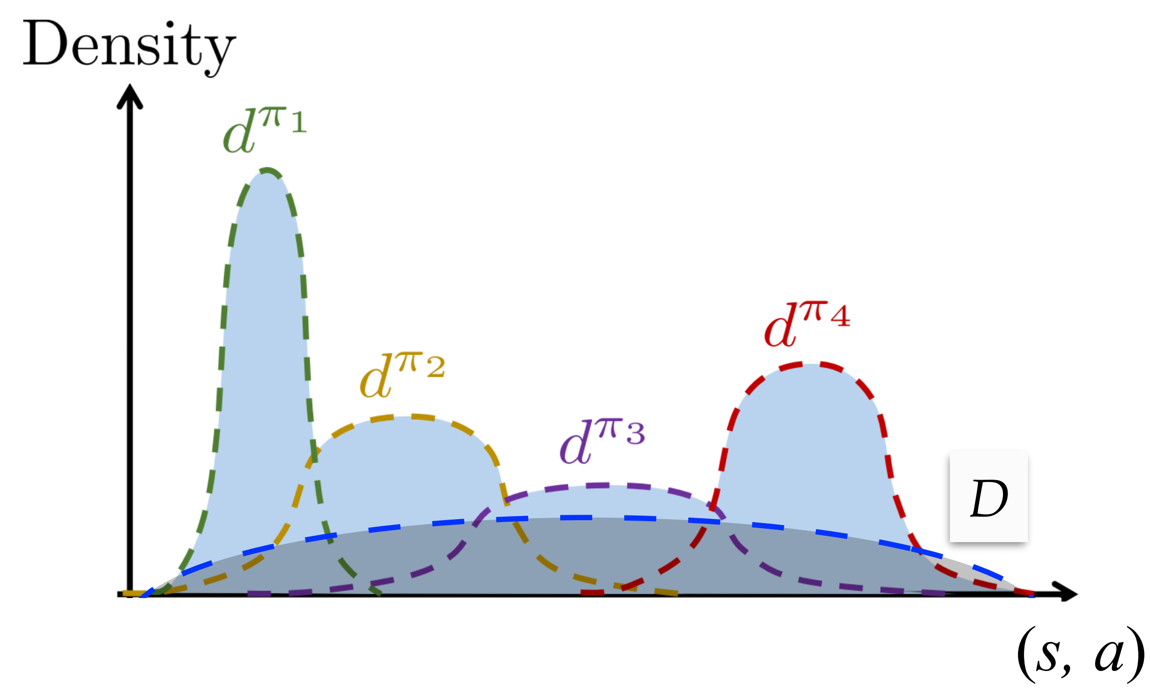
Generative learning?



Coverability [XFBJK'23]

- Online vs. Offline: Unification
  - Not all MDPs admit such D
  - Those who do can be explored efficiently

Longterm directions

- RL (theory) so far: mostly single-agent & Markovian

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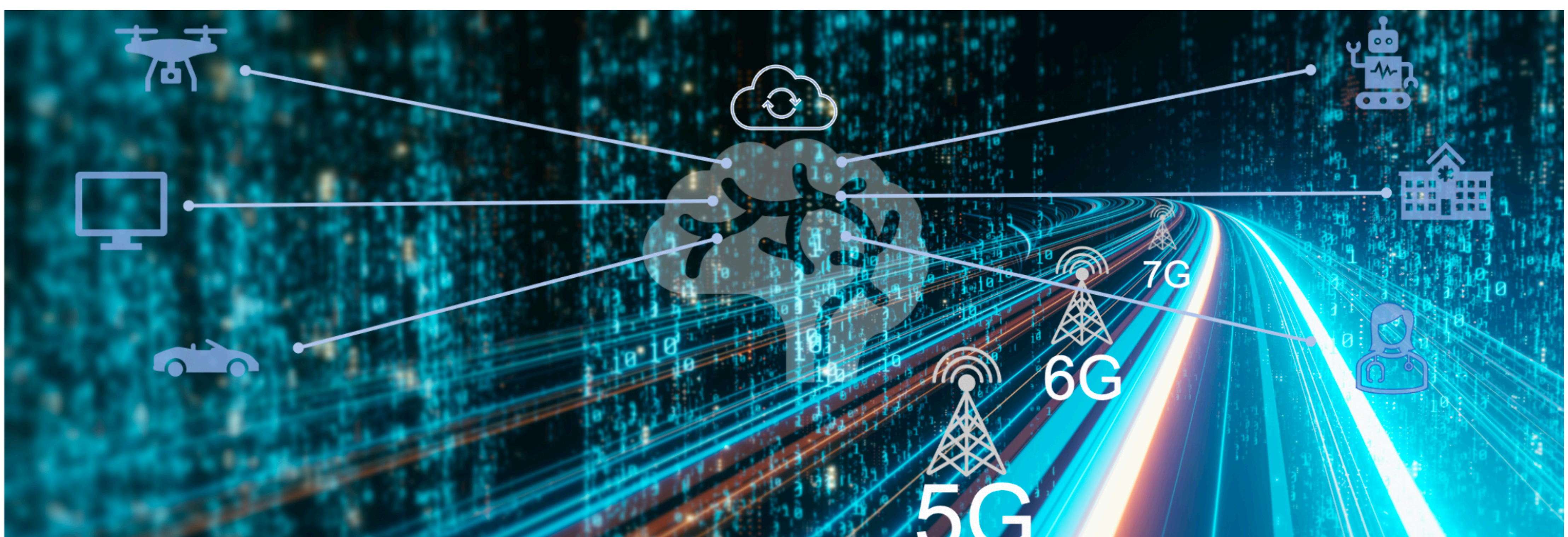

Longterm directions

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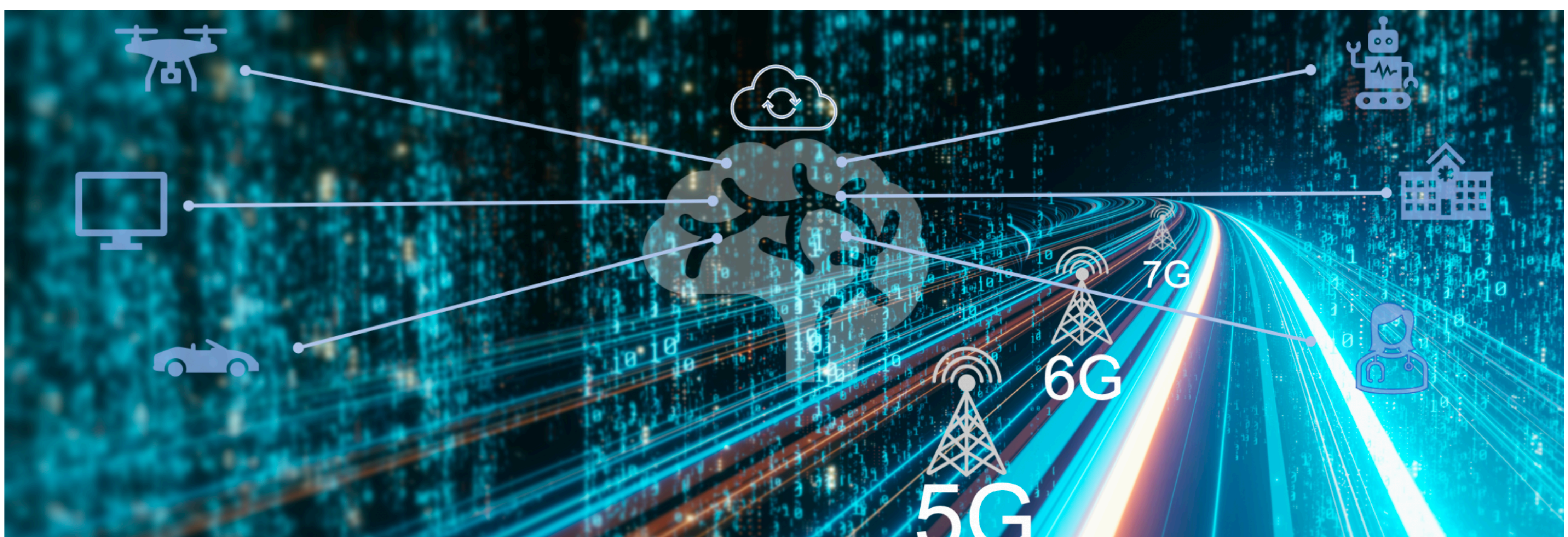
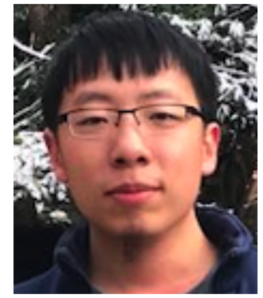


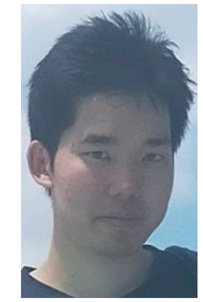
PhD Alumni

Tengyang Xie (Assistant Professor, Wisconsin-Madison, 24)



Jingling Chen (Research Engineer, TikTok)

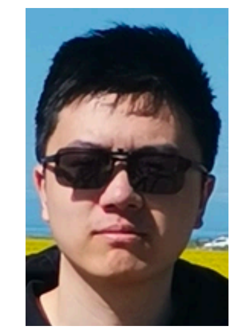
Alumini (visiting student & MS)



Masatoshi Uehara (Assistant Professor, Wisconsin-Madison, 25)

Siyuan Zhang

Priyank Agarwal (Columbia PhD)



Jiawei Huang (ETH Zurich PhD)

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Audrey Huang

Yuheng Zhang

Wei Xiong

Collaborators (Reverse Chronological): Dylan Foster, Ching-An Cheng, Akshay Krishnamurthy, Yu Bai, Alekh Agarwal, Sham Kakade, Chengchun Shi, Wen Sun, Aditya Modi, Paul Mineiro, John Langford, Jason Lee, Cameron Voloshin, Hoang M. Le, Yisong Yue, Gellert Weisz, Csaba Szepesvári, Nathan Kallus, Yu-Xiang Wang, Simon Du, Miroslav Dudík, Christoph Dann, Robert Schapire, Alex Kulesza, Satinder Singh, Ambuj Tewari, Hal Daumé III

Thank you! Questions?

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