The Surprising Power of Belief Propagation

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January 8, 2015

Why does BP Works??

- ML algorithm work well in practice.
- But Why?
- Understand linear algebra algorithms (spectral, singular values ...) well.
- Non-linear algorithms?
- Especially when they do better even in practice! than all algorithm we know.
- Today: Belief Propagation.

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- Goal of Belief Propagation: Compute marginals:

$$p(x_v = a)??$$

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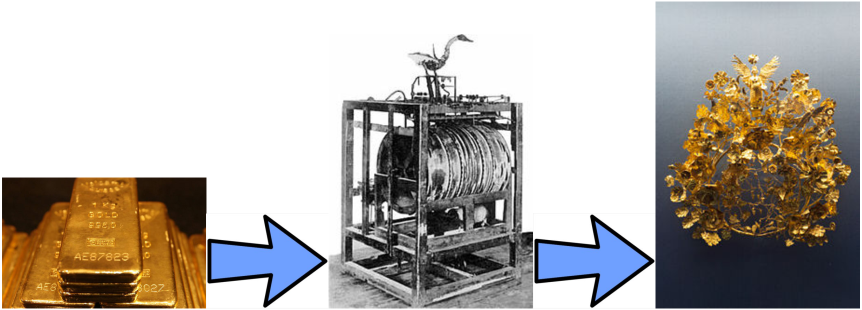
- $$p(x_v = c)??$$

Belief Propagation on Trees



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- Example of 3-coloring:

$$\eta^a_{v \to w} = \frac{\prod_{u \in N(v) \setminus w} (1 - \eta^a_{u \to v})}{\sum_{b=1}^{3} \prod_{u \in N(v) \setminus w} (1 - \eta^b_{u \to v})}$$

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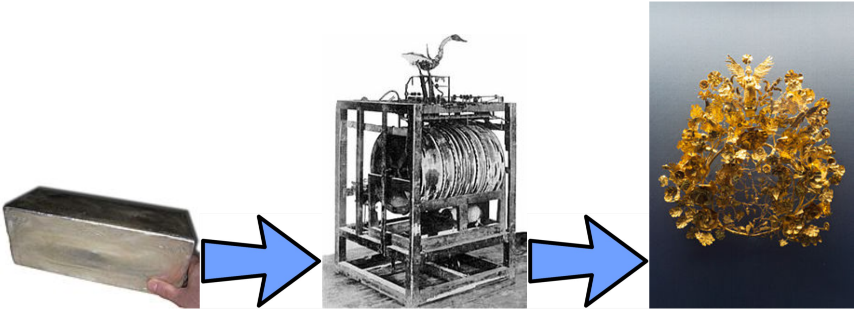
- If G is not a forest then $T(G)$ is infinite ...

BP on tree-like graphs and local information



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  1. locally tree-like and
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- Spielman-00,Richardson-Shokrollahi-Urbanke-01.

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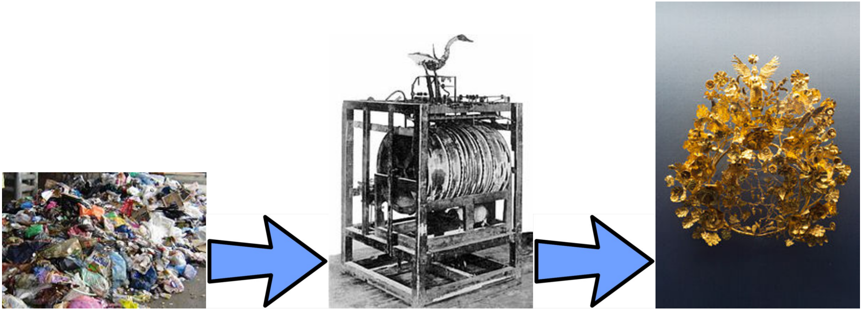
- (Why) Does BP work in other cases?
- In particular, how does it work when there is no way to initialize the messages?

BP on tree-like graphs without local information



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- and ... possible to do better than random iff $(a-b)^2 > 2(a+b)$.

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- Note: graph is very sparse - cannot hope to recover clusters exactly.

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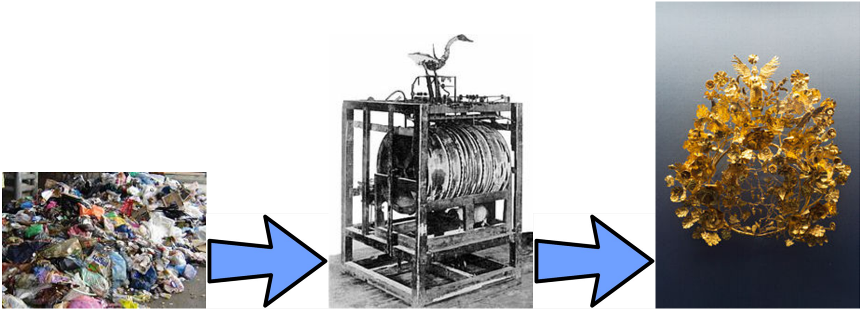
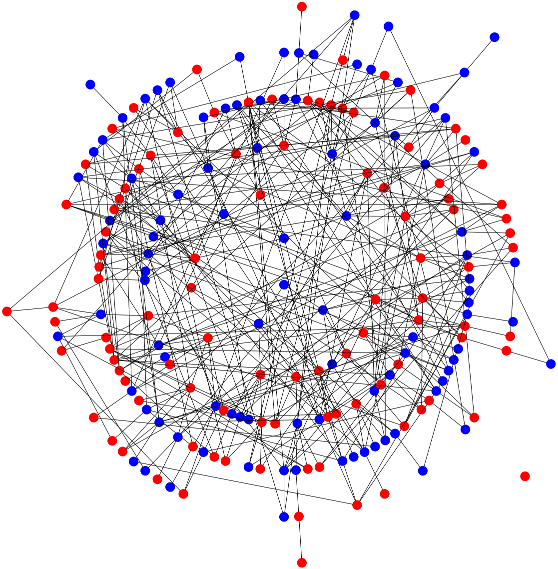


- Note: initializing correctly $(1/2, 1/2)$ is a fixed point.
- Instead initialize randomly ??

The Block Model in pictures

A sample from the model



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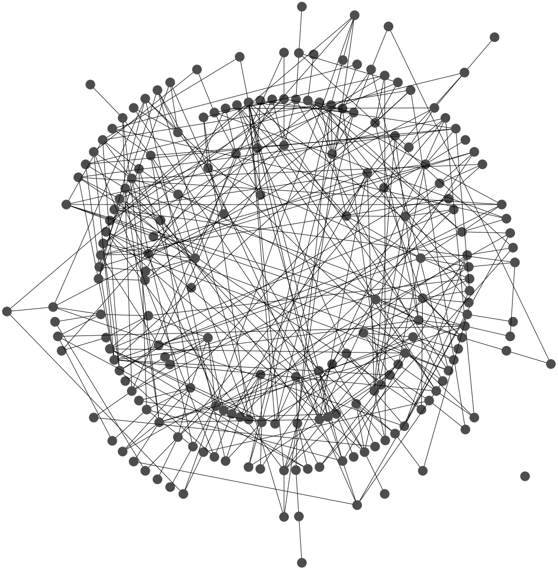
The data (one sample!)



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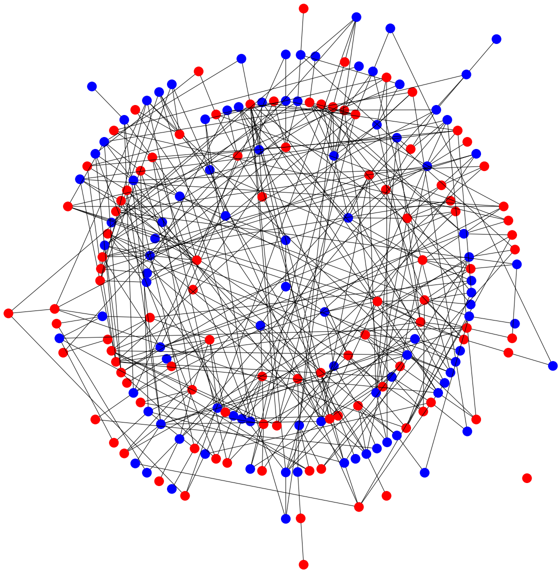
What we want to Infer



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- Note: Thm 4 improves on a very long line of research in computer science and statistics including Boppana (87) Dyer and Freeze (89), Jerrum and Sorkin (89), Carson and Impagliazzo (01) and Condon and Karp (01).

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- KMMNSZZ established connections between N and Belief Propagation

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- Study it and conjecture it's optimality.

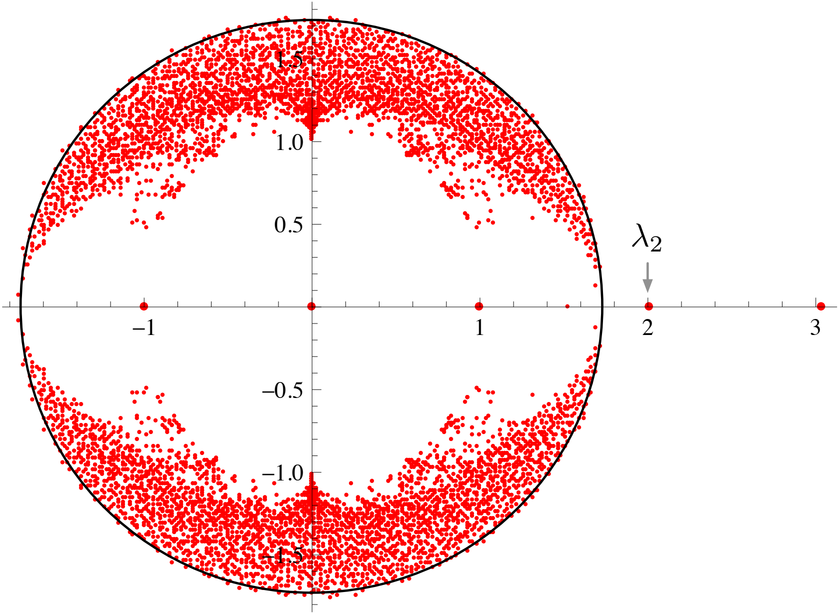
Zeta functions on graphs

1. Hashimoto-89: Introduced a graph analogue of Zeta functions of p-adic algebraic varieties:

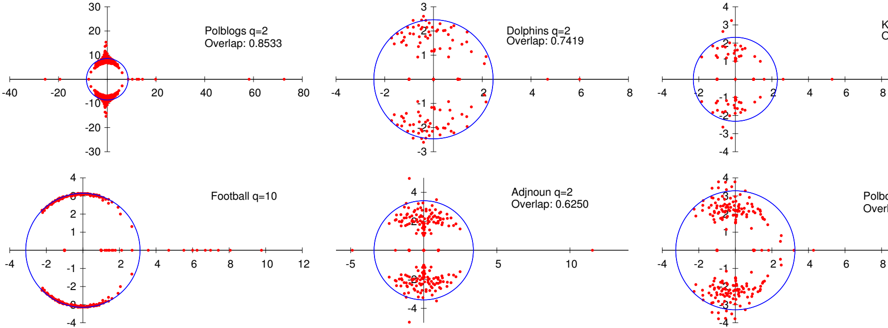
   $$Z(u,f) = \exp\Big(\sum_{\ell=1}^{\infty}\sum_{C\in X_\ell}\frac{f(C)}{\ell}u^\ell\Big),$$

   where $X_\ell =$ set of closed non backtracking loops of length ℓ and $f(C) = \prod_{e\in C} f(e)$.
2. Proved that $Z(f,u)$ is a rational function of u.
3. Asked: how much $Z(f,u)$ is revealing about the graph ...

The Spectrum of N



The spectrum on real networks



Performance on Real Networks

- $R = N$.
- $L =$ normalized laplacian (random walk matrix).

network name	BP overlap	sign of vector 2 of $\mathbf{R}$	k-means of $\mathbf{R}$	sign of vector 2 of $\mathbf{L}_{sym}$	k-means of $\mathbf{L}_{sym}$
words	*	**0.9107**	0.875	0.5625	0.5714
political blogs	0.5167	0.9313	0.6383	**0.9542**	0.9476
karate club	0.5588	**1**	**1**	0.9706	**1**
dophin	**0.9838**	0.8710	0.96774	0.9677	**0.9839**
brsmall	*	0.6548	**0.69345**	0.6235	0.6687
brcorp	*	0.6993	0.72631	**0.7332**	0.6993
adjnoun	0.5625	0.8125	**0.8214**	0.5446	0.5357

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- Massoulie gets symmetric matrix. MNS - almost linear time.

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- More challenging: BP and Survey Propagation for satisfiability problems.
- How to let linear algebra algorithms utilize local information?

A New Type of Phase Transition Question

- **Thm 1** (M-Neeman-Sly 12): f $(a-b)^2 \leq 2(a+b)$ the impossible to infer better than random.
- **Thm 3** (M-Neeman-Sly, 14): If $(a-b)^2 > 100(a+b)$ then Belief Propagation is optimal for detection.
- Proofs via phase transitions for broadcasting on trees. Thm 3 requires a new phase transition!

Broadcasting on trees

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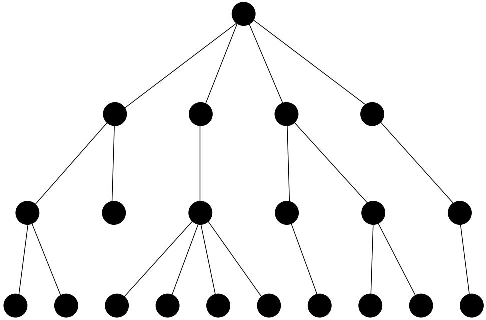

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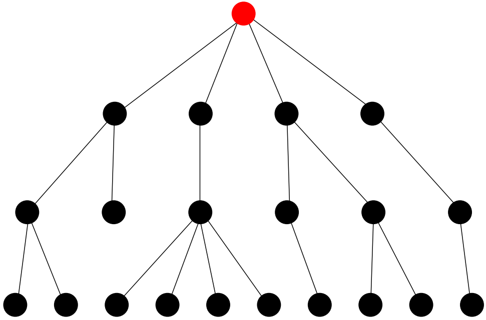

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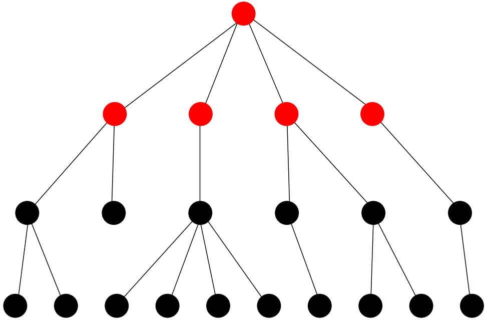

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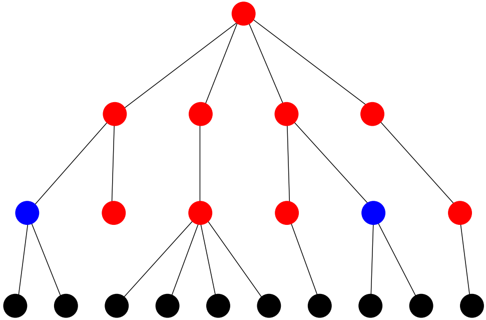

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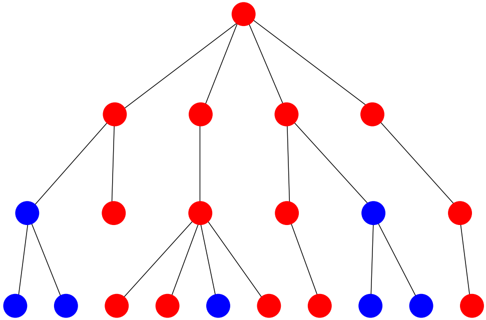

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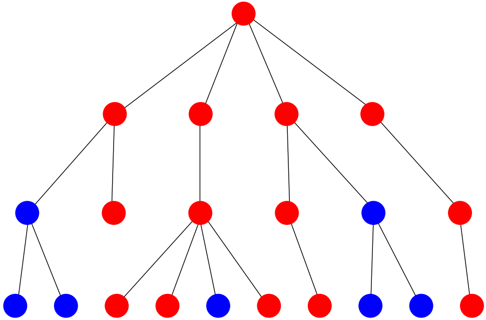

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Broadcasting on trees

Take a tree. Fix $\epsilon \in (0, 1)$.

Color the root randomly.

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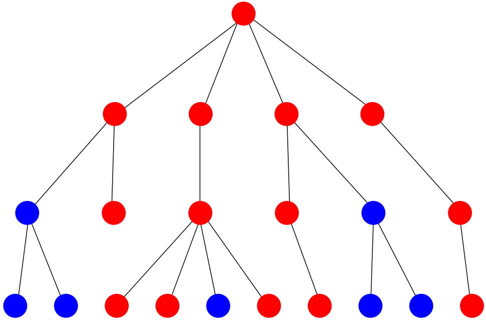

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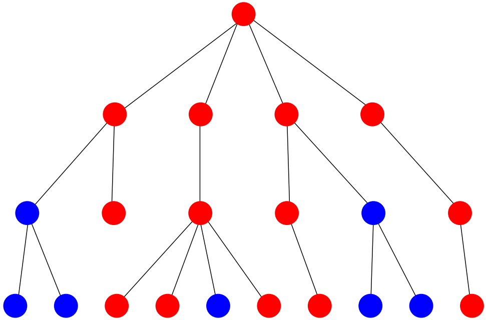

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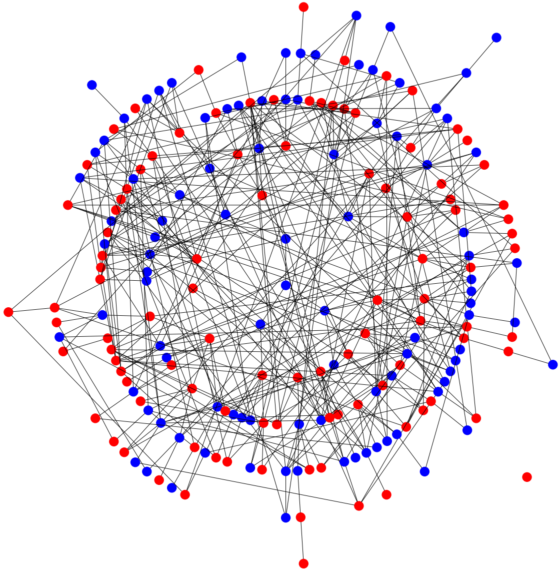
$\implies$ Thm 1 with $\varepsilon = a/(a + b), d = (a + b)/2$.

Back to the original problem



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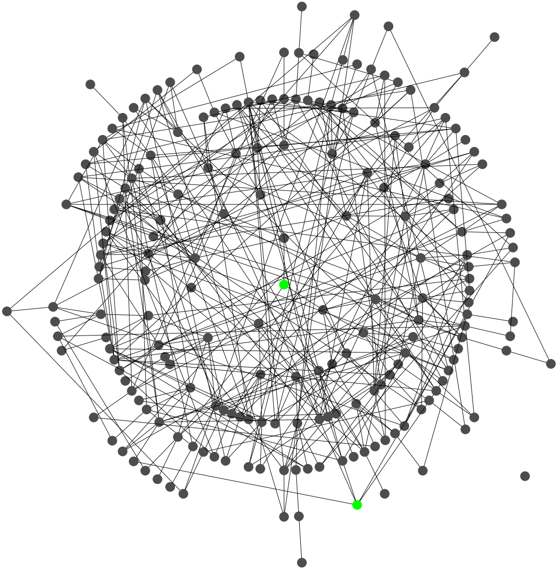

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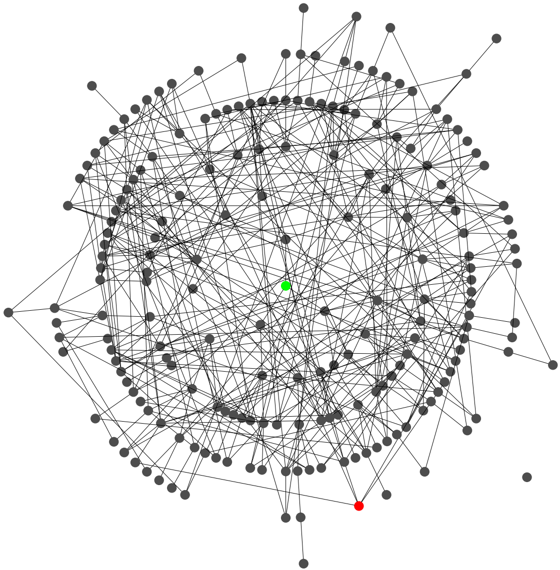

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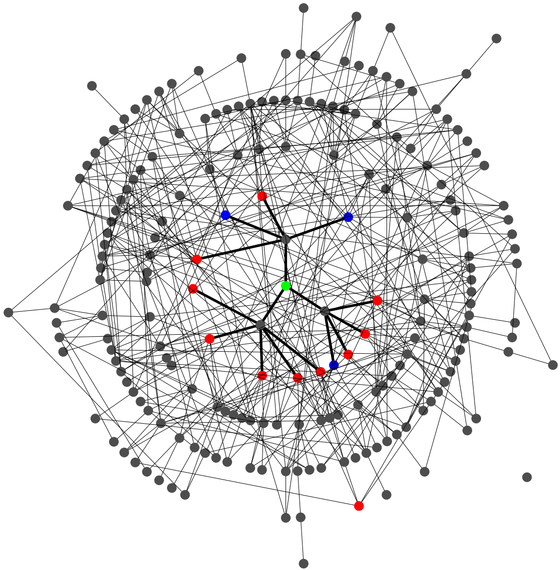

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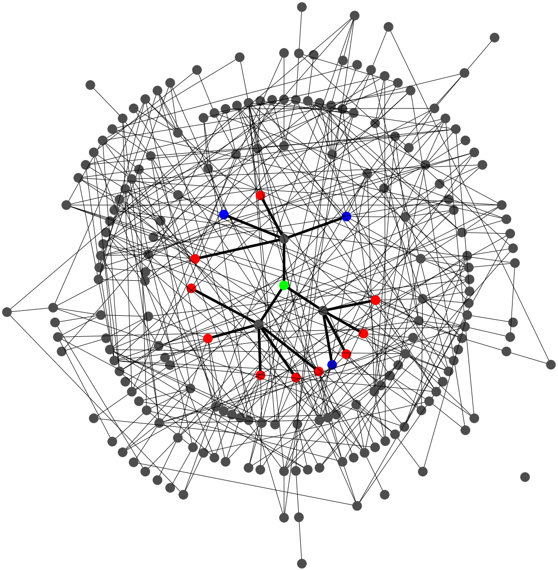

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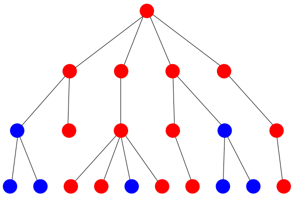

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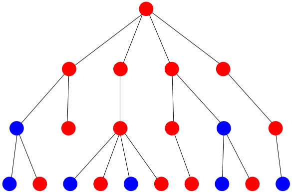

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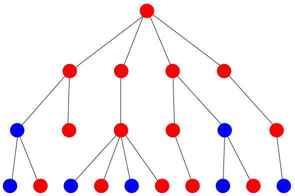

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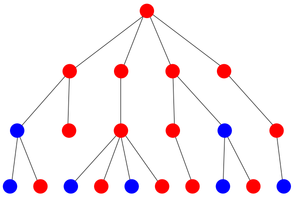

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Strong property of a non-linear dynamical system (stronger than non-ergodicity, "robust reconstruction" etc. (Janson-M-04).

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- Nice to work with physicists and/or statisticians.

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- More challenging: BP and Survey Propagation for satisfiability problems.
- How to let linear algebra algorithms utilize local information?