RW2: Liquidity in Credit Networks

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Credit Network

- Decentralized payment infrastructure introduced by [DeFigueiredo, Barr, 2005] and [Ghosh et. al., 2007]
- Do not need banks, common currency
- Models trust in networked interactions
- A robust “reputation system” for transaction oriented social networks
Barter and Currency

- **Barter:** If I need a goat from you, I had better have the blanket that you are looking for. Low liquidity.

- **Centralized banks:** Issue currencies, which are essentially IOUs from the bank. Very high liquidity; allows strangers to trade freely.

- **Credit Networks:** Bilateral exchange of IOUs among friends.
Illustration: Credit Networks
Illustration: Credit Networks

OBELIX, I TRUST YOU FOR 10 IOUs
Illustration: Credit Networks

ASTERIX, I TRUST YOU FOR 90 IOUs
Illustration: Credit Networks

I NEED 10 IOUs WORTH OF STUFF
Illustration: Credit Networks

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New Trust Values…
Illustration: Credit Networks

Interaction at a Distance
Illustration: Credit Networks
Interaction at a Distance
Illustration: Credit Networks
Interaction at a Distance

NEED A FAVOR FROM CACOPHONIX
…!#$@%...
Illustration: Credit Networks
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Interaction at a Distance
What is a Credit Network?

- Graph $G(V, E)$ represents a network (social network, p2p network, etc.)
- **Nodes:** (non-rational) agents/players; print their own currency
- **Edges:** credit limits $c_{uv} > 0$ extended by nodes to each other\(^1\)
- Payments made by passing IOUs along a chain of trust. Same as augmentation of *single-commodity* flow along the chain
- Credit gets replenished when payments are made in the other direction

**Robustness:** Every node is vulnerable to default only from its own neighbors, and only for the amount it directly trusts them for.

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\(^1\) assume all currency exchange ratios to be unity
Research Questions

- Liquidity: Can credit networks sustain transactions for a long time, or does every node quickly get isolated?
- Network Formation: How do rational agents decide how much trust to assign to each other?
Liquidity Model

- Edges have integer capacity $c > 0$ (summing up both directions)
- Transaction rate matrix $\Lambda = \{\lambda_{uv} : u, v \in V, \lambda_{uu} = 0\}$
- Repeated transactions; at each time step choose $(s, t)$ with prob. $\lambda_{st}$
- Try to route a unit payment from $t$ to $s$ via the shortest feasible path; **update edge capacities** along the path
- Transaction fails if no path exists
Liquidity Model

The Random Walk

Failure rate = Stationary probability of making a transition to the same state
Definition

Let \( S \) and \( S' \) be two states of the network. We say that \( S' \) is **cycle-reachable** from \( S \) if the network can be transformed from state \( S \) to state \( S' \) by routing a sequence of payments along feasible cycles (i.e. from a node to itself along a feasible path).
Analysis
Steady-State

*Cycle-reachability* partitions all possible states of the credit network into equivalence classes.
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Steady-State

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**Theorem**

*If the transaction rates are symmetric, then the network has a uniform steady-state distribution over all reachable equivalence classes.*
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**Theorem**

If the transaction rates are symmetric, then the network has a uniform steady-state distribution over all reachable equivalence classes.

Consequence: Yields a complete characterization of success probabilities in trees, cycles, or complete graphs; estimate for Erdös-Rényi graphs
Assume capacity $c$. Then we have $c + 1$ states; each in a different equivalence class.

Success probability for a transaction is $c/(c + 1)$. 
No cycles. Hence, all states are equally likely.

Let $c_1, c_2, \ldots, c_L$ be the capacities along the path from $s$ to $t$ in the tree. Then, success probability is

$$
\prod_{i=1}^{L} \frac{c_i}{c_i + 1}.
$$
Analysis

Example: Bankruptcy probability in general graphs

Assume capacity $c = 1$ on each edge, and the Markov chain is ergodic. Let $d_v$ denote the degree of node $v$. Then the stationary probability that $v$ is bankrupt is at most $1/(1 + d_v)$. 
Analysis
Centralized Payment Infrastructure

NEED A FAVOR FROM CACOFONIX...!#@%...
Analysis

Centralized Payment Infrastructure
Analysis

Centralized Payment Infrastructure
Convert Credit Network → Centralized Model

∀ u, \( c_{ru} = \sum_v c_{vu} \)
Convert Credit Network → Centralized Model

∀ \ u, c_{ru} = \sum_v c_{vu}

⇒ Total credit in the system is conserved during conversion
Analysis
Centralized Payment Infrastructure

**Convert Credit Network \(\rightarrow\) Centralized Model**

\[ \forall u, c_{ru} = \sum_v c_{vu} \]

\[ \implies \text{Total credit in the system is conserved during conversion} \]

Slight variant of the liquidity analysis gives steady state distribution and success probabilities.
## Bankruptcy probability

<table>
<thead>
<tr>
<th>Graph class</th>
<th>Credit Network</th>
<th>Centralized System</th>
</tr>
</thead>
<tbody>
<tr>
<td>General graphs</td>
<td>$\leq 1/(d_v + 1)$</td>
<td>$\approx 1/(d_{AVG} + 1)$</td>
</tr>
</tbody>
</table>

## Transaction failure probability

<table>
<thead>
<tr>
<th>Graph class</th>
<th>Credit Network</th>
<th>Centralized System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Star-network</td>
<td>$\Theta(1/c)$</td>
<td>$\Theta(1/c)$</td>
</tr>
<tr>
<td>Complete Graph</td>
<td>$\Theta(1/nc)$</td>
<td>$\Theta(1/nc)$</td>
</tr>
<tr>
<td>$G_c(n, p)$</td>
<td>$\Theta(1/npc)$</td>
<td>$\Theta(1/npc)$</td>
</tr>
</tbody>
</table>

*(simulation/estimate)*

### Summary

Many credit networks have liquidity which is almost the same as that in centralized currency systems.
Random Forests

An Interesting Connection

- $G = (V, E)$, a multi-graph,
- RF-connectivity between two vertices $u$ and $v = \Pr(u$ is connected to $v$ in a uniformly chosen random forest of $G$).

Prop: Liquidity in a Credit Network = Average RF-connectivity in the underlying graph (via [Kleitman and Winston, 1981])
Def: Expansion of a graph is

\[ h(G) = \min_{S \subseteq V : 0 \leq |S| \leq |V|/2} \frac{|E(S, \bar{S})|}{|S|} \]
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$$h(G) = \min_{S \subseteq V: 0 \leq |S| \leq |V|/2} \frac{|E(S, \bar{S})|}{|S|}$$

For graphs with expansion $h(G)$,

Thm (Main): Average RF-connectivity over any two vertices

$$\geq 1 - \frac{2}{h(G)}.$$ 

Thm: Average RF-connectivity between one vertex and all other vertices

$$\geq 1 - \frac{\log n + 2}{h(G) + 1}.$$
Corollaries: In a uniformly random forest,

- Expected size of largest component $\geq n - \frac{2n}{h(G)}$.
- Expected number of components $\leq 1 + \frac{2n}{h(G)}$.
- $\Pr(\text{largest component } \leq \frac{n}{2}) \leq \frac{2}{h(G)}$. 
RF-connectivity on Expanding Subgraphs

**Thm:** Let $S$ be any subset of vertices and $G_S$ be the induced subgraph. Then $\Phi_S(G) \geq 1 - \frac{2}{h(G_S)}$. 

**The Monotonicity Conjecture:** RF-connectivity can not decrease if we add a new edge in the graph. Equivalent to Negative Correlation (known for random spanning trees).
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Equivalent to Negative Correlation (known for random spanning trees).
Open Problems

- The Monotonicity conjecture
- Approximately sampling a random forest from a graph
- Rationality: how do nodes initialize and update trust values (in general settings)?
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