Algorithms for Visualization: Constructing Contour Trees [1]

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[1] Hamish Carr, Jack Snoeyink, and Ulrike Axen. "Computing contour trees in all dimensions." Computational Geometry 24.2 (2003): 75-94.

Introduction

Scientific Visualization

[2] http://www.cc.gatech.edu/scivis/tutorial/linked/whatisscivis.html

Scientific Visualization

Scientific Visualization (SciVis)[2] is the representation of data graphically as a means of gaining understanding and insight into the scientific data.

[2] http://www.cc.gatech.edu/scivis/tutorial/linked/whatisscivis.html

Why algorithms for SciVis?

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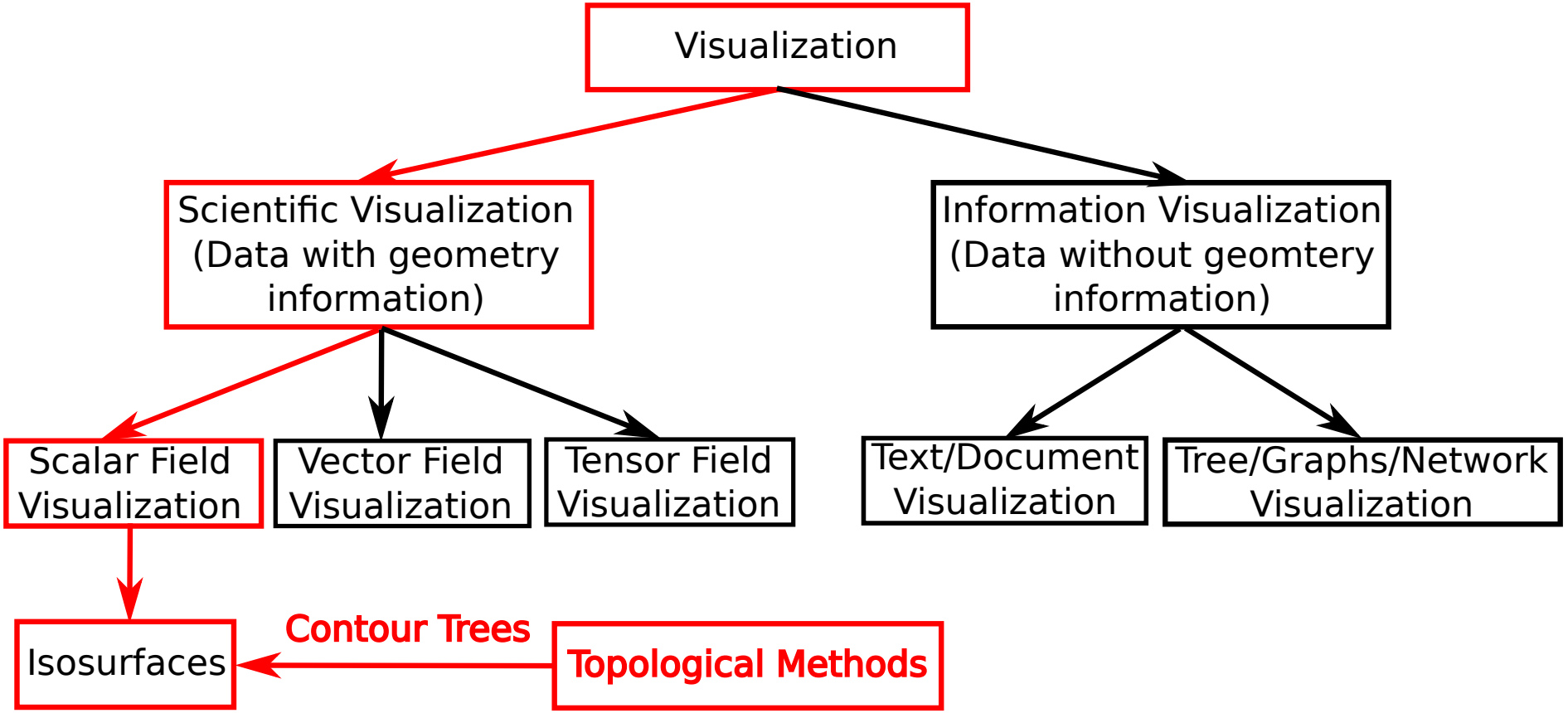
- Automate the process/minimize human intervention.
- Spatio-temporal nature of data.
  - Huge data size.
- Real-time visualization.

Today's Agenda



Background

Some background

- Scalar fields.
  - $f : \mathbb{R}^{2,3} \longrightarrow \mathbb{R}$
- Isovalue.
  - Real value v in the range.
- Level sets.
  - All points $p \in \mathbb{R}^{2,3}$ s.t. $f(p) = v$
  - Can have many components.
- Isosurfaces.
  - When the domain is $\mathbb{R}^3$
- Topological Structures.
  - Stick figures.
  - Succinct representation of the functions.

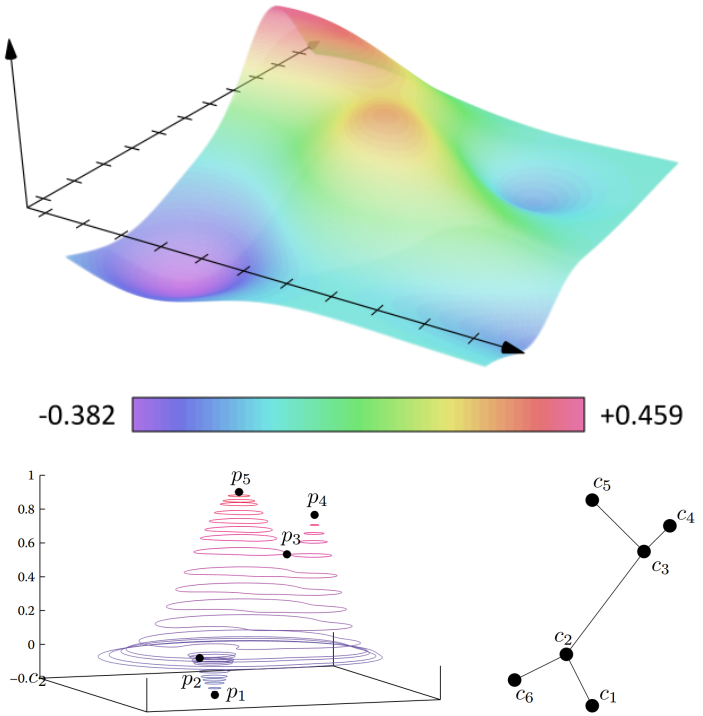


Figure: **Top** Scalar field defined on $\mathbb{R}^2$ [a] **Middle** Isocontours and contour tree[b]

[a] from Wikipedia

[b] from Rephael Wagner, "Isosurfaces: geometry, topology, and algorithms", 2013.

The data

- Continuity?
- Data representation.
- Dividing cubes into tetrahedra.
- Triangulation.
- Linear interpolation.

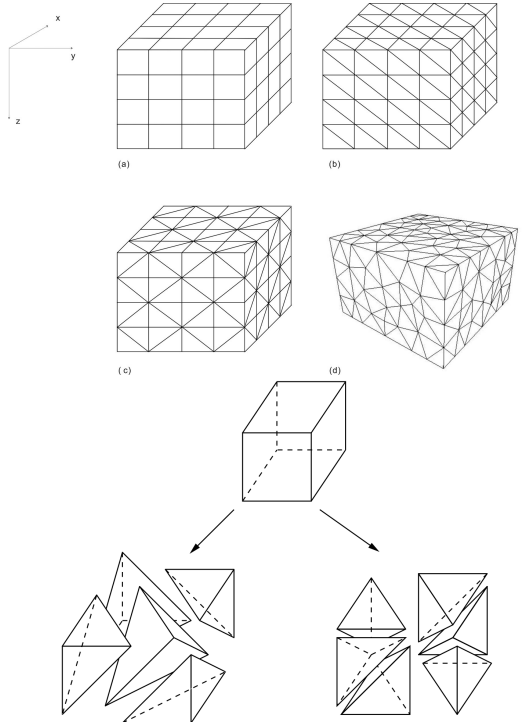


Figure: **Top** Example mesh/grid [a] **Bottom** Dividing a cube[b]

[a] from Wang et al. "Three-dimensional DC anisotropic resistivity modelling using finite elements on unstructured grids". Geophys J Int 2013.

[b] from Rephael Wagner

More about Isosurfaces

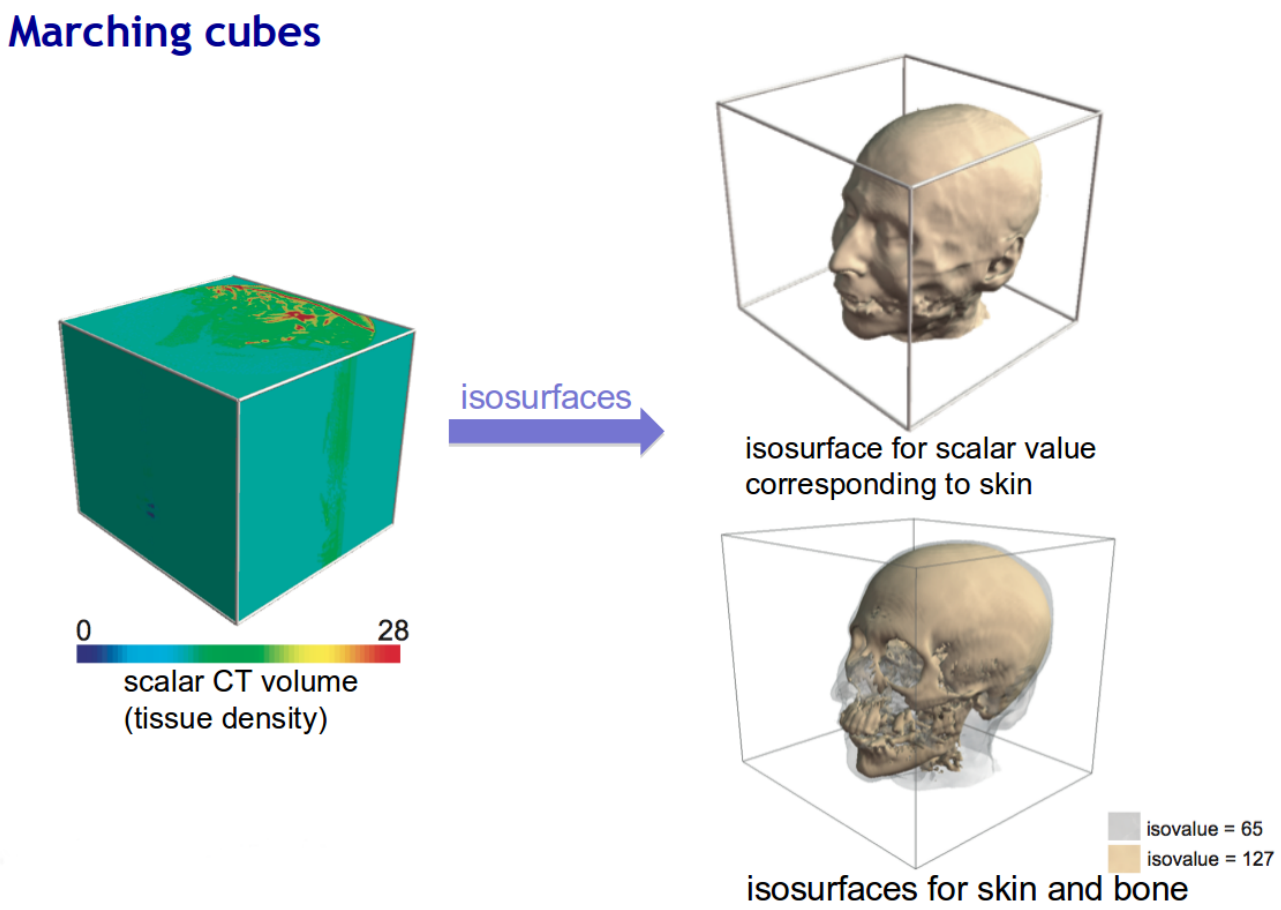


Figure: **Left:** Data set. **Right:**Isosurfaces.[3]

[3] from Alexandru Telea, "Data visualization: principles and practice". CRC Press, 2014.

More about Isosurfaces

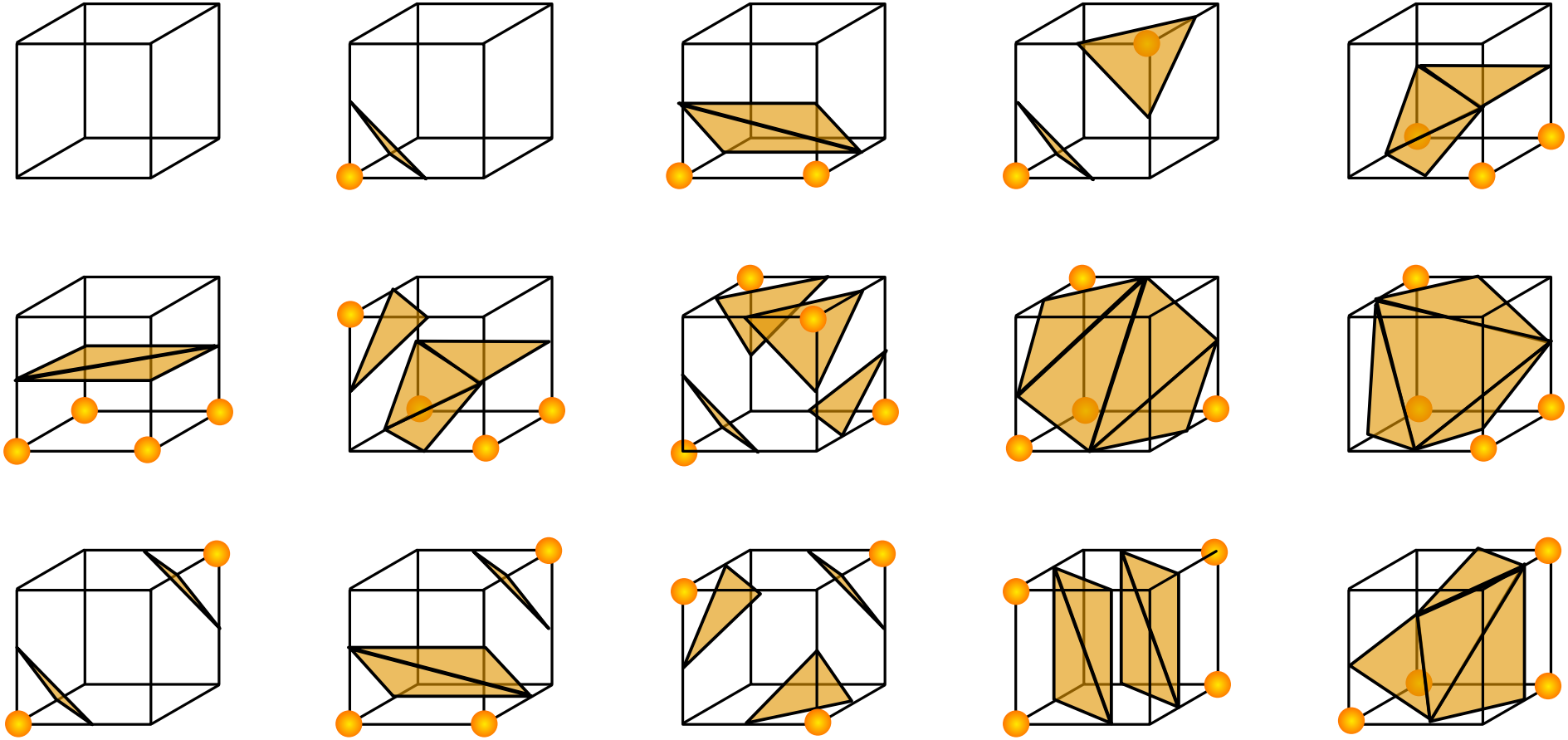
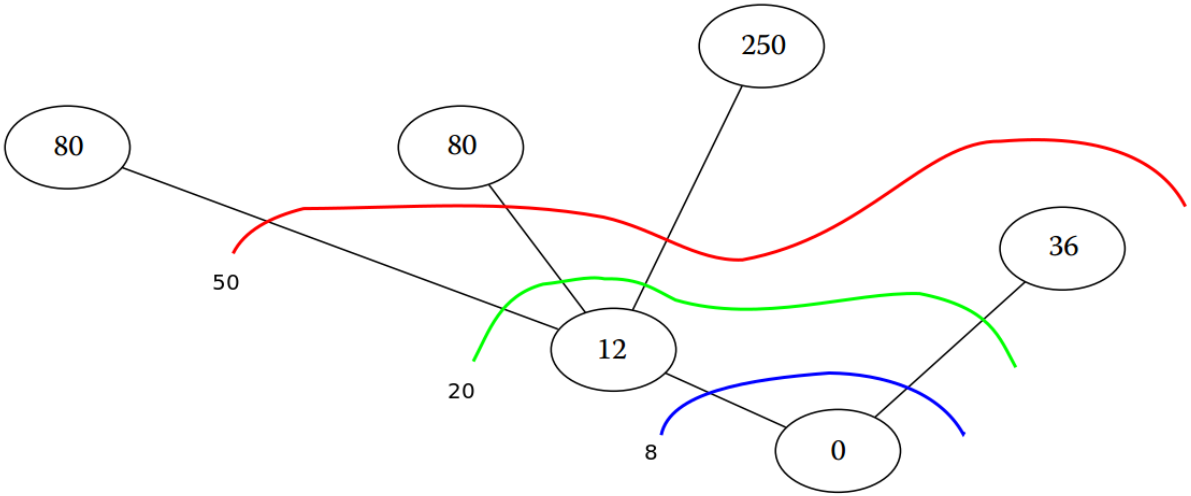


Figure: Marching cubes lookup table.[4]

[4] from Wikipedia

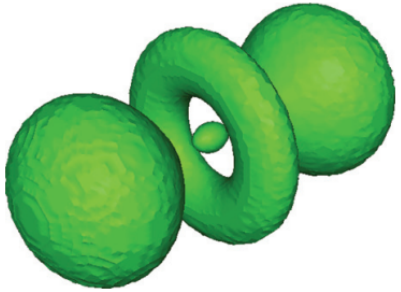
Contour trees

Problems/Why contour trees?

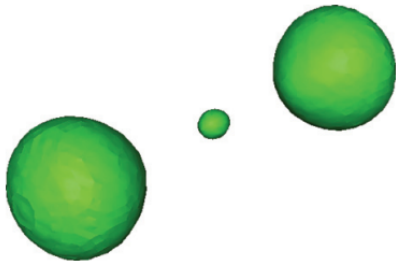


(a) Contour tree.

(b) Isovalue 8.



(c) Isovalue 20.



(c) Isovalue 50.

Figure: Need for contour tree.[5]

[5] from Rephael Wagner, "Isosurfaces: geometry, topology, and algorithms", 2013.

What are contour trees?

We can see how the level sets evolve by changing the isovalues

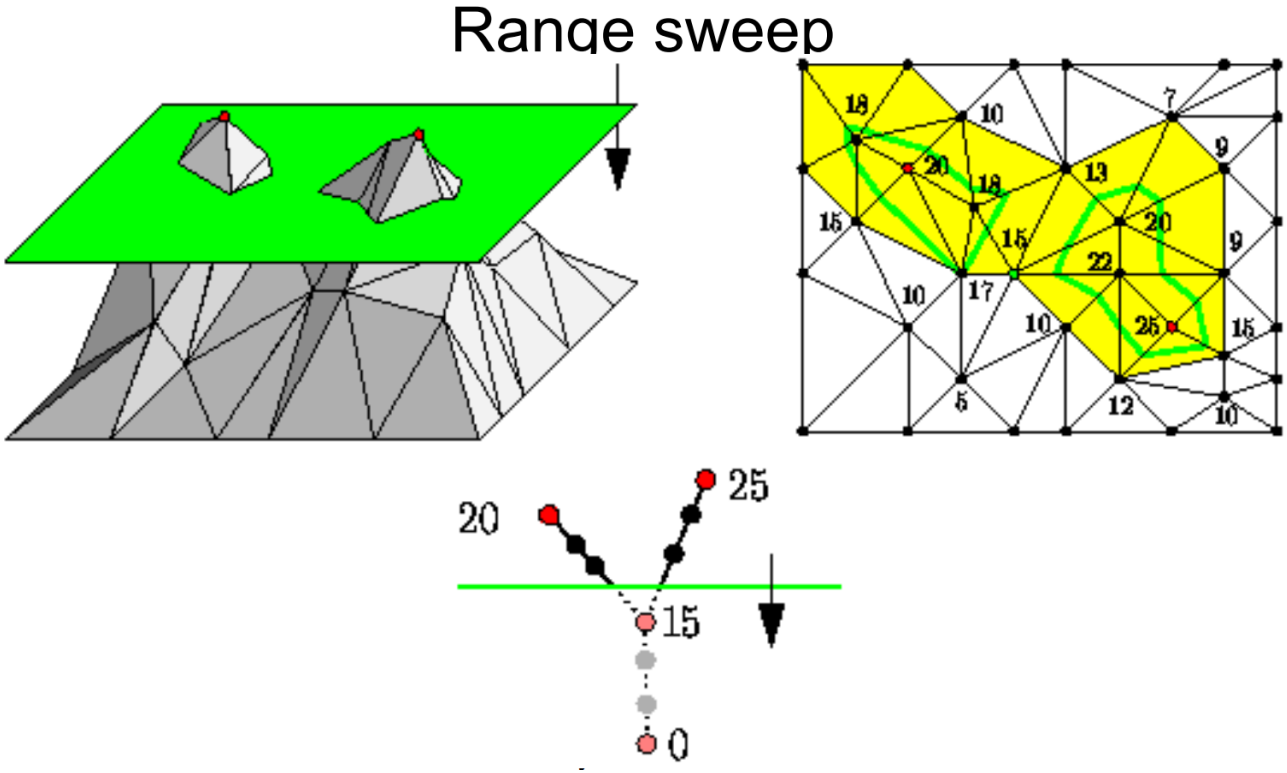


Figure: Evolution of level sets[6]

[6] from J.Snoeyink,C Bajaj

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- It has set of nodes where connectivity of level sets change.
- It has set of edges which connects these nodes in a meaningful manner.

What are contour trees?

Running Example

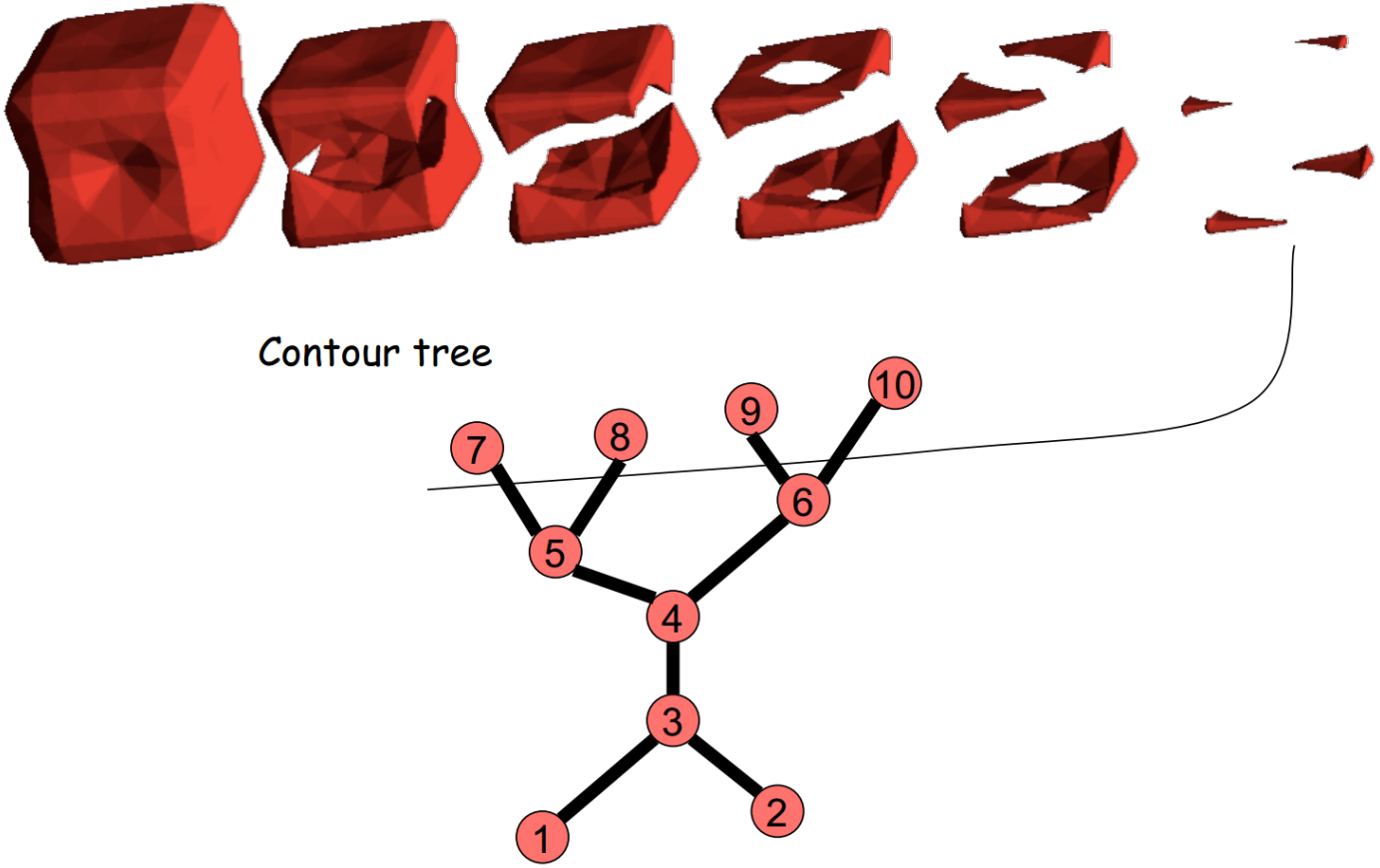

Figure: Evolution of level sets[7]

[7] from Hamish Carr et al.

What are contour trees?

Running Example

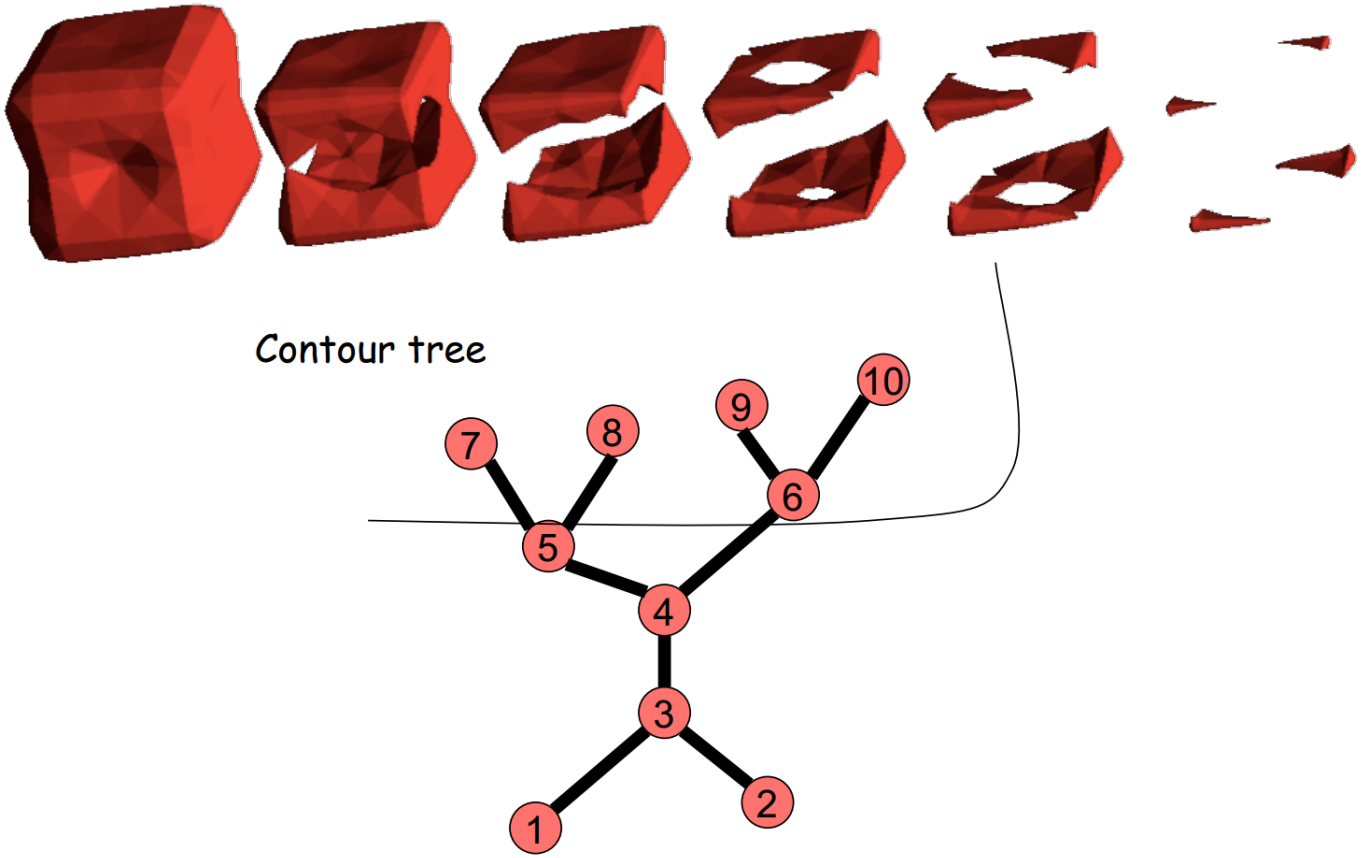

Figure: Evolution of level sets[8]

[8] from Hamish Carr et al.

What are contour trees?

Running Example

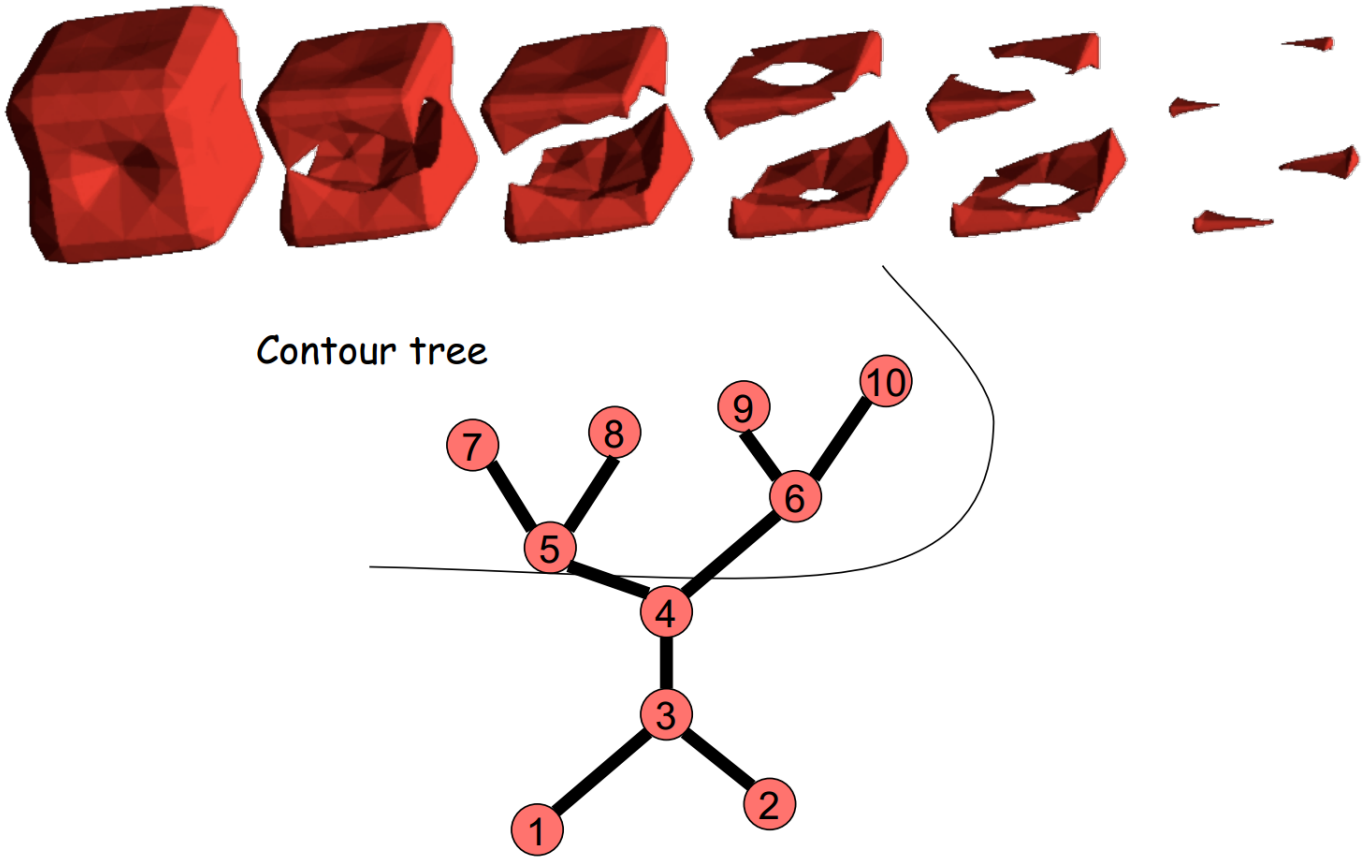

Figure: Evolution of level sets[9]

[9] from Hamish Carr et al.

What are contour trees?

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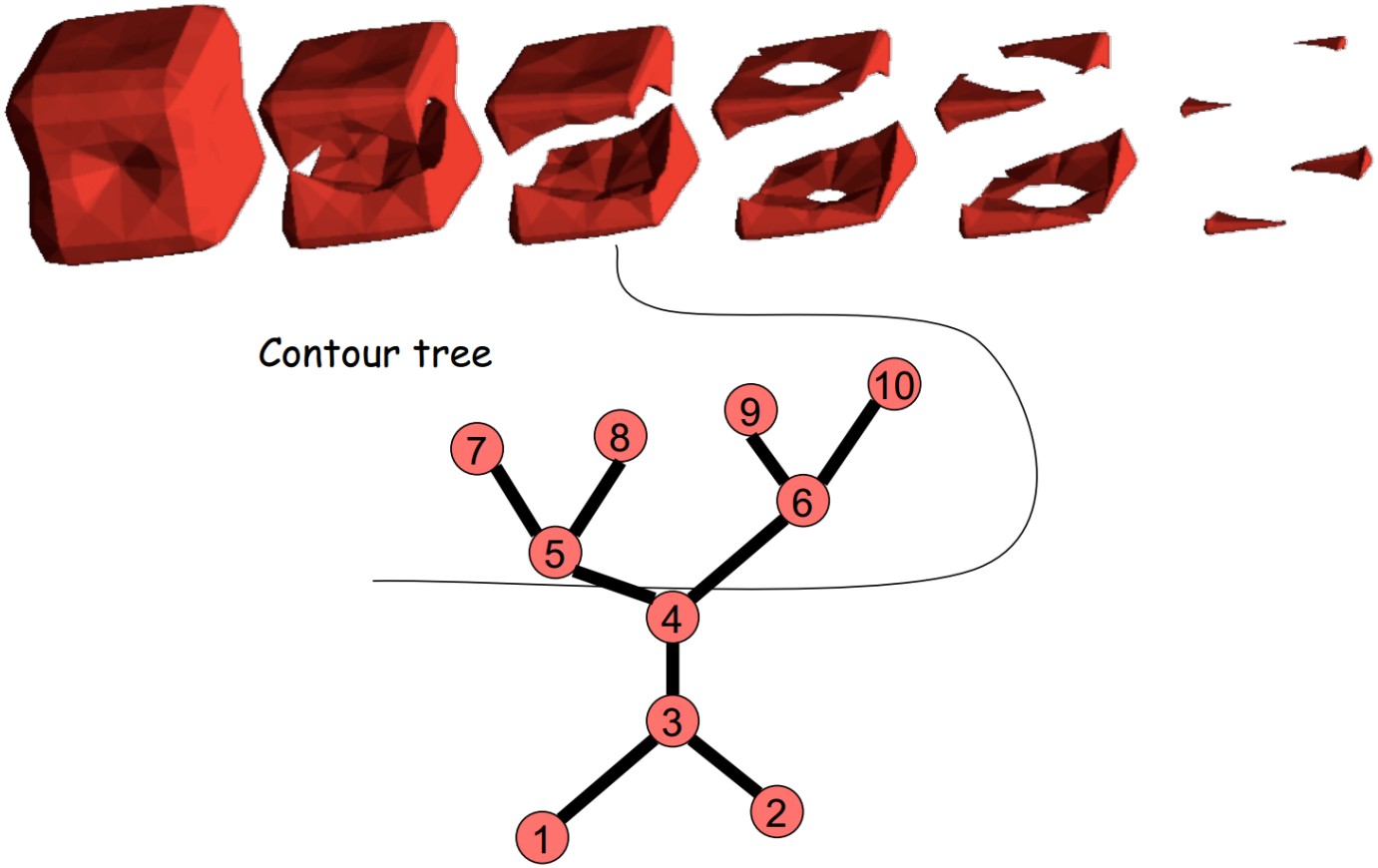

Figure: Evolution of level sets[10]

[10] from Hamish Carr et al.

What are contour trees?

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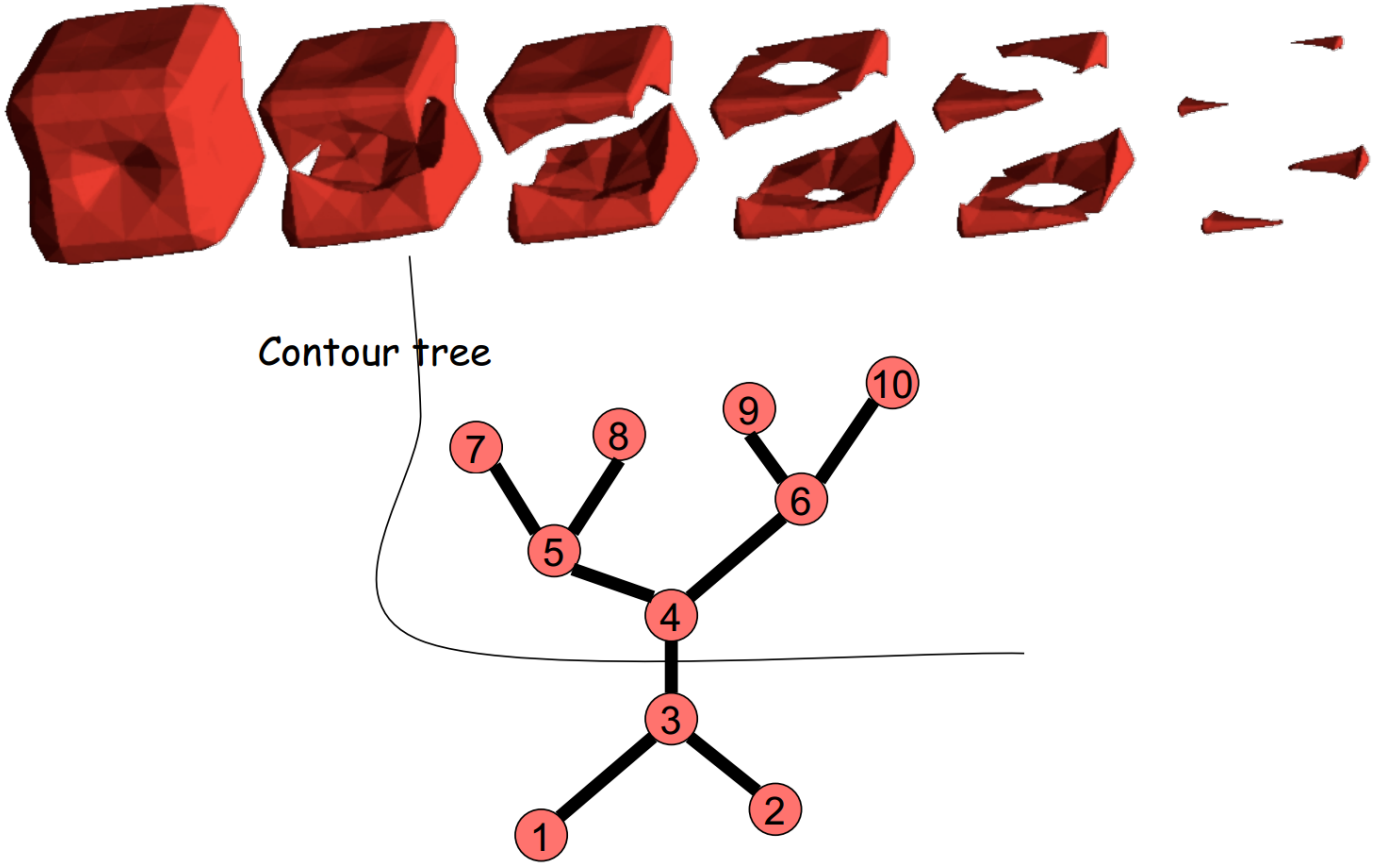

Figure: Evolution of level sets[11]

[11] from Hamish Carr et al.

What are contour trees?

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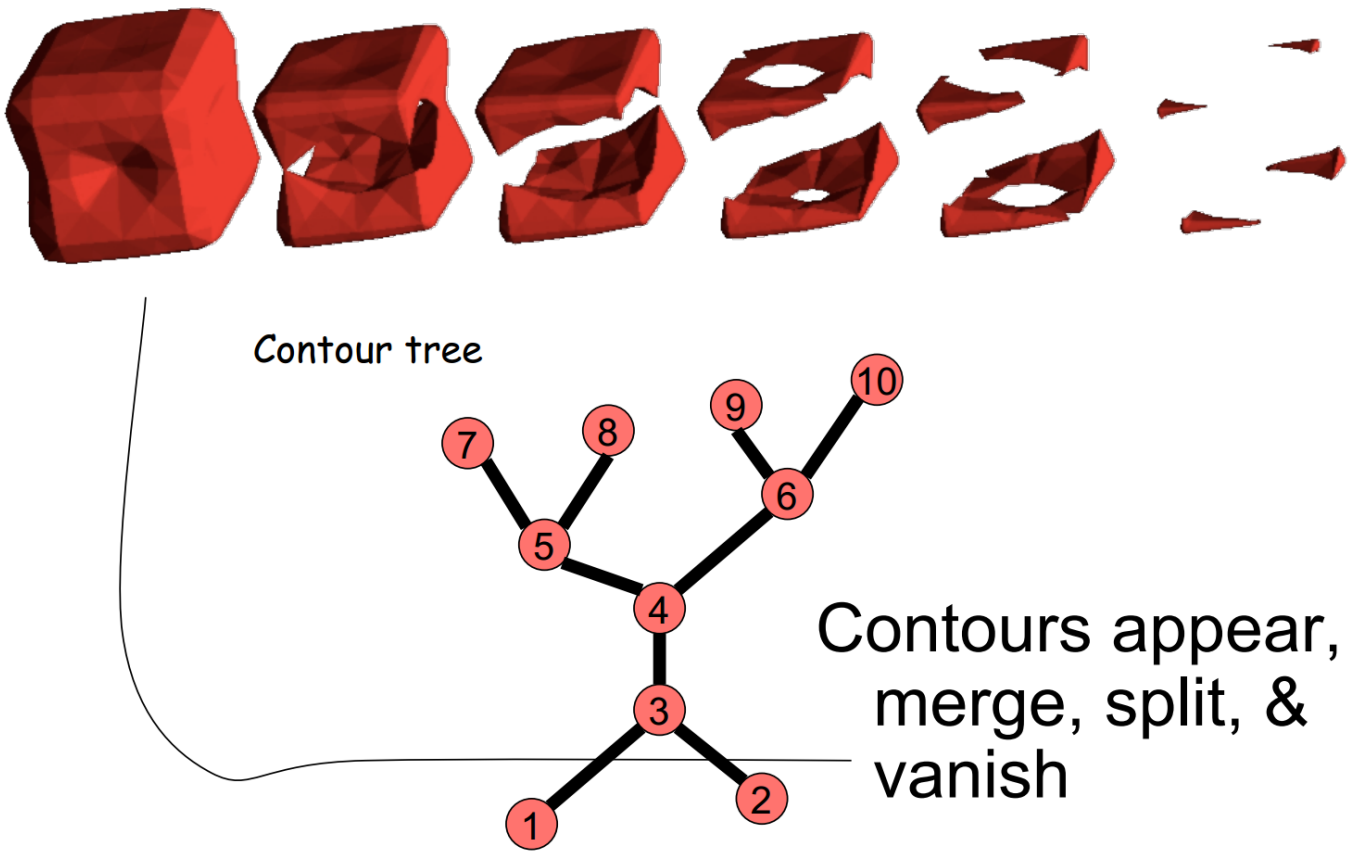


Figure: Evolution of level sets[12]

[12] from Hamish Carr et al.

More background

- Link.
  - Lower link (LL).
  - Upper link (UL).
- Critical points.
  - Maxima (Empty UL).
  - Minima (Empty LL).
  - Saddle (UL/LL not simply connected).
- Identifying by Lower links
- Nodes of Contour tree are all critical points.

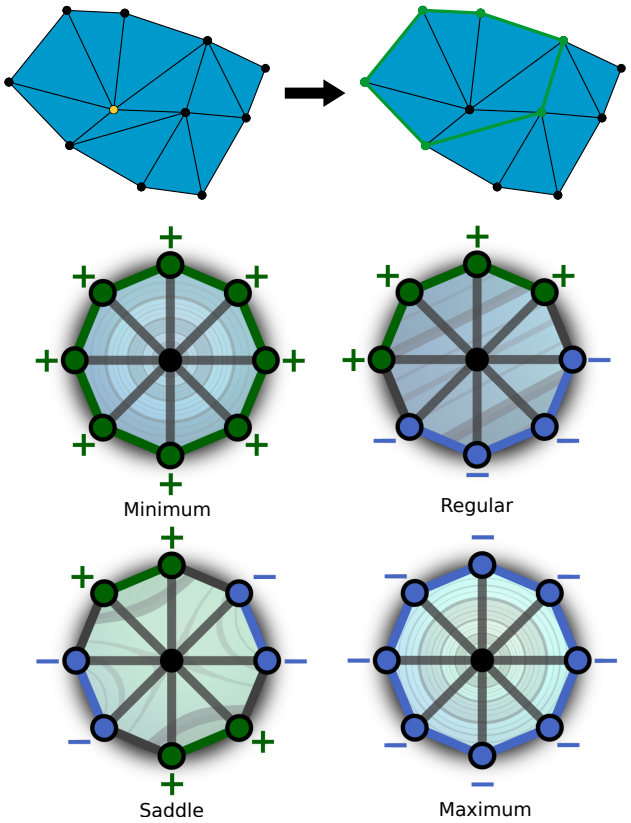


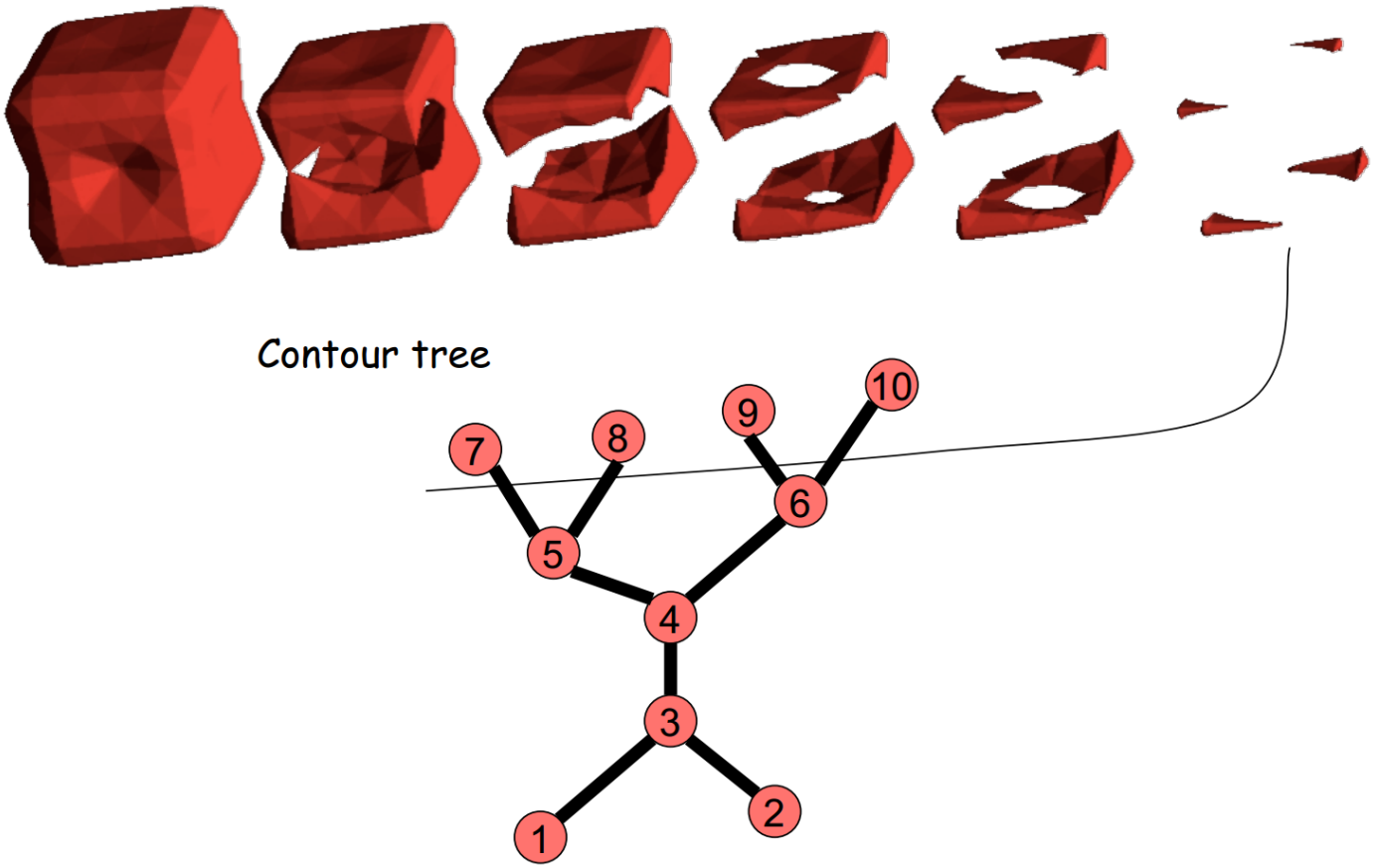
Figure: **Top** Link in Triagulations[a]
Middle/Bottom Classification based on LL[b]

[a] from Wikipedia

[b] from slides by Julian Tierny

Evolution in terms of Critical points

- Minima $\Longrightarrow$ Level set component is created.
- Maxima $\Longrightarrow$ Level set component is destroyed.
- Saddle $\Longrightarrow$ Level set components split or join.



Algorithm

Join tree + Split tree = Contour Tree

- Join tree.
- Split tree.

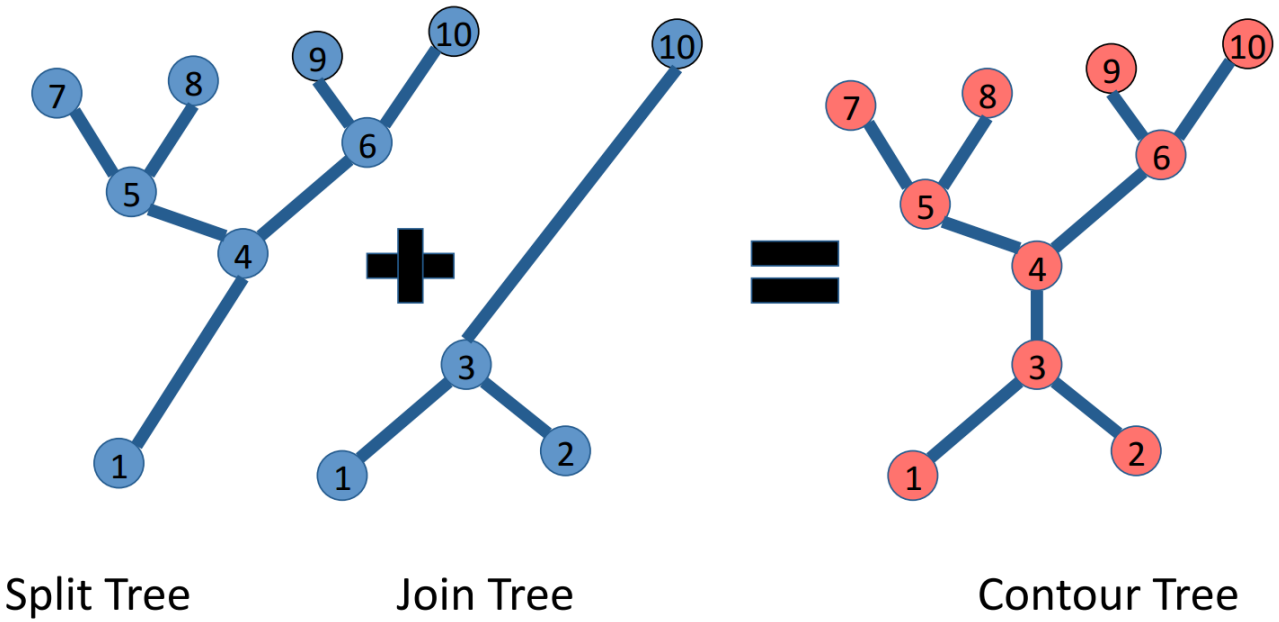


Figure: Join tree + Split tree = Contour Tree[14]

[14] from Hamish Carr et al.

Algorithm

[15] from Hamish Carr et al.

Algorithm

- Construct Join tree.

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- Merge them to get Contour tree.

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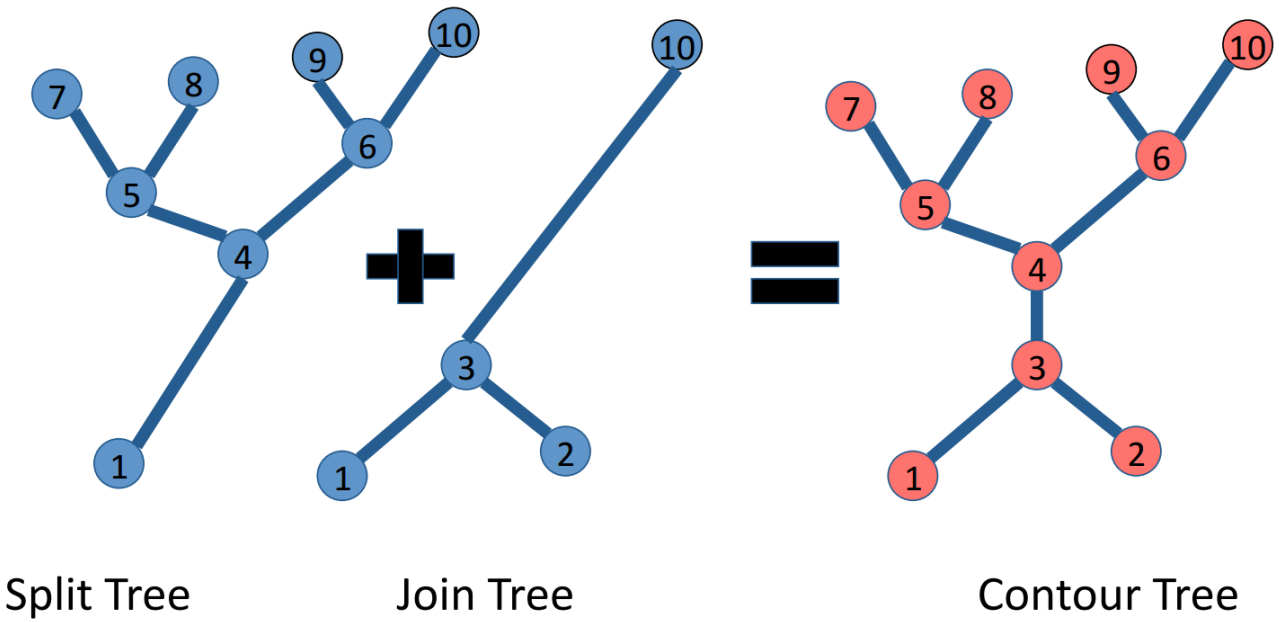

Figure: Join tree + Split tree = Contour Tree[15]

[15] from Hamish Carr et al.

Some observations and details

- Joins are easy to track but splits aren't. Why/how?

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- Notice that Split tree is actually Join tree for the function swept in opposite direction.
- Use Union-Find (UF) to track joins.

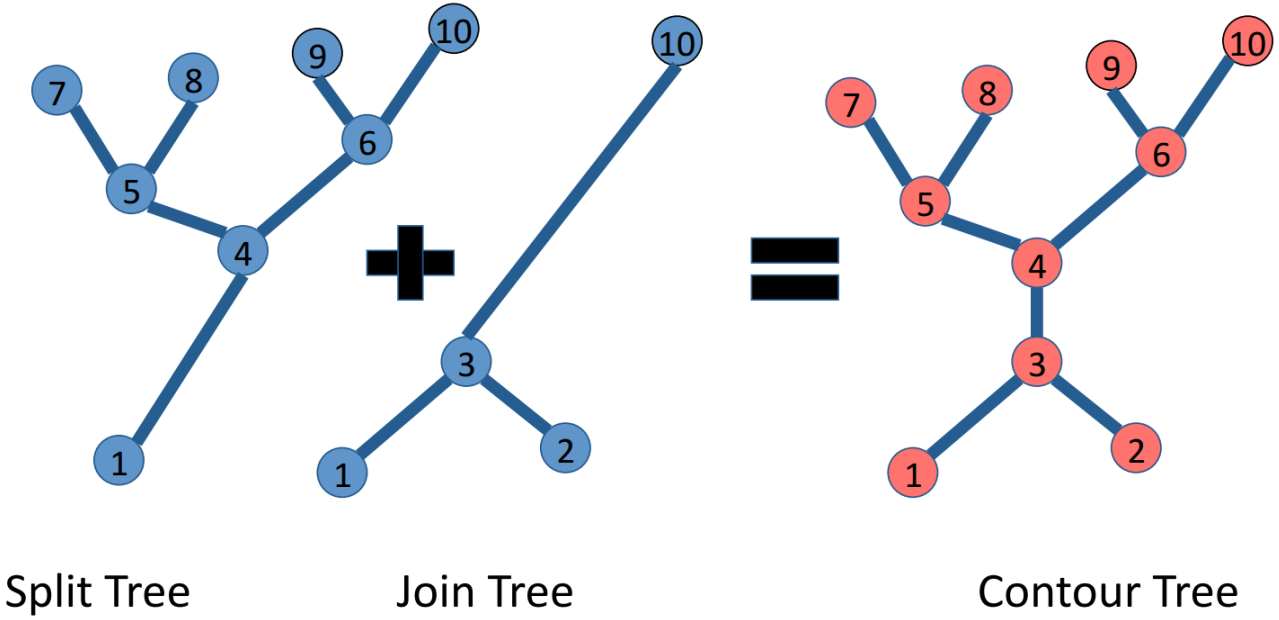

Figure: Join tree + Split tree = Contour Tree[16]

[16]

Algorithm for Join/Split Tree

- Sort all the vertices in the domain based on function values.

```
JoinTree(V, E)
JT = NewTree()
UF = NewUF()
for i = 0 to n − 1
    AddNode(JT, i)
    if LL(v_i) = ∅
        NewSet(UF, i)
    else
        for v_j ∈ LL(v_i)
            i' = Find(UF, i)
            j' = Find(UF, j)
            if j' ≠ i'
                AddEdge(JT, i', j')
                Union(UF, i', j')
    return JT
```

Figure: Algorithm[17]

[17] from Hamish Carr et al.

The output

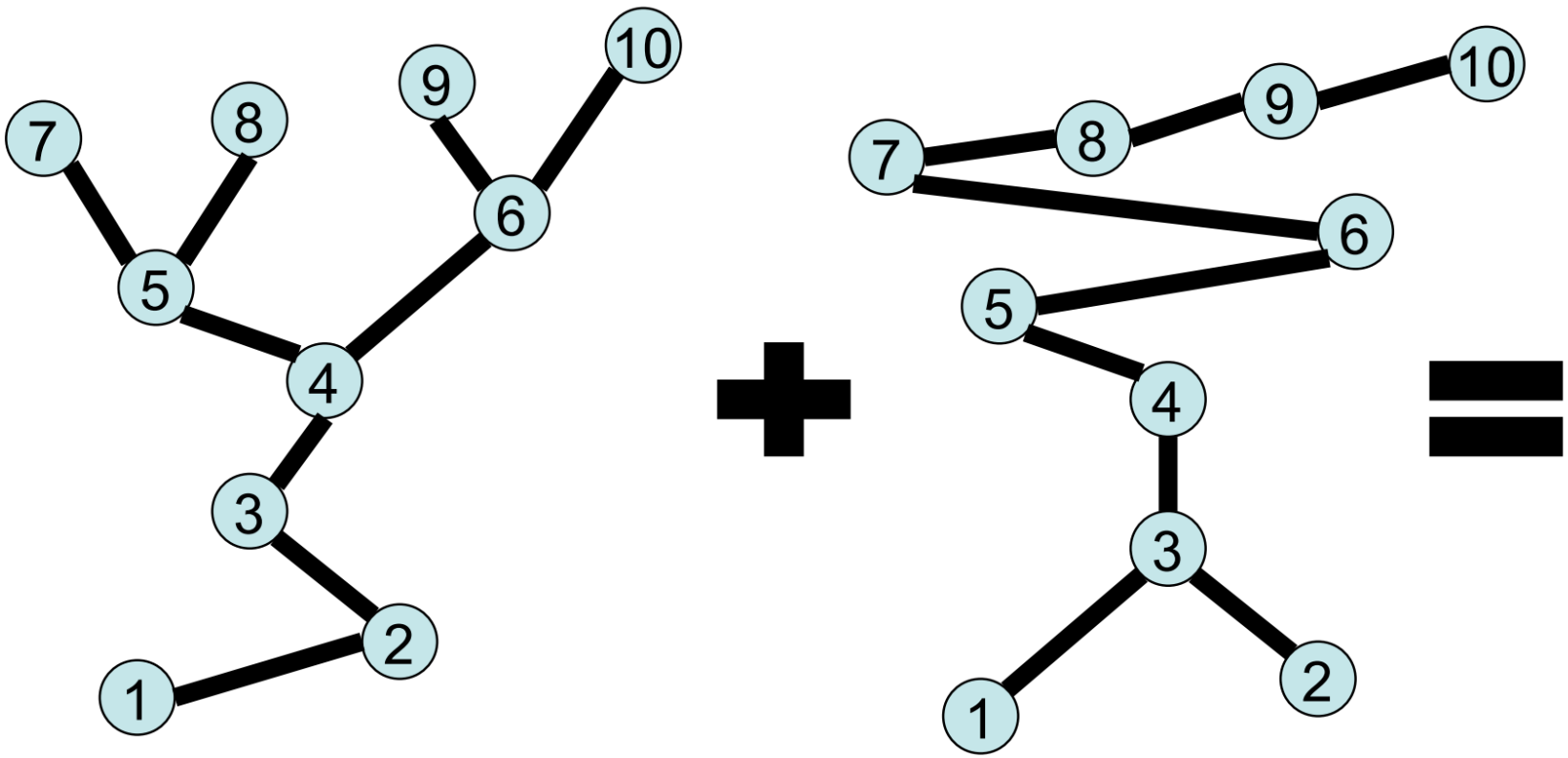

Figure: Algorithm[18]

[18] from Hamish Carr et al.

Merging trees

How to merge?

[19] from Hamish Carr et al.

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- Remember retrieving binary trees using two traversals?

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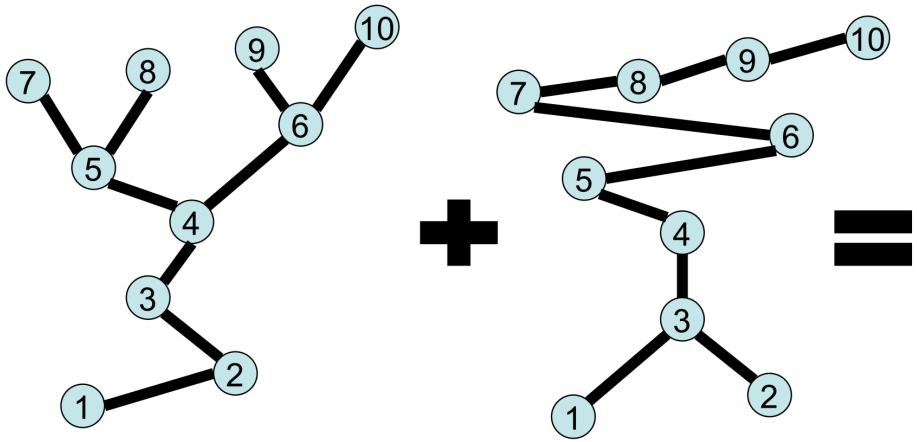

Figure: Observations for merging.[19]

[19] from Hamish Carr et al.

Merge Algorithm

- Initialize CT to be an empty graph on vertices of JT.
- Construct a queue of leaves.
  - if $updegree(v_i)$ in $ST + lowdegree(v_i)$ in $JT \leq 1$ enqueue.
- While $QueueSize > 1$
  - Dequeue first vertex v_i from the Queue.
  - If v_i is the upper leaf, find incident arc $(v_i, v_j) \in ST$
  - Else find incident arc $(v_i, v_j) \in JT$
  - Add (v_i, v_j) to CT
  - Remove v_i from both JT and ST.
  - If v_j is a leaf then Enqueue v_j.

Merge Algorithm

Running Example

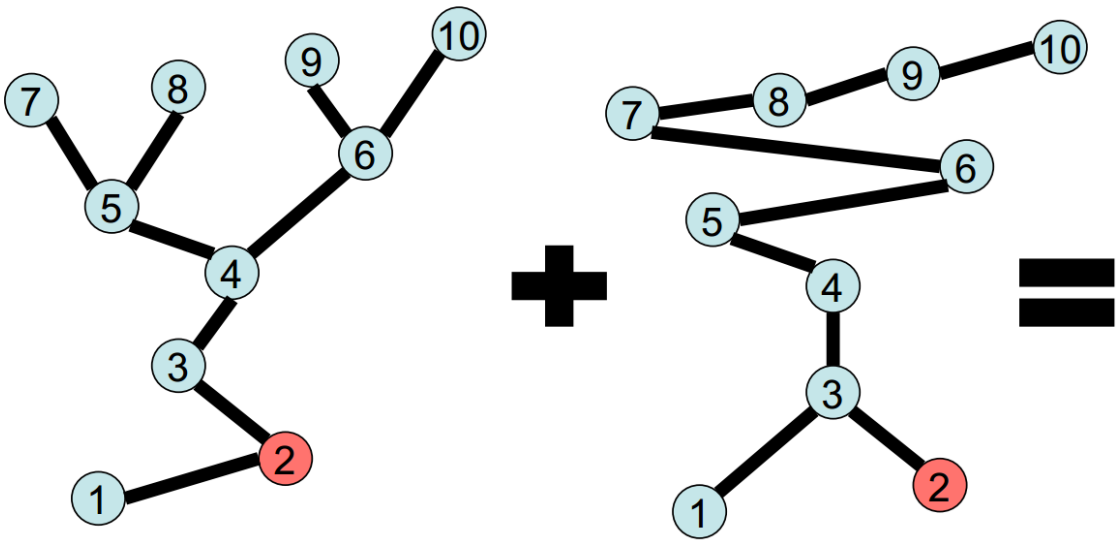

Figure: Working of Merge algorithm[20]

[20] from Hamish Carr et al.

Merge Algorithm

Running Example

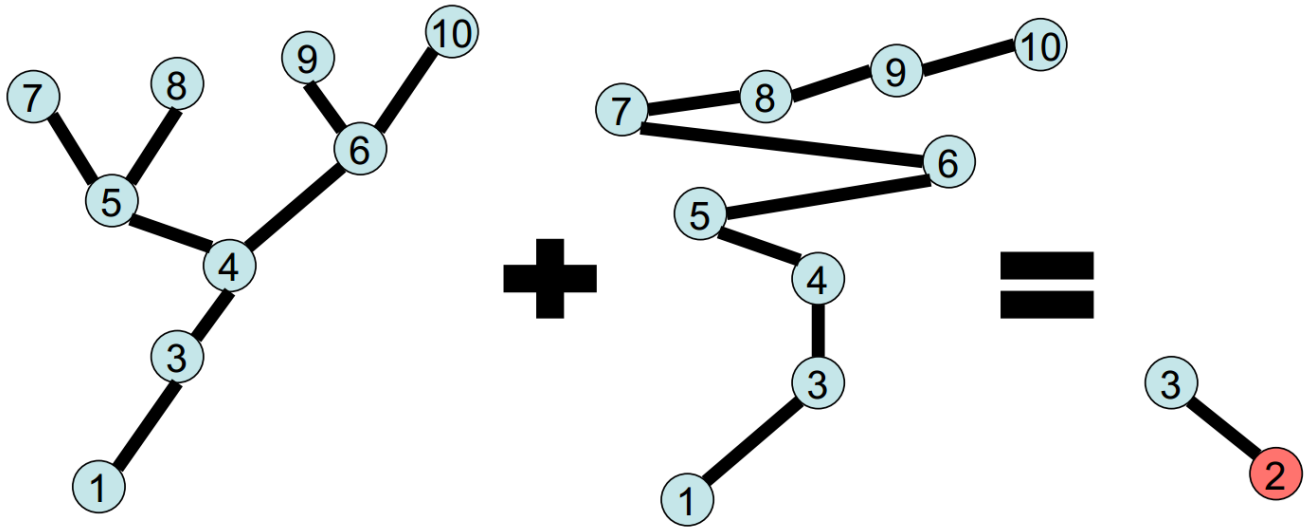

Figure: Working of Merge algorithm[21]

[21] from Hamish Carr et al.

Merge Algorithm

Running Example

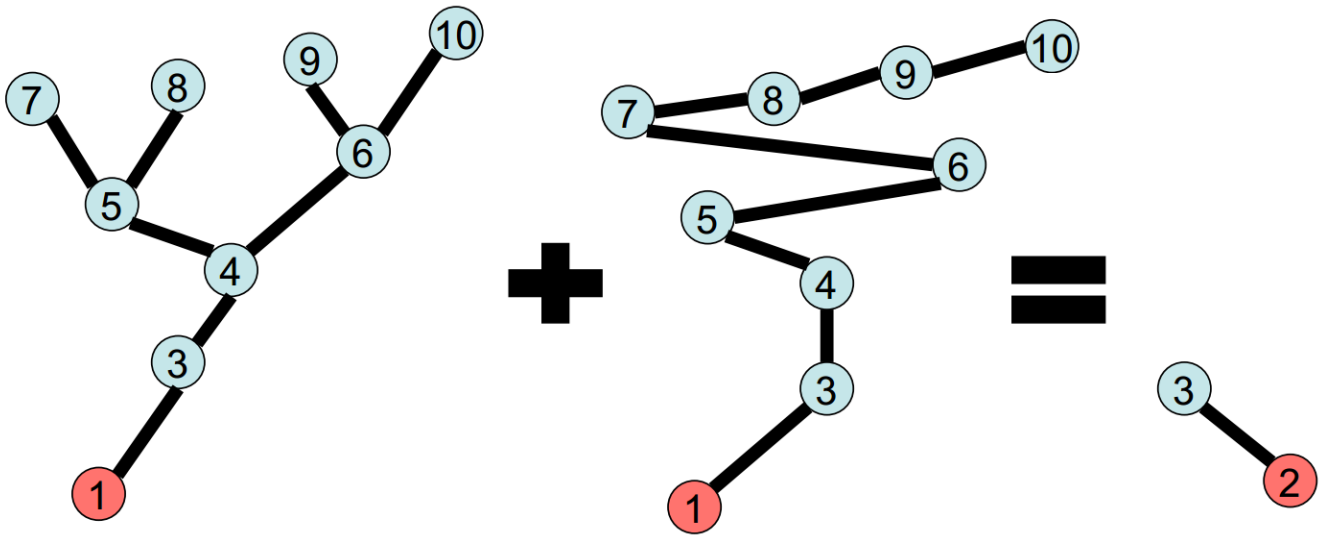

Figure: Working of Merge algorithm[22]

[22] from Hamish Carr et al.

Merge Algorithm

Running Example

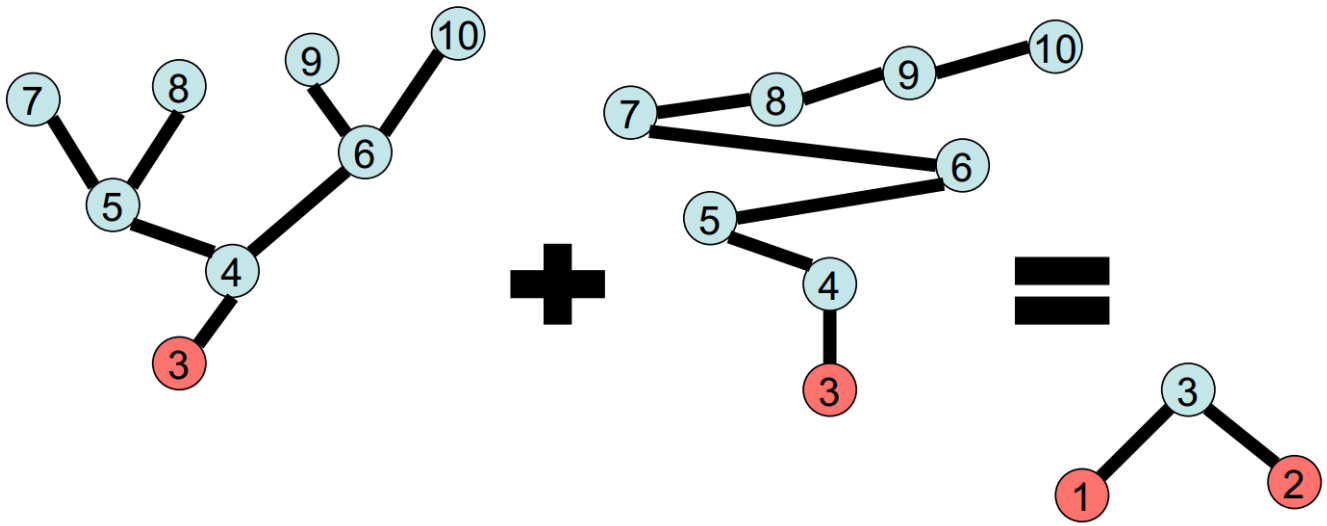

Figure: Working of Merge algorithm[23]

[23] from Hamish Carr et al.

Merge Algorithm

Running Example

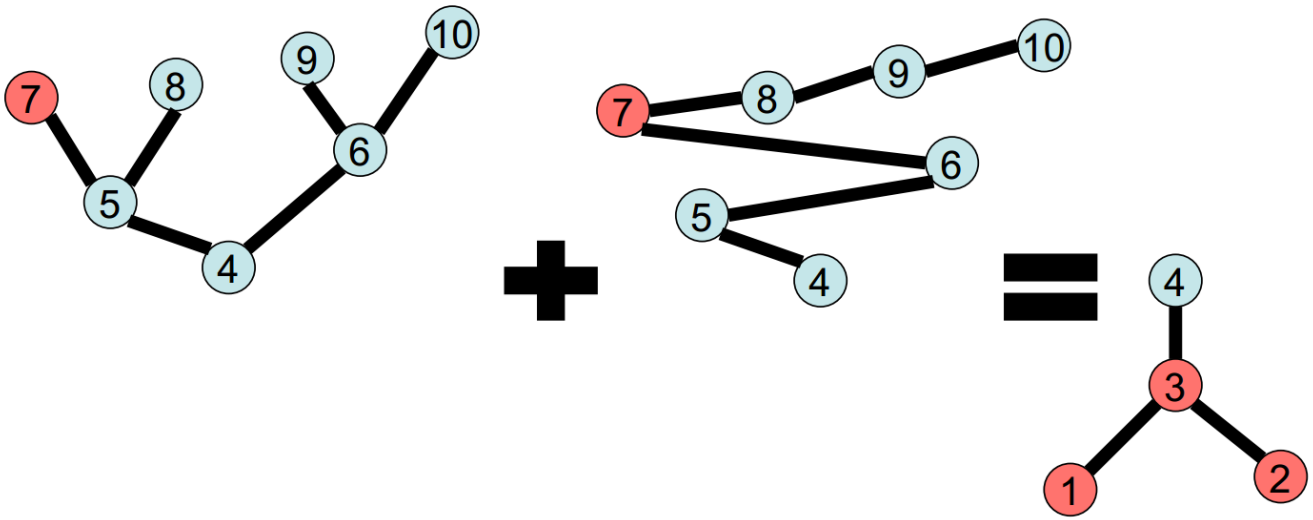

Figure: Working of Merge algorithm[24]

[24] from Hamish Carr et al.

Merge Algorithm

Running Example

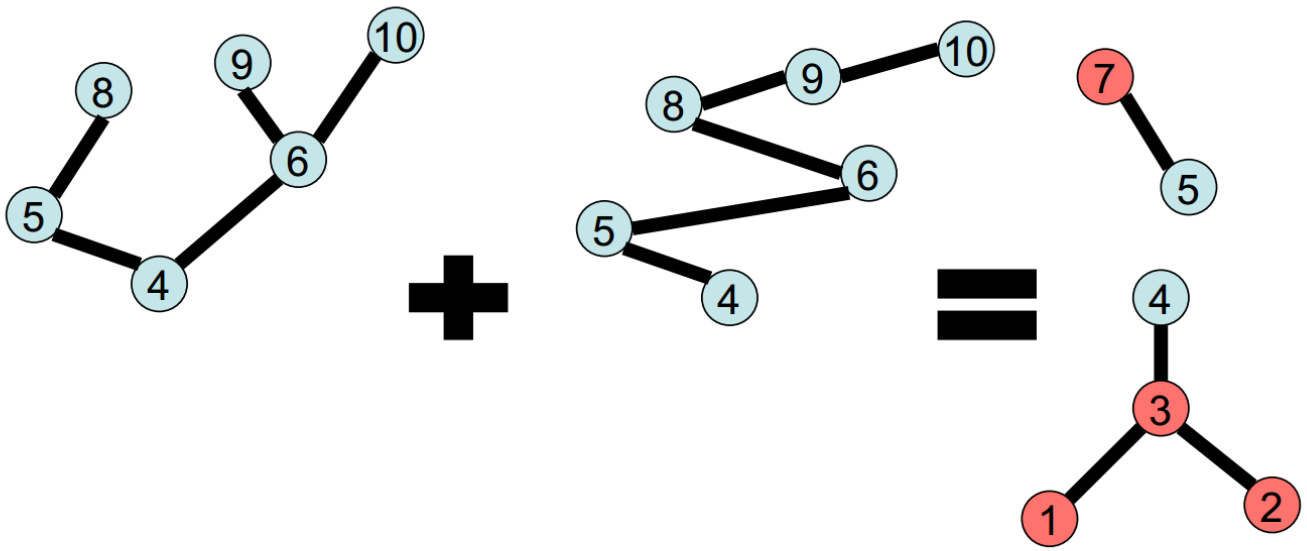

Figure: Working of Merge algorithm[25]

[25] from Hamish Carr et al.

Merge Algorithm

Running Example

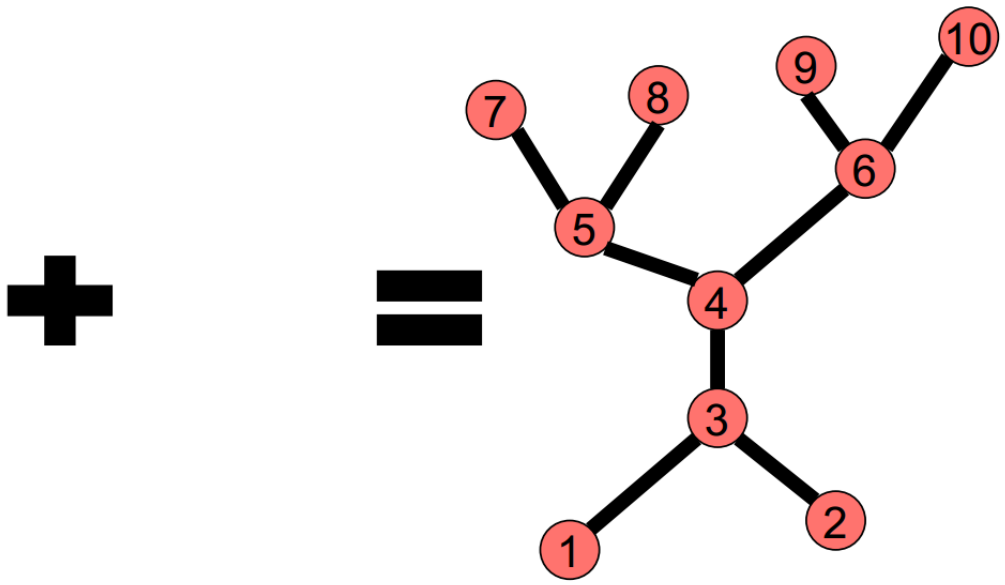

Figure: Working of Merge algorithm[26]

[26] from Hamish Carr et al.

Analysis

Runtime Analysis

Running time for data containing n vertices and N simplices/triangles, is given by $O(n \log n + N\alpha(N))$.

- $O(n \log n)$ for the initial sorting of the vertices based on function values.
- $O(N\alpha(N))$ is for the union find algorithm.

What about correctness?

- Skipped due to time constraint.

Application

One application

Flexible Isosurfaces

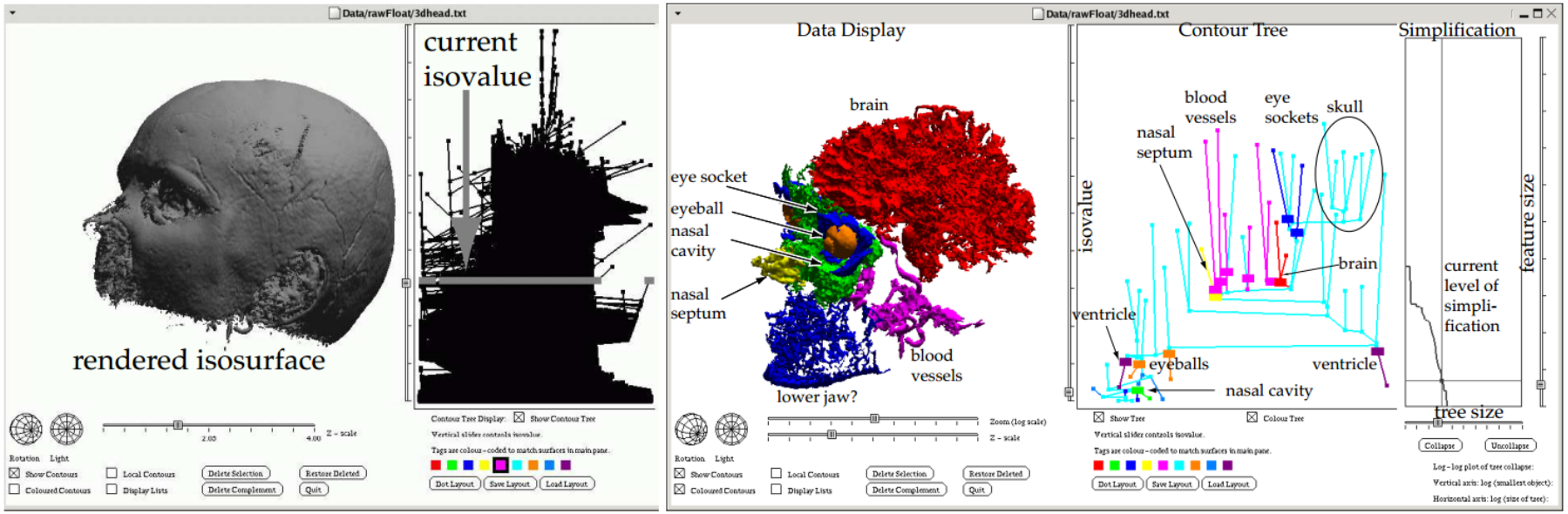

Figure: Flexible Isosurfaces[27]

[27] from Hamish Carr et al.

Conclusion

Conclusions

Conclusions

- We have a simple algorithm for constructing contour trees.
- Works for all dimensions.
- based on split/join trees followed by a merging algorithm.

Thank You